Inventory System of Perishable Products under Trade Credit and Time Discounting with Time Depending Demand Function

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Abstract

We know that most of the products such as food products, flowers, vegetables, medicines, chemicals, drugs, dairy products etc. are instantaneously perishable products. So producer offers credit period to his customers to increase the demand of these products. Sometimes it may be risky for the producer. In this article, an inventory system has been developed for perishable products having demand rate as weibull function of time and constant deterioration under the assumptions of trade credit and time discounting. Here shortages are not allowed. The aim of this study is to minimize the total variable cost and to maximize the profit of the retailer.

Keywords: Inventory system, Weibull Demand, Trade Credit, Time Discounting, Deterioration, Minimizing Total Cost

Subject Classification: 90B05

1. Introduction

Generally it is assumed that the retailer must be paid for the items at the time of delivery. But in real life, the supplier may offer delay period to the retailer, which is called the trade credit period for the payment of purchasing cost to simulate his products. During this period, retailer can sell the products and can earn the interest. It is beneficial for the retailer to delay the settlement of the replenishment account up to the last moment of the credit period allowed by the supplier.

Chapman et al. [1] obtained the optimal replenishment policies under different considerations such as paying during a credit period, and paying after a fixed period. He derived an EOQ model which was more sensitive to the demand and less sensitive to the ordering cost than the traditional EOQ models. Chang and Dye [2] presented an inventory model for deteriorating items with time varying demand and deterioration rate when the credit period depends on the retailer’s ordering quantity. Goyal and Giri [3] extended lot-size model with the constant rate of deterioration when supplier offers trade credit for settling the account for the purchase quantity. Ouyang et al. [4] and Huang and Chung [5] extended Goyal’s model [6] to the case of two part trade credit. Chung and Huang [7] discussed the payment rule. They extended Goyal’s model [6] with the assumption that the retailer must pay a partial amount of total purchasing cost at the end of the credit period and the remaining amount would be paid by loan from the bank. Chang and Chen [8] derived an EOQ model with deteriorating items in a periodic review environment, with shortages, under inflation when supplier credits linked to order quantity. H.F. Huang [9] developed on EPQ model under trade credit by considering higher selling price than purchasing cost. Hari Kishan, Megha Rani and Shiv Raj Singh [10] discussed an inventory model for deteriorating products under the supplier's partial trade credit policy. They considered the time varying demand rate and constant deterioration rate and also discussed some different cases depending upon trade credit policy. Hari Kishan, Megha Rani and Vipin Kumar [11] derived and inventory model of deteriorating products with non uniform demand rules under life time and trade credit. Pratima Patil and P. N. Mishra [12] proposed an inventory model of perishable products with periodic demand by considering both the cases with and without shortages. Mihir S. Suthar et al. [13] derived mathematical formula to minimize the total cost of an inventory system by considering constant deterioration rate and demand dependent production. Mihir S. Suthar and Kunal Shukla [14] have formulate an ordering policy in general for ramp-type demand for retailers by assuming non-instantaneous deterioration rate. The main objective of this study was to maximize the total profit of the retailer over the period of time and increase ordering quantity of the inventory system.

In this article, an inventory system of perishable products with life time has been considered. Demand rate has been taken as weibull function of time that is $\alpha\beta t^{\beta-1}$. Deterioration rate has been taken as constant. Here model has been developed under trade credit and time discounting. Model considered only without shortages.

For modelling the inventory system, we will use the following assumptions and notations.

2. Assumptions

The following assumptions are considered in this chapter:

(1) The demand rate is weibull function of time and is given by $\alpha\beta t^{\beta-1}$.
(2) The deterioration rate θ is constant.
(3) Time horizon H is finite and distributed into n equal parts.
(4) Shortages are not allowed.
(5) During the time, the account is not settled. The generated sales revenue is deposited in an interest bearing account.

When $T \geq M$, the account is settled at $T=M$ and we start paying for the interest charges on items in stock.

When $T \leq M$, the account is settled at $T=M$ and we need not to pay any interest charge.

