BI-ARCHIMEDIAN SEMI GROUP

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ABSTRACT

In this paper, an archimedian semi group where $x^n = yz$, where n being non-zero, is checked for a new formulation when n is 2, the Bi-Archimedian Semigroup. The newly constructed structure is observed to be zero-symmetric and completely prime. Homomorphism is found to preserve the structure of a bi-archimedian semi group. It is also observed to refrain to be as other existing sub structures under the involvement of other detonations like, regularity, quasi weak commutativity, homomorphism, etc. A Bi-archimedian semi group is found to be completely semi prime, trivial and commutative added with some other properties and conditions.

Keywords: Archimedian semi group, Zero-symmetric, completely prime, homomorphism, regular, quasi weak commutative.

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1. INTRODUCTION

Semi group structures has found it's own way of development in recent years. The name semi group originates in the fact that a semi group generalizes a group by preserving only as associativity and closure under the binary operation from the axioms defining a group. In mathematics, semigroup is an algebraic structure consisting of a set together with an associative internal binary operation on it. Commutative semigroups can be considered as the core semigroups since groups are regular semigroups with a unique idempotent. The idempotents and regularity play a predominant role in the structure of commutative semigroups. Several authors have extensively studied about its properties, the most important and fascinated structure is the Insertion of Factors Property and their co structures.

2. PRELIMINARIES

Definition 2.1

G is said to be a *Semigroup* if it is closed and associative with respect to addition "+" and Multiplication ".".

Definition 2.2

G is said to be *commutative* if ab = ba and G is said to be *Boolean* if $a^2 = a$ for all a, b from the semi group G.

Definition 2.3

Semi group G is *left (right) singular* if ab = a (ab = b) and G is said to be *regular* if aba = ab for all a, b, c belonging to the semi group G.

Definition 2.4

A mapping f: G ⟶ G' is said to be a *homomorphism*, where G and G' are semi groups, when the following conditions are satisfied:

i. g(m+n) = g(m) + g(n)
ii. g(mn) = g(m)g(n) where m, n belongs to the semi group G.

Definition 2.5

G supports *left duo* property if every left ideal I is also a right ideal and G is *completely prime* if xy ∈ I ⇒x ∈ I and y ∈ I.

Definition 2.6

A semi group G is said to be *quasi weak commutative* if abc = bac for all a, b, c belonging to the semi group G.

MAIN RESULT

Theorem 3.1

Every Bi-archimedian semi group is Zero Symmetric.

Proof:

Let G be the bi-archimedian semi group.

Then $x^2 = yz$ for all x, y, $z \in G$.

When $z = 0$, for every y in G,

$\Rightarrow x^2 = 0$.

Thus, the Bi-archimedian semi group G is Zero-symmetric.

Theorem 3.2

Homomorphic image of a Bi-archimedian Semigroup is also a Bi-archimedian Semigroup.

Proof:

Define g: G to G' be the homomorphism between the Bi-Archimedian semi groups G and G' such that $f(y) = y$ and $f(z) = z$, for every elements from the semi group.

Consider $x^2 = yz$.

Then, $f(x^2) = x^2$

$= yz$

$= f(yz)$

$= f(y)\, f(z)$

$\Rightarrow f(x^2) = f(y)\, f(z)$

Thus, homomorphism preserves the structure of a Bi-archimedian semigroup.

Theorem 3.3

Every Bi-archimedian left duo semi group is completely semi prime.

Proof:

Let G be the Bi-archimedian semi group.

Defines, $x^2 = yz$ for every x, y, z from G.

Let I be the ideal in G.

Now, $y, z \in I$.

$\Rightarrow yz \in I$

$\Rightarrow x^2 \in I$

$\Rightarrow xx \in I$

$\Rightarrow x \in I$

Since G is a left duo, it makes G to be Completely Semi Prime.

Hence the proof.

Proposition 3.4

Every quasi weak commutative bi-archimedian semi group is commutative in general.

Proof:

Let G be the bi-archimedian semigroup which is quasi weak commutative.

Then, $x^2 = yz$ for every x, y, z from G.

$\Rightarrow xxx = yzx$

$= zyx$

$\Rightarrow x^3 = zyx$

And this completes the proof.

Theorem 3.5

Every bi-archimedian semigroup is completely prime.

Proof:

Let G be the Bi-archimedian semi group.

Then, $x^2 = yz$ for every x, y, z from G.

Consider $yz \in I$.

$\Rightarrow x^2 \in I$

$\Rightarrow y \in I$ and $z \in I$, making G to be Completely prime.

Hence the proof.

Theorem 3.6

A Bi-archimedian semigroup is

(i) trivial

(ii) commutative

whenever regularity holds good.

Proof:

Let G be the Bi-archimedian semi group.

And let x, y, z ∈ G, then, $x^2 = yz$.

(i) Taking G to be regular, yz = yzy

= yy where G is also observed to be left singular

$= y^2$

$\Rightarrow x^2 = y^2$

$\Rightarrow x^2 = y^2 = yz$

$\Rightarrow y = z$

Thus, G is a trivial bi-archimedian semigroup.

(ii) Taking G to be regular, yz = yzy

= zy where G is also right singular

$\Rightarrow yz = zy$

$\Rightarrow x^2 = zy$, making G to be commutative.

Theorem 3.7

Every regular quasi weak commutative bi-archimedian semigroup is an idempotent semigroup.

Proof:

Let G be the quasi weak commutative bi-archimedian semigroup.

Then, $x^2 = yz = yzy$, where G is regular.

$= zyy$, since G is quasi weak commutative

$= zy^2$

$\Rightarrow yz = zy^2$

$= zy^2z$

$= y^2zz$

$= y^2z^2$

Thus, $yz = y^2z^2$

Hence G is Boolean.

Proposition 3.8

In a bi-archimedian quasi weak commutative regular semi group,

(i) $y^nz^n = y^{2n}z^{2n}$

(ii) $z^ny^{2n} = z^{2n}y^{4n}$

for every Even n.

Proof:

Let G be the bi-archimedian regular semi group.

Then, from the previous theorem 3.7, for every x, y, z from G, $x^2 = y^2z^2$ ----- I

$= y^2z^2y^2$, since G is regular

And, $y^2z^2y^2 = z^2y^2y^2$

$= z^2y^4$

$\Rightarrow x^2 = z^2y^4$

$= z^2y^4\,z^2$

$= z^2y^4$

$\Rightarrow x^2 = z^2y^4$ ----------- i

$= z^2y^4z^2$

$= y^4z^2z^2$

$= y^4z^4$

$\Rightarrow x^2 = y^4z^4$ ----------- II

Proceeding this way gives, $x^2 = z^4y^8$ ------- ii

$x^2 = y^8z^8$ -------- III

$x^2 = z^8y^{16}$ -------- iii and so on.

Thus, without loss of generalization, we get, (i) $y^nz^n = y^{2n}z^{2n}$ (ii) $z^ny^{2n} = z^{2n}y^{4n}$ for every n being an even number.

This completes the proof.

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