Symmetric Key Generation with using of Pythagorean Triple and Polygonal Numbers

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Abstract:

Key generation and Secure is critical to the security of a Cryptosystem. In fact, key generation and key exchange is the most challenging part of cryptography. In this paper, a scheme for symmetric key generation based on Polygonal numbers and Pythagorean triples has been presented. i.e, focus on generating symmetric keys with using of k-gonal numbers P (k, n) is $\frac{n}{2}[(k-3)(n-1)+(n+1)]$ *for* $k>2$, $n\geq 0$ *and* to Generate a Pythagorean Triple

(x_1, x_2, x_3) for each x_1,there exists at least one x_2 and at least one x_3 , with

$$x_2 = \left|ax_1^2 - \frac{1}{4a}\right| , x_3 = \left|ax_1^2 + \frac{1}{4a}\right| ,\text{where } a = \left\{\begin{matrix} \left\{\frac{1}{2p}, p \text{ is a factor of } x_1^2, if\ x_1 \text{ is an odd} \right. \\ \left\{\frac{1}{4p}, p \text{ is a factor of } \left(\frac{x_1}{2}\right)^2, if\ x_1 \text{ is an even}\right. \end{matrix}\right\}.$$

Also, introduce to define additive (+) and multiplicative (*) operations on Sets of k-gonal numbers as follows;

$$\boldsymbol{p(k,m)+p(k,n)=p(k,m+n)-(k-2)mn} \text{ forsome integers } \boldsymbol{k>2, n\geq 0}.$$

$$\boldsymbol{p(k,m)*p(k,n)=p(k,mn)+\frac{(k-4)(k-2)mn}{4}(m-1)(n-1)}$$

forsome integers $\boldsymbol{k>2, n\geq 0\ m\geq 0}$.

Keywords: Polygonal number, Triangular numbers, Residues, Non- Residues, k-gonal numbers.

Introduction:

Key generation and Secure is critical to the security of a Cryptosystem. In fact key generation and key exchange is the most challenging part of cryptography. In this paper, a scheme for symmetric key generation based on Polygonal numbers has been presented. The proposed scheme incorporates a Key Distribution center (KDC) for user authentication and secure exchange of secret information to generate keys. The KDC operation involves a request from a user for initiation. The KDC authenticates authenticate and secure exchange of secret information to generate keys. The KDC authenticates the initiator. If the authentication is successful, KDC generates and sends an encrypted timestamp to both the initiator and responder. The proposed system is based on a novel mechanism to determine Polygonal numbers to generate keys. The formula uses factors of x to generate y and z such that x , y , z satisfy the Pythagorean theorem.

Case1:The following notation has been used to Pythagorean triple calculation

x- input to calculate Pythagorean triple

p_1 - First prime factor of x, p_2 - Second Prime factor of x

y and z – Key Pair , Suppose, If x is odd then y = $\frac{|x^2-p_1^2|}{2p_1}$ and z= $\frac{|x^2+p_1^2|}{2p_1}$ the final key is computed by XORing y and z. i.e. p = y ⊕ z.

Case 2: The following notation has been used to k-gonal calculation

x- choose some non- negative integers k, m, n as input to calculate polygonal number

y and z – Key Pair with using of addition and multiplication binary operation on a Set of Polygonal numbers,

y = $\boldsymbol{p(k,m) + p(k,n) = p(k,m+n) - (k-2)mn}$

forsome integers $\boldsymbol{k > 2, n \geq 0 , m \geq 0}$ and

z = $\boldsymbol{p(k,m) * p(k,n) = p(k,mn) + \frac{(k-4)(k-2)mn}{4}(m-1)(n-1)}$

forsome integers $\boldsymbol{k > 2, n \geq 0 , m \geq 0}$

In the proposed system, three parties are involved in key exchange process. i.e

Key distribution center (KDC), source (A) and destination (B). If A wants to communicate with B using symmetric key encryption, a session must be created between them. A secret session key shared between and B is required for encryption of data in this session.