3. Notations

The following notations have been used in this chapter:

(1) θ = the deterioration rate of on hand inventory, where $0 \leq \theta < 1$
(2) $q(t)$ = the inventory level at time $t, 0 \leq t \leq T$
(3) C_r = total replenishment cost
(4) C_0 = the ordering cost per order
(5) C_h = total holding cost
(6) C_p = total purchasing cost per unit
(7) h = unit holding cost per unit time
(8) p = the selling price per unit

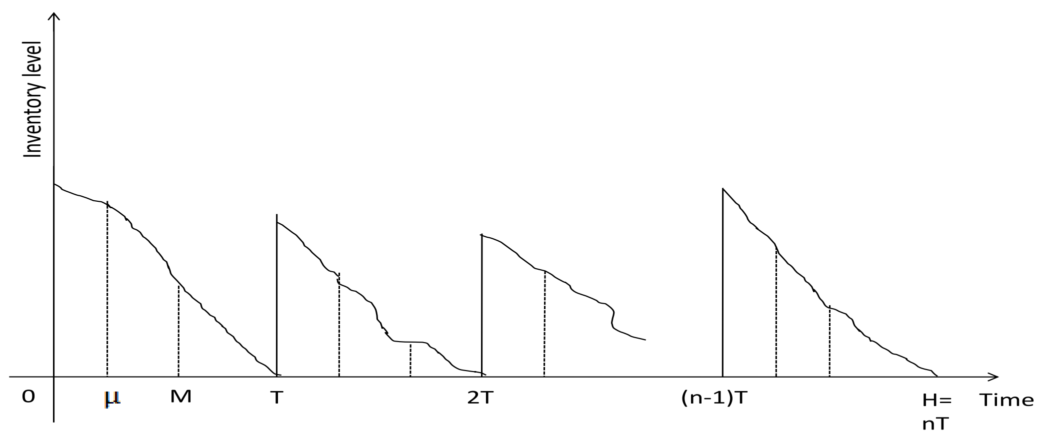
(9) C = the unit purchasing price
(10) I_r = the interest charged per Re per year
(11) I_e = the interest earned per Re per year
(12) Q = the maximum inventory level
(13) T = the replenishment cycle time
(14) M = the trade credit period
(15) H = length of planning horizon
(16) n = number of replenishment during the planning horizon
(17) μ = the life time
(18) TVC = the annual total relevant cost
(19) T^* = the optimal cycle time of TVC
(20) Q^* = the optimal order quantity

4. Model formulation

Let the demand rate is weibull function of time and is given by $\alpha\beta t^{\beta-1}$ and deterioration rate θ is constant. The total time horizon H has been divided in n equal parts of length T. So that $T = \frac{H}{n}$. Therefore the reorder times over the planning horizon H are given by

$$T_j = jT, (j = 0,1,2, \dots \dots \dots n - 1).$$

This model is shown as in figure given below.



Let $q(t)$ be the inventory level during the first replenishment cycle. This inventory level is depleted to zero due to demand and deterioration during the replenishment cycle.

Hence, the variation of inventory level with respect to time during the period $0 \leq t \leq T$ can be represented by the following differential equation.

$$\frac{dq}{dt} = -\alpha\beta t^{\beta-1},\ 0 \le t \le \mu \tag{1}$$

$$\frac{dq}{dt} = -\alpha\beta t^{\beta-1} - \theta q, \qquad \mu \le t \le T \tag{2}$$

with boundary conditions,

$$q(0) = Q \tag{3}$$

$$q(T) = 0 \tag{4}$$

5. Analysis

Solving equation (1) and using boundary condition (3), the solution of (1) may be represented by,

$$q(t) = Q - \alpha t^{\beta} \tag{5}$$

Solving equation (2) with boundary condition (4), we get

$$q(t) = -\alpha\beta\left[\frac{t^{\beta}}{\beta} + \frac{\theta t^{\beta+1}}{\beta+1}\right]e^{-\theta t} + \alpha\beta\left[\frac{T^{\beta}}{\beta} + \frac{\theta T^{\beta+1}}{\beta+1}\right]e^{-\theta t} \tag{6}$$

Now, from (5) and (6) we have

$$Q = \alpha\mu^{\beta} - \alpha\beta\left[\frac{\mu^{\beta}}{\beta} + \frac{\theta\mu^{\beta+1}}{\beta+1}\right]e^{-\theta\mu} + \alpha\beta\left[\frac{T^{\beta}}{\beta} + \frac{\theta T^{\beta+1}}{\beta+1}\right]e^{-\theta\mu} \tag{7}$$

