Empirical Modeling of a Rayleigh Fading Channel and Computation of Channel Capacity for Multi-Path Wireless Propagation

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Abstract

This work presents the empirical modeling of wireless communication channels and reviews the performance metrics of the multi-path propagation model with the help of the Singular-Value-Decomposition (SVD) method to enhance the channel capacity in 4G and 5G wireless communication systems. MIMO has established a rapid improvement in data rate by lowering the Signal-to-Noise ratio (SNR). Fading is a serious issue in the multi-path propagation model, that decreases the overall average Bit-Error-Rate (BER). In this research, the data rate is analyzed in terms of enhancing the channel capacity of the MIMO system and is analyzed by reducing the SNR using SVD. An Empirical model of the MIMO is postulated, and the Rayleigh-Fading system is modeled to compute the channel capacity. It is found that the maximum channel capacity is achieved up to 6.9 bits/sec/Hz.

Keywords: *Multi-Input-Multi-Output, Fading, Antenna Diversity, Rayleigh Distribution, Singular Value Decomposition, Bit Error Rate.*

1. Introduction

The 5G technology has received wide attention and a lot of research from global enterprises, research institutes, and universities to improve spectrum utilization through channel capacity improvement in wireless communication systems. Therefore, the communication system demands proper utilization of the bandwidth resources efficiently. Due to the shortage of spectrum resources, it is important to improve the spectrum utilization of the system for future communication technologies [1]. In MIMO, multiple antennas are utilized on both the transmitting and receiving ends to improve transmission reliability.

1.1 Basics of MIMO

MIMO was a paradigm shift in wireless transmission because it broke fundamental barriers to increasing data rates. Before the invention of MIMO, the channel capacity was improved primarily by either enhancing the link speed or higher transmit power. But they suffer from their perspectives as both bandwidth and power were insurmountable barriers. MIMO multiplied data rates many folds without the need for increased bandwidth or transmit power [2]. MIMO is today the core enabler of 4G and 5G mobile and Wi-Fi networks. The 6G standards are currently being developed and it is expected that the use of MIMO and AI together may promise even more higher data rates and efficient communication delivery.

1.2 Comparison Between SISO and MIMO System

In a conventional SISO system, the data rate can be increased by either increasing the transmission bandwidth or transmitting power. But both techniques are not fruitful as the frequency spectrum is a valuable resource and on the other hand, we cannot provide large power to the antennas as it reduces the battery life.

Whereas, MIMO increases the spectrum efficiency without increasing the transmission power and bandwidth. Rate gain and diversity gain are the two popular measures used in MIMO. For parallel MIMO channels, if NR is the number of received antennas and NT is the number of transmit antennas, then the minimum rate gain from that of the SISO system is popularly known as multiplexing gain. Therefore, in spatial multiplexing, different data streams are sent through parallel channels with the help of a serial-to-parallel converter to provide a higher transmission rate. The maximum number of independent paths travelled by each signal can be at the maximum number of transmit and receive antennas. In the multi-path propagation scenario, not all the paths may be highly faded but in such a scenario, the same data is sent through all the transmit antennas [3-6]. If any of the paths is completely down, the other paths will still be working. It makes the receiver efficient to use this technique for decoding the data accordingly, which gives higher link reliability.

1.3 Key Concept of Different Fading Issues

Wireless transmission has completely transformed the way of communication still, several issues exist. When there are variations in the signal strength, a serious issue termed fading is observed. The main offenders of log-normal shadowing, or gradual fading, are trees, buildings, and topographical features. As the name implies, slow fading causes gradual variations in signal strength over comparatively long periods. This kind of fading is more common in outdoor environments when there are a lot of obstacles in the way of signal propagation. Robust error correction and detection techniques are necessary to ensure reliable communication due to the features of slow fading, particularly in situations where the received signal power varies gradually over time.

The effects of slow fading on wireless transmission are significant. To offset the signal loss caused by gradual fading, complex modulation techniques, and signal amplification techniques are frequently used as key metrics in their mitigation. Furthermore, adaptive antenna systems are utilized to dynamically modify the signal transmission to counteract slow fading and guarantee dependable connectivity throughout the different types of terrain. In contrast to slow fading, fast fading is also characterized by sudden variations in signal strength that typically occur over short time intervals. This phenomenon is sometimes attributed to multipath propagation, in which the supplied signal travels over multiple paths before reaching the receiver as shown in Figure 1.

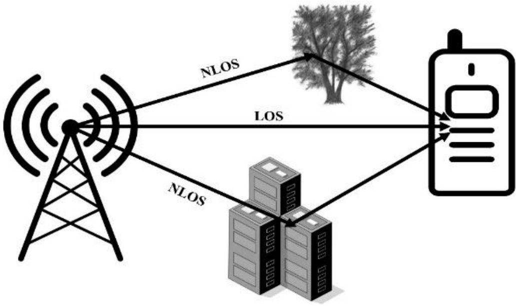

Figure 1: LoS and NLoS propagation of antenna element

Fast fading is a significant issue in wireless communication as it causes error and distortion due to sudden changes in signal strength. Adaptive modulation, coding techniques, and dynamic channel equalization are used to offset the fast signal fluctuations and decrease the impact of rapid fading. Variations in the attenuation and delay of the transmitted signal cause frequency-selective fading, which is also referred to as frequency- or time-dispersive fading.

Wireless systems use methods like orthogonal frequency division multiplexing (OFDM) and frequency-domain equalization to counteract the effects of frequency-selective fading. These approaches help to reduce distortions related to frequency and guarantee reliable communication across a range of frequency components. Inter-symbol interference and spectral distortion are brought on by frequency-selective fading, which makes it more challenging to reliably recover the supplied data [7-12].

One important way to lessen the impact of frequency-selective fading is to use equalization techniques to counteract the frequency-dependent channel defects. Adaptive equalizers minimize distortion and provide accurate signal recovery from frequency-selective fading. Furthermore, orthogonal frequency-division multiplexing (OFDM) modulation mitigates frequency-selective fading and efficiently lowers the effects of frequency-dependent channel variations by splitting the transmission into several narrowband subcarriers.

2. Antenna Diversity

In SISO, Low-Density-Parity-Check-Codes and Turbo Codes with iterative decoding algorithms are the capacity boosters. Whereas, in Single-Input-Multiple-Output (SIMO), receiver diversity techniques such as Equal-Gain-Combining (EGC), Selection Combining (SC), and Maximal-Ratio-Combining (MRC) are utilized to combat the phenomenon such as multi-path fading. MRC is optimal in terms of SNR but complex to implement in terms of the other combining schemes. For Multi-Input-Single-Output (MISO), the receiver diversity is not cost impressive, Instead, transmit diversity at the base station is a better choice. Whereas, in the MIMO system both the transmit and received diversity techniques are utilized rather than the individual techniques for improving the channel capacity and reliability [13-16]. The channel capacity of a MIMO system increases with a higher