Main work:

In this paper, the total work is classified into Three parts:

Part-A: Introduce to define additive and multiplicative operations on Sets of Polygonal numbers.

Part-B: Introduce to generate Pythagorean Triple.

Part-A:

Introduce to study Additive and Multiplicative operations on Set of k-gonal numbers

Now, I can go to introduce to some inherent properties of Sets of Polygonal numbers. Now we can go to generate sets of numbers of square, pentagonal, Hexagonal, Heptagonal, and octagonal with using of Triangular number $p(3, n-1)$. Also, by applying recursive results of Triangular numbers, generated k-gonal numbers are $\frac{n}{2}[(k-3)(n-1)+(n+1)]$ for some integers $k > 2, n \geq 0$,.

We obtain following Polygonal numbers by replacing k as 3, 4 ,5 ,6 ,7 ,8.

For some positive integer n, the Triangular number is $\frac{n(n+1)}{2}(if\ k=3)$;

the square number is $n^2(if\ k=4)$;

the Pentagonal number is $\frac{n(3n-1)}{2}\ (if\ k=5)$;

the Hexagonal number is $n(2n-1)\ (if\ k=6)$;

the Heptagonal number is $\frac{n(5n-3)}{2}(if\ k=7)$; and

the octagonal number is $n(3n-2)(\ if\ k=8)$.

Also, The formation of triangular numbers is 1,1+2,1+2+3,…etc. The formation of square numbers is 1,1+3,1+3+5,……etc. It implies, successive addition of Arithmetic Progression the formation of Pentagonal numbers1,1+4,1+4+7,….etc. it follows that k-gonal numbers

have having common difference **k-2**.

In particular, concerning addition (+), Binary operation of addition on Set of Polygonal numbers as follows

$p(k,m)+p(k,n)=p(k,m+n)-(k-2)mn$ forsome integers $k>2, n \geq 0, m \geq 0$ **.**

Also, concerning multiplication (*), Binary operation of multiplication on Set of Polygonal numbers as follows

$$p(k,m) * p(k,n) = p(k,mn) + \frac{(k-4)(k-2)mn}{4}(m-1)(n-1) \text{ forsome integers}$$

$k>2, n \geq 0, m \geq 0$.

Also, verified these properties are well defined for following Sets of Polygonal numbers by replacing k values for 3,4,5,6,7 and 8.

From **Reference [1]**, Binary operations on a set of Triangular numbers are well defined

That is $\boldsymbol{T_m + T_n = T_{m+n} - mn, T_m * T_n = T_{mn} - \frac{mn}{4}(n-1)(m-1).}$ Now I can go to extend this property to all k-gonal numbers as follows:

Case 1: Now we can introduce an additive ('+') operation on a **set of Triangular numbers** as follows $\boldsymbol{p(3,m) + p(3,n) = p(3,m+n) - mn}$

Table 1: Verification of additive operation on a Set of Triangular numbers:

m	n	$p(3,m)$	$p(3,n)$	$p(3,m+n)$	mn	$p(3,m)+p(3,n)$	$p(3,m+n)-mn$
1	2	1	3	6	2	4	4
2	3	3	6	15	6	9	9
4	5	10	15	45	20	25	25

Now we can introduce a multiplicative ('*') operation on a Set of Triangular numbers as follows, for some positive integers a, b

$$\boldsymbol{p(3,m) * p(3,n) = p(3,mn) - \frac{mn}{4}(m-1)(n-1).}$$

Table 2: Verification of multiplicative operation on a Set of Triangular numbers:

m	n	$p(3,m)$	$p(3,n)$	$\frac{mn}{4}(m-1)(n-1)$	$p(3,mn)$	$p(3,mn)-\frac{mn}{4}(m-1)(n-1)$	$p(3,m)*p(3,n)$
2	3	3	6	3	21	21-3 =18	18
3	4	6	10	18	78	78-18 = 60	60
4	5	10	15	60	210	210-60 = 150	150
5	6	15	21	150	465	465-150 = 315	315

Now we can extend this methodology to remaining all other polygonal numbers.