Since, there are n equal parts in the total time horizon H. Therefore the present value of the total replenishment cost is given by,

$$C_r = C_0 \sum_{j=0}^{n-1} e^{-jRT}$$

$$\therefore C_r = C_0\left(\frac{1-e^{-RH}}{1-e^{-\frac{RH}{n}}}\right) \tag{8}$$

The present value of the total purchasing cost is given by,

$$C_p = C\sum_{j=0}^{n-1} q(0)\, e^{-jRT}$$

$$= C\sum_{j=0}^{n-1} Q\ e^{-jRT}$$

$$\therefore C_p = C\left[\alpha\mu^{\beta} - \alpha\beta\left[\frac{\mu^{\beta}}{\beta} + \frac{\theta\mu^{\beta+1}}{\beta+1}\right]e^{-\theta\mu} + \alpha\beta\left[\frac{T^{\beta}}{\beta} + \frac{\theta T^{\beta+1}}{\beta+1}\right]e^{-\theta\mu}\right]\left(\frac{1-e^{-RH}}{1-e^{-\frac{RH}{n}}}\right) \tag{9}$$

The present value of the holding cost during the first replenishment cycle is given by,

$$h_1 = h\int_0^T q(t)e^{-RT}dt$$

$$\Longrightarrow h_1 = h\left[\int_0^{\mu} q(t)e^{-RT}dt + \int_{\mu}^{T} q(t)e^{-RT}dt\right]$$

$$\Longrightarrow h_1 = h\left[\int_0^{\mu}(Q-\alpha t^{\beta})e^{-RT}dt + \int_{\mu}^{T}\left(-\alpha\beta\left[\frac{t^{\beta}}{\beta}+\frac{\theta t^{\beta+1}}{\beta+1}\right]e^{-\theta t}+\alpha\beta\left[\frac{T^{\beta}}{\beta}+\frac{\theta T^{\beta+1}}{\beta+1}\right]e^{-\theta t}\right)e^{-RT}dt\right]$$

$$\therefore h_1 = \left\{\frac{Q}{R}(1-e^{-R\mu}) - \alpha\left[\frac{T^{\beta+1}}{\beta+1}+\frac{\theta(T^{\beta+2}-\mu^{\beta+2})}{(\beta+1)(\beta+2)}-\frac{\beta\theta(\theta+R)(T^{\beta+3}-\mu^{\beta+3})}{(\beta+1)(\beta+3)}-\frac{RT^{\beta+2}}{\beta+2}\right]+\alpha\left(T^{\beta}-\frac{\beta\theta T^{\beta+1}}{\beta+1}\right)\left((T-\mu)-\frac{(\theta+R)(T^2-\mu^2)}{R}\right)\right\} \tag{10}$$

Therefore the total holding cost over the time horizon H is given by,

$$C_h = h_1\sum_{j=0}^{n-1}e^{-jRT}$$

$$\therefore C_h = h\left\{\frac{Q}{R}(1-e^{-R\mu}) - \alpha\left[\frac{T^{\beta+1}}{\beta+1}+\frac{\theta(T^{\beta+2}-\mu^{\beta+2})}{(\beta+1)(\beta+2)}-\frac{\beta\theta(\theta+R)(T^{\beta+3}-\mu^{\beta+3})}{(\beta+1)(\beta+3)}-\frac{RT^{\beta+2}}{\beta+2}\right]+\alpha\left(T^{\beta}-\frac{\beta\theta T^{\beta+1}}{\beta+1}\right)\left((T-\mu)-\frac{(\theta+R)(T^2-\mu^2)}{2}\right)\right\}.\left(\frac{1-e^{-RH}}{1-e^{-\frac{RH}{n}}}\right) \tag{11}$$

Since the inventory model considers the effect of delay in payments, there are two different cases in the inventory system.

Case- I: $\mu \le \boldsymbol{M} \le \boldsymbol{T}$

In this case, the interest payable for the first replenishment cycle is given by,

$$I_{p_1}^1 = CI_r\int_M^T q(t)\,e^{-Rt}dt$$

$$\therefore I_{p_1}^1 = \alpha CI_r\left\{\frac{(-1)(T^{\beta+1}-\mu^{\beta+1})}{\beta+1}+\frac{(\theta+(\beta+1)R)(T^{\beta+2}-\mu^{\beta+2})}{(\beta+1)(\beta+2)}+\frac{\beta\theta(\theta+R)(T^{\beta+3}-\mu^{\beta+3})}{(\beta+1)(\beta+3)}+\left(T^{\beta}-\frac{\beta\theta T^{\beta+1}}{\beta+1}\right)\left((T-\mu)-\frac{(\theta+R)(T^2-\mu^2)}{2}\right)\right\} \tag{12}$$

Therefore the present value of the total interest payable over the time horizon H is given by,

$$I_{p_1}^H = \sum_{j=0}^{n-1} I_{p1}^1 . e^{-jRT}$$

$$\therefore I_{p_1}^H = \alpha CI_r\left\{\frac{(-1)(T^{\beta+1}-\mu^{\beta+1})}{\beta+1}+\frac{(\theta+(\beta+1)R)(T^{\beta+2}-\mu^{\beta+2})}{(\beta+1)(\beta+2)}+\frac{\beta\theta(\theta+R)(T^{\beta+3}-\mu^{\beta+3})}{(\beta+1)(\beta+3)}+\left(T^{\beta}-\frac{\beta\theta T^{\beta+1}}{\beta+1}\right)\left((T-\mu)-\frac{(\theta+R)(T^2-\mu^2)}{2}\right)\right\}.\left(\frac{1-e^{-RH}}{1-e^{-\frac{RH}{n}}}\right) \tag{13}$$

Now, the present value of the total interest earned during the first replenishment cycle is given by,

$$I_{e_1}^1 = pI_e \int_0^T \left(\alpha\beta t^{\beta-1}\right)t.e^{-Rt}dt$$

$$= \alpha\beta pI_e \int_0^T t^{\beta}.e^{-Rt}dt$$

$$= \alpha\beta pI_e \int_0^T t^{\beta}.(1-Rt)dt$$

$$= \alpha\beta pI_e \left[\frac{T^{\beta+1}}{\beta+1} - \frac{RT^{\beta+2}}{\beta+2}\right] \tag{14}$$

The present value of the total interest earned over the time horizon H is given by,

$$I_{e_1}^H = \sum_{j=0}^{n-1} I_{e1}^1.e^{-jRT}$$

$$= \alpha\beta pI_e \left[\frac{T^{\beta+1}}{\beta+1} - \frac{RT^{\beta+2}}{\beta+2}\right].\left(\frac{1-e^{-RH}}{1-e^{-\frac{RH}{n}}}\right) \tag{15}$$

The total present value of the cost over the time horizon H is given by,

$$TVC_1(n) = C_r + C_p + C_h + I_{p_1}^H - I_{e_1}^H$$

$$= \Bigg\{C_0 + C\left[\alpha\mu^{\beta} - \alpha\beta\left(\frac{\mu^{\beta}}{\beta} + \frac{\theta\mu^{\beta+1}}{\beta+1}\right)e^{-\theta\mu} + \alpha\beta\left(\frac{T^{\beta}}{\beta} + \frac{\theta T^{\beta+1}}{\beta+1}\right)e^{-\theta\mu}\right] + h\Bigg[\frac{Q}{R}(1 - e^{-R\mu}) - \alpha\left(\frac{T^{\beta+1}}{\beta+1} + \frac{\theta\left(T^{\beta+2}-\mu^{\beta+2}\right)}{(\beta+1)(\beta+2)} - \frac{\beta\theta(\theta+R)\left(T^{\beta+3}-\mu^{\beta+3}\right)}{(\beta+1)(\beta+3)} - \frac{RT^{\beta+2}}{\beta+2}\right) + \alpha\left(T^{\beta} - \frac{\beta\theta T^{\beta+1}}{\beta+1}\right)\left((T-\mu) - \frac{(\theta+R)(T^2-\mu^2)}{2}\right)\Bigg] + \alpha CI_r\Bigg[\frac{(-1)\left(T^{\beta+1}-\mu^{\beta+1}\right)}{\beta+1} + \frac{(\theta+(\beta+1)R)\left(T^{\beta+2}-\mu^{\beta+2}\right)}{(\beta+1)(\beta+2)} - \frac{\beta\theta(\theta+R)\left(T^{\beta+3}-\mu^{\beta+3}\right)}{(\beta+1)(\beta+3)} + \left(T^{\beta} - \frac{\beta\theta T^{\beta+1}}{\beta+1}\right)\Bigg] - \alpha\beta pI_e\left[\frac{T^{\beta+1}}{\beta+1} - \frac{RT^{\beta+2}}{\beta+2}\right]\Bigg\} \tag{16}$$