Case 2: Now we can go to define two types of Operations on a Set of Square numbers as follows

$\boldsymbol{S_n = n^2, S_m = m^2}$ then addition (+) and multiplication (*) operation are defined as follows $\boldsymbol{S_n + S_m = S_{m+n} - 2mn}$ and $\boldsymbol{S_n * S_m = S_{mn}}$.

Proof: Consider $\boldsymbol{S_{m+n} - 2mn = (n+m)^2 - 2mn = n^2 + m^2 = S_m + S_n}$.

Hence $\boldsymbol{S_n + S_m = S_{m+n} - 2mn}$.

Table 3: Verification of additive operation on a Set of square numbers:

n	m	S_n	S_m	S_{n+m}	$S_{n+m} - 2mn$	$S_n + S_m$
1	2	1	4	9	5	5
2	3	4	9	25	13	13
4	5	16	25	81	41	41

Again consider $\boldsymbol{S_n} * \boldsymbol{S_m} = n^2.m^2 = (nm)^2 = \boldsymbol{S_{nm}}$.Hence $\boldsymbol{S_n * S_m = S_{nm}}$.

Table 4: Verification of multiplicative binary operation on a Set of square numbers

n	m	S_n	S_m	S_{nm}	$S_n * S_m$
1	2	1	4	4	4
2	3	4	9	36	36
4	5	16	25	400	400

Under these operations, we can verify easily algebraic structure of $(\boldsymbol{S_n},*)$ is becomes as a Monoid, since this Nonempty Set is satisfying Closure axiom, Associate and Existence of Identity element is 1 concerning multiplication. Also, '*' is a binary operation on $\boldsymbol{S_n}$.

i.e Closure Axiom:$\boldsymbol{S_a * S_b = S_{ab} \in S_n}$,for all $\boldsymbol{S_a \in S_n, S_b \in S_n}$.

Associative axiom**:** $\boldsymbol{(S_a * S_b) * S_c = S_a * (S_b * S_c) = S_{abc}}$

Existence of identity**:** identity element $\boldsymbol{S_1 = 1 \in S_n}$**,such that** $\boldsymbol{S_n * S_1 = S_1 * S_n = S_n}$.

Hence $(\boldsymbol{S_n},*)$ is becomes a Monoid.

Case 3: Now we can go to define two types of Operations on a Set of Pentagonal numbers as follows

$P_n = p(5,n) = \frac{n(3n-1)}{2}$, $P_m = p(5,m) = \frac{m(3m-1)}{2}$ then additive (+), multiplicative (*) operations are defined as follows $P_n + P_m = P_{n+m} - 3mn$, $P_n * P_m = P_{nm} + \frac{3mn}{4}(n-1)(m-1)$

Proof: Consider $P_{n+m} - 3mn = \frac{(n+m)(3(n+m)-1)}{2} - 3mn = \frac{3(n+m)^2-(n+m)-6mn}{2}$

$= \frac{n(3n-1)}{2} + \frac{m(3m-1)}{2} = P_n + P_m$. Hence $P_n + P_m = P_{n+m} - 3mn$

Table 5: Verification of additive operation on a Set of Pentagonal numbers:

n	m	P_n	P_m	P_{n+m}	$P_{n+m} - 3mn$	$P_n + P_m = P_{n+m} - 3mn$
1	2	1	5	12	6	6
2	3	5	12	35	17	17
4	5	22	35	117	57	57

Again consider, $P_n * P_m - P_{nm} = \frac{n(3n-1)}{2}\frac{m(3m-1)}{2} - \frac{nm(3nm-1)}{2}$

$$= \frac{nm}{2}\left[\frac{9nm-3n-3m+1}{2} - \frac{3nm-1}{1}\right] = \frac{3mn}{4}\ (n-1)(m-1)$$

Hence $P_n * P_m = P_{nm} + \frac{3mn}{4}\ (n-1)(m-1)$.