<u>Case- II:</u> $\boldsymbol{M > T}$

In this case, the present value of the total interest earned during the first replenishment cycle is given by,

$$I_{e_2}^1 = pI_e\left(\int_0^T \left(\alpha\beta t^{\beta-1}\right)t.e^{-Rt}dt + (M-T)\int_0^T \left(\alpha\beta t^{\beta-1}\right)tdt\right)$$

$$\therefore I_{e_2}^1 = \alpha\beta pI_e\left[\frac{T^{\beta+1}}{\beta+1} - \frac{RT^{\beta+2}}{\beta+2} + (M-T)\frac{T^{\beta+1}}{\beta+1}\right] \tag{17}$$

Hence the present value of the total interest earned over the time horizon H is given by,

$$I_{e_2}^H = \sum_{j=0}^{n-1} I_{e2}^1.e^{-jRT}$$

$$= \alpha\beta pI_e\left[\frac{T^{\beta+1}}{\beta+1} - \frac{RT^{\beta+2}}{\beta+2} + (M-T)\frac{T^{\beta+1}}{\beta+1}\right]\left(\frac{1-e^{-RH}}{1-e^{-\frac{RH}{n}}}\right) \tag{18}$$

Therefore the total present value of the cost over the time horizon H is given by,

$$TVC_2(n) = C_r + C_p + C_h - I_{e_2}^H$$

$$= \left\{C_0 + C\left[\alpha\mu^\beta - \alpha\beta\left(\frac{\mu^\beta}{\beta} + \frac{\theta\mu^{\beta+1}}{\beta+1}\right)e^{-\theta\mu} + \alpha\beta\left(\frac{T^\beta}{\beta} + \frac{\theta T^{\beta+1}}{\beta+1}\right)e^{-\theta\mu}\right] + h\left[\frac{Q}{R}(1 - e^{-R\mu}) - \alpha\left(\frac{T^{\beta+1}}{\beta+1} + \frac{\theta(T^{\beta+2}-\mu^{\beta+2})}{(\beta+1)(\beta+2)} - \frac{\beta\theta(\theta+R)(T^{\beta+3}-\mu^{\beta+3})}{(\beta+1)(\beta+3)} - \frac{RT^{\beta+2}}{\beta+2}\right) + \alpha\left(T^\beta - \frac{\beta\theta T^{\beta+1}}{\beta+1}\right)\left((T-\mu) - \frac{(\theta+R)(T^2-\mu^2)}{2}\right)\right]\right\}.\left(\frac{1-e^{-RH}}{1-e^{-\frac{RH}{n}}}\right) \quad (19)$$

At $M = T = \frac{H}{n}$, we have $TVC_1(n) = TVC_2(n)$.

Therefore we have,

$$TVC(n) = \begin{cases} TVC_1 & T > M \\ TVC_2(n), & T < M \\ TVC_1(n) = TVC_2(n), & T = M \end{cases}$$

6. Algorithm

(Step-1) Start by choosing a discrete value of $n \geq 1$.

(Step-2) If $T \geq M$ for different integral values of n then $TVC_1(n)$ is derived from expression (16).

If $T \leq M$ for different integral values of n then $TVC_2(n)$ is derived from expression (19).

(Step-3) Step-1 and 2 are repeated for all possible value of n with $T \geq M$ until the minimum value of $TVC_1(n)$ is found from equation (16). Let $n = n_1^*$ be such value of n. For all possible value of n with $T \leq M$ until the minimum value of $TVC_2(n)$ is found from equation (19). Let $n = n_2^*$ be such value of n. The values $n_1^*, n_2^*, TVC_1(n^*)$ and $TVC_2(n^*)$ constitute the optimal solution.

(Step-4) The optimal numbers of replenishment n^* is selected such that

$$TVC(n^*) = \begin{cases} TVC_1(n) & T = \frac{H}{n^*} > M \\ TVC_2(n), & T = \frac{H}{n^*} < M \\ TVC_1(n) = TVC_2(n), & T = \frac{H}{n^*} = M \end{cases}$$

The optimal value of ordered quantity Q^* is derived by substituting n^* in the equation (7) and optimal cycle time T^* is given by $T = \frac{H}{n^*}$.

7. Conclusion

In this article, we have developed an inventory model for perishable products under the assumptions of trade credit and time discounting. The demand rate has been taken as weibull function of time and deterioration rate is considered as a constant over units in the system. Shortages are not allowed during the cycle time. The optimal total variable cost has been evaluated with the help of fundamental calculus. Computational algorithm is given to evaluate the optimal value of ordered quantity and the optimal cycle time, based on which retailer can decide permissible period to be offered to the customer. It has been also observed that the various parameters such as holding cost, ordering cost, demand pattern can affect the system.

References

[1] *Chapman C.B. and ward S.C. Cooper, D.F. and Page, M.J., "Credit Policy and Inventory Control", Journal of the Operational Research Society, 35 (12),* ***(1984)*** *1055-1065, 87.*

[2] *Chang, H.J. and Dye C.Y., "The effect of credit period on the optimal lot size for deteriorating items with time varying demand and deterioration rates", Asia-Pacific Journal of Operational Research, 22,* ***(2001)*** *211-227.*

[3] *Goyal S. K., B. C. Giri, "Recent trends in modeling of deteriorating inventory", European Journal of Operation Research, 134* ***(2001)*** *1-16.*

[4] *L. Ouyang, M. Chen, and K. Chuang, "Economic order quantity model under cash discount and payment delay", International Journal of Information and Management Sciences, vol. 13, no. 1,* ***(2002)*** *1–10.*

[5] *Y.F. Huang and K.J. Chung, "Optimal replenishment and payment policies in the EOQ model under cash discount and trade credit", Asia-Pacific Journal of Operational Research, vol. 20, no. 2,* ***(2003)*** *177–190.*

[6] *Goyal, S.K., "Economic Order Quantity under Conditions of Permissible Delay in Payments", Journal of Operation Research Society, 36,* ***(1985)****335-338.*

[7] *K.J. Chung and Y.F. Huang, "Economic ordering policies for items under permissible delay in payments", Journal of Information and Optimization Sciences, vol. 24, no. 2,* ***(2003)*** *329–344.*

[8] *Chang, C.T, "An EOQ model with deteriorating items under inflation when supplier credits linked to order quantity", International Journal of Production Economics, 88(3),* ***(2004)*** *307-316.*

[9] *Y.F. Huang and H.F. Huang, "Optimal inventory replenishment policy for the EPQ model under trade credit derived without derivatives", International Journal of Systems Science, vol. 39, no. 5,* ***(2008)*** *539–546.*

[10] *Hari Kishan, Megha Rani, and Shiv Raj Singh, "Inventory model deteriorating product under supplier's partial trade credit policy", Inter.Tran. Mathematical Science and computer Vol.4 (1),* ***(2011)****143-156.*

[11] *Hari Kishan, Megha Rani and Vipin Kumar, "Inventory model of deteriorating products with non-uniform demand rates under life time and trade credit", International Journal of Mathematics and Mathematical Science, 2* ***(2015)*** *84-94.*

[12] *Pratima Patil and P. N. Mishra, "Inventory Model of Perishable Products with Periodic Demand", International Journal of Engineering Science and Computing, Vol. 7, no.11,* ***(2017)*** *15636-15639.*

[13] *Mihir S. Suthar, Kunal T. Shukla and Nikhilkumar D. Abhanghi, "Production Inventory System for Deteriorating Items with Trapezoidal-type Demand", Bull. Pure Appl. Sci. Sect. E Math. Stat. 38E(2),* ***(2019)*** *540-549.*

[14] *Mihir S. Suthar and Kunal Shukla, "General Approach of Retailer's Credit Period and Ordering Policy for Non-instantaneous Deteriorating Items having Ramp-Type Demand Pattern", Journal of Supply Chain Management Systems 9 (2 & 3)* ***(2020)****, 20-27.*