Table 6: Verification of multiplicative operation on a Set of pentagonal numbers:

n	m	P_n	P_m	$P_{nm} + \frac{3mn}{4}\ (n-1)\ (m-1)$	$P_n * P_m$
1	2	1	5	5	5
2	3	5	12	60	60
4	5	22	35	770	770

It follows that the above binary operations are well-defined.

Case 4: Now we can go to define two types of Operation on a Set of Hexagonal numbers as follows

$Hx_n = n(2n-1)$, $Hx_m = m(2m-1)$ then additive (+), multiplicative (*) operations are defined as follows

$Hx_n + Hx_m = Hx_{n+m} - 4mn$ and $Hx_n * Hx_m = Hx_{nm} + 2mn(m-1)(n-1)$.

Proof: Consider $Hx_n + Hx_m - Hx_{n+m}$

$= n(2n-1) + m(2m-1) - (n+m)(2(n+m)-1) = -4mn$

Hence $Hx_n + Hx_m = Hx_{n+m} - 4mn$.

Table 7: Verification of additive binary operation on a Set of Hexagonal numbers:

n	m	Hx_n	Hx_m	Hx_{n+m}	$Hx_{n+m} - 4mn$	$Hx_n + Hx_m$.
1	2	1	6	15	7	7
2	3	6	15	45	21	21
4	5	28	45	153	73	73

Again consider $Hx_n * Hx_m - Hx_{nm}$

$= n(2n-1)\,m(2m-1) - mn(2mn-1)$

$= mn\,[(2n-1)\,(2m-1) - (2mn-1)] \; = 2mn(m-1)(n-1)$

Hence $Hx_n * Hx_m = Hx_{nm} + 2mn(m-1)(n-1)$

Table 8: Verification of multiplicative operation on a Set of Hexagonal numbers:

n	m	Hx_n	Hx_m	$Hx_{nm} + 2mn(m-1)$ $(n-1)$	$Hx_n * Hx_m$
1	2	1	6	6	6
2	3	6	15	90	90
4	5	28	45	1260	1260

Case 5: Now we can go to define two types of Operations on a Set of Heptagonal numbers as follows

$Hp_n = \frac{n(5n-3)}{2}, Hp_m = \frac{m(5m-3)}{2}$ then additive (+), multiplicative (*) operations are defined as follows

$Hp_n + Hp_m = Hp_{n+m} - 5mn$ and $Hp_n * Hp_m = Hp_{nm} + \frac{15mn}{4}(n-1)(m-1)$.

Proof: Consider $Hp_n + Hp_m - Hp_{n+m} = \frac{n(5n-3)}{2} + \frac{m(5m-3)}{2} - \frac{(n+m)(5(n+m)-3)}{2}$ = -5mn

Hence $Hp_n + Hp_m = Hp_{n+m} - 5mn$

Table 9: Verification of additive operation on a Set of Heptagonal numbers:

n	m	Hp_n	Hp_m	Hp_{n+m}	$Hp_{n+m} - 5mn$	$Hp_n + Hp_m$.
1	2	1	7	18	8	8
2	3	7	18	55	25	25
4	5	34	55	189	89	89

Again consider

$$Hp_n * Hp_m - Hp_{nm} = \frac{n(5n-3)}{2}\,\frac{m(5m-3)}{2} - \frac{mn(5mn-3)}{2}$$
$$= \frac{nm}{2}\left[\frac{25nm-15n-15m+9}{2} - \frac{5nm-3}{1}\right]$$

$= \frac{15mn}{4}\,(n-1)(m-1)$Hence $Hp_n * Hp_m = Hp_{nm} + \frac{15mn}{4}(n-1)(m-1)$

Table 10: Verification of multiplicative operation on a Set of Heptagonal numbers

n	m	Hp_n	Hp_m	Hp_{nm}	$Hp_{nm} + \frac{15mn}{4}(n-1)\ (m-1)$	$Hp_n * Hp_m$
1	2	1	7	7	7	7
2	3	7	18	81	126	126
4	5	34	55	970	1870	1870

Case 6: Now we can go to define two types of Operations on a Set of Octagonal numbers as follows $O_n = n(3n-2)$, $O_m = m(3m-2)$ then additive (+), multiplicative (*) operations are defined as follows

$O_n + O_m = O_{n+m} - 6mn$ and $O_n * O_m = O_{nm} + 6mn(m-1)(n-1)$

Proof: Consider $O_n + O_m - O_{n+m} = n(3n-2) + m(3m-2) - (n+m)(3(n+m) - 2) = -6mn.$

Hence $O_n + O_m = O_{n+m} - 6mn.$

Table 11: Verification of additive operation on a Set of Octagonal numbers :

n	m	O_n	O_m	O_{n+m}	$O_{n+m} - 6mn$	$O_n + O_m = O_{n+m} - 6mn$
1	2	1	8	21	9	9
2	3	8	21	65	29	29
4	5	40	65	225	105	105

Again consider $O_n * O_m - O_{nm} = n(3n-2)m(3m-2) - nm(3nm-2)$

$= mn\ [(3n-2)\ (3m-2) - (3mn-2)] = 6mn(m-1)(n-1)$

Hence $O_n * O_m = O_{nm} + 6mn(m-1)(n-1)$.

Table 12: Verification of multiplicative operation on a Set of Octagonal numbers:

n	m	O_n	O_m	O_{nm}	$O_{nm} + 6mn(m-1)$ $(n-1)$	$O_n * O_m$ $= O_{nm} + 6mn(m-1)(n-1)$
1	2	1	8	8	8	8
2	3	8	21	96	168	168
4	5	40	65	1160	2600	2600

Hence, by observation,

$\boldsymbol{p(k,m) + p(k,n) = p(k,m+n) - (k-2)mn}$ forsome integer $\boldsymbol{k > 2.}$

Also, concerning multiplication (*), $\boldsymbol{p(k,m) * p(k,n) = p(k,mn) + \frac{(k-4)(k-2)mn}{4}(m-1)(n-1)}$ forsome integer $\boldsymbol{k > 2.}$

Case 7: summation of any two different k-gonal is $\boldsymbol{p(k_1,n) + p(k_2,n) = n(n+1) + \frac{n(n-1)}{2}[(k_1+k_2)-6].}$

Proof: we know that $\boldsymbol{p(k,n) = \frac{n}{2}[(k-3)(n-1)+(n+1)]\ for\ k > 2.}$

Consider

$$\boldsymbol{p(k_1,n) + p(k_2,n) = \frac{n}{2}[(k_1-3)(n-1)+(n+1)] + \frac{n}{2}[(k_2-3)(n-1)+(n+1)]}$$

$$\boldsymbol{= n(n+1) + \frac{n(n-1)}{2}[(k_1+k_2)-6] = 2T_n + [(k_1+k_2)-6]T_{n-1}.}$$

E. g: Verify above result by taking some values of n, k_1, k_2.

Let $k_1 = 4, k_2 = 5\ and\ n = 4$. Hence $T_4 = 10, T_3 = 6, S_4 = 16\ and\ P_4 = 22$ implice $S_4 + P_4 = 38$.

Also, $n(n+1) + \frac{n(n-1)}{2}[(k_1+k_2)-6] = 20 + 18 = 38.$

Also,$2T_n + [(k_1+k_2)-6]T_{n-1} = 20 + 18 = 38$

Part-B: Introduce to generate Pythagorean Triple.

Case1: to Generate a Pythagorean Triple (x_1, x_2, x_3) for each x_1,there exists at least one x_2 and at least one x_3 , with $\mathrm{x}_2 = \left|\mathrm{a}x_1^2 - \frac{1}{4\mathrm{a}}\right|$, $x_3 = \left|ax_1^2 + \frac{1}{4a}\right|$,

where $\mathrm{a} = \begin{Bmatrix} \left\{\frac{1}{2p}, p \text{ is a factor of } x_1^2, if\ x_1 \text{ is an odd} \right. \\ \left\{\frac{1}{4p}, p \text{ is a factor of } \left(\frac{x_1}{2}\right)^2, if\ x_1 \text{ is an even}\right. \end{Bmatrix}$

Case1.1: **If $\boldsymbol{x_1}$ is an odd**

Consider $x_2 = \left|ax_1^2 - \frac{1}{4a}\right|$ since p is a factor of x_1^2.

Therefore $x_1^2 = np$ where n is an integer

$x_2 = \left|\frac{np}{2p} - \frac{1}{4\left(\frac{1}{2p}\right)}\right| = \left|\frac{n-p}{2}\right|$, both n and p odd numbers $\frac{\mathrm{n}-p}{2}$ become an integer.

Similarly, $x_3 = \left|ax_1^2 + \frac{1}{4a}\right|$ since p is a factor of x_1^2. Therefore

$x_1^2 = np$ where n is an integer.

$x_3 = \left|\frac{np}{2p} + \frac{1}{4\left(\frac{1}{2p}\right)}\right| = \left|\frac{n+p}{2}\right|$, both n and p odd numbers $\frac{n+p}{2}$ become an integer.

Hence (x_1, x_2, x_3) becomes a Pythagorean Triple.

Table 13: Some results are represented below:

x_1	Choose p (Factor of x_1^2)	$n = \frac{x_1^2}{p}$	$x_2 = \left\|ax_1^2 - \frac{1}{4a}\right\| = \frac{n-p}{2}$	$x_3 = \left\|ax_1^2 + \frac{1}{4a}\right\| = \frac{n+p}{2}$	(x_1, x_2, x_3)
3	1	9	4	5	(3,4,5)
15	1	225	112	113	(15,112,113)
15	3	75	36	39	(15,36,39)

Case 1.2: **If x_1 is an even**.

Consider $x_2 = \left|ax_1^2 - \frac{1}{4a}\right|$ since p is a factor of $\left(\frac{x_1}{2}\right)^2$.

There fore $x_1^2 = 4np$ where n is an integer.$x_2 = \left|\frac{4np}{4p} - \frac{1}{4\left(\frac{1}{4p}\right)}\right| = |n - p|$, both *n* and *p* are integers, hence x_2 is also an integer. Similarly, $x_3 = \left|ax_1^2 + \frac{1}{4a}\right|$ since *p* is a factor of$\left(\frac{x_1}{2}\right)^2$.

Therefore $x_1^2 = 4np$ where n is an integer

$x_3 = \left|\frac{np}{4p} + \frac{1}{4\left(\frac{1}{4p}\right)}\right| = |n + p|$, both *n* and *p* are integers, hence x_3 is also an integer.

Hence (x_1, x_2, x_3) becomes a Pythagorean Triple.

Table 14: Some results are represented below:

x_1	Choose *p* (Factor of $\left(\frac{x_1}{2}\right)^2$)	$n = \frac{x_1^2}{4p}$	$x_2 = \left\|ax_1^2 - \frac{1}{4a}\right\| = n - p$	$x_3 = \left\|ax_1^2 + \frac{1}{4a}\right\| = n + p$	(x_1, x_2, x_3)
4	1	4	3	5	(4,3,5)
6	3	3	0	6	(6,0,6)
6	1	9	8	10	(6,8,10)
8	1	16	15	17	(8,15,17)
8	2	8	6	10	(8,6,10)

Theorem 1: Generalisation of k-gonal numbers with using of triangular numbers is $\boldsymbol{\frac{n}{2}[(k-3)(n-1)+(n+1)]}$ **for k > 2.** Successive Replacement of k values 3,4,5,….etc ,we obtain Triangular numbers, Square Numbers, Pentagonal numbers… etc.

Proof: We know that n[th] term of Triangular number $\boldsymbol{p(3,n)=\frac{n(n+1)}{2}}$,

Also, generate all other Polygonal numbers with using of Triangular numbers as follows:

Square number $S_n = T_{n-1} + T_n$, Pentagonal number $P_n = T_{n-1} + S_n = 2T_{n-1} + T_n$

Hexagonal numbers $Hx_n = T_{n-1} + P_n = 3T_{n-1} + T_n$,

Heptagonal numbers $Hp_n = T_{n-1} + Hx_n = 4T_{n-1} + T_n$

Octagonal numbers $O_n = T_{n-1} + Hp_n = 5T_{n-1} + T_n$.

Hence, generalised k-gonal numbers $k_n = (k-3)T_{n-1} + T_n = \frac{n}{2}[(k-3)(n-1) + (n+1)]$ (if k>2).

Conclusion: In this paper proposed to generate some set of Polygonal numbers is $\frac{n}{2}[(k-3)(n-1)+(n+1)]\,for\ k>2, n \geq 0$. Also, another form to generate k-gonal numbers is $\frac{(k-2)n^2-(k-4)n}{2}$ for k >2, $n \geq 0$.Also introduced Binary operations under addition and multiplication on a Set of Polygonal numbers as follows :

On a set of Triangular numbers $T_m + T_n = T_{m+n} - mn$,

$$T_m * T_n = T_{mn} - \frac{mn}{4}(n-1)(m-1).$$

On a Set of Square numbers $S_n + S_m = S_{n+m} - 2\sqrt{S_n S_m}$, $S_n * S_m = S_{nm}$.

On a Set of Pentagonal numbers $P_n + P_m = P_{n+m} - 3mn$,

$P_n * P_m = P_{nm} + \frac{3mn}{4}(n-1)(m-1)$.

On a Set of Hexagonal numbers

$Hx_n + Hx_m = Hx_{n+m} - 4mn$, $Hx_n * Hx_m = Hx_{nm} + 2mn(m-1)(n-1)$.

On a Set of Heptagonal numbers $Hp_n + Hp_m = Hp_{n+m} - 5mn$,

$Hp_n * Hp_m = Hp_{nm} + \frac{15mn}{4}(n-1)(m-1)$.

On a Set of Octagonal numbers $O_n + O_m = O_{n+m} - 6mn$ and

$O_n * O_m = O_{nm} + 6mn(m-1)(n-1)$.

particularly concerning addition, Binary operation of k-gonal numbers is

$k_m + k_n = k_{m+n} - (k-2)mn$.

Also, concerning multiplication, Binary operation of k-gonal numbers is

$k_n * k_m = k_{nm} + \frac{(k-4)(k-2)mn}{4}(n-1)(m-1)$. And to generate a Pythagorean Triple (x_1, x_2, x_3) for each x_1,there exists at least one x_2 and at least one x_3 , with $\mathrm{x}_2 = \left|\mathrm{ax}_1^2 - \frac{1}{4\mathrm{a}}\right|$,

$x_3 = \left|ax_1^2 + \frac{1}{4a}\right|$,where $\mathrm{a} = \begin{Bmatrix} \left\{\frac{1}{2p}, p \text{ is a factor of } x_1^2, if\ x_1 \text{ is an odd}\right. \\ \left\{\frac{1}{4p}, p \text{ is a factor of } \left(\frac{x_1}{2}\right)^2, if\ x_1 \text{ is an even}\right. \end{Bmatrix}$.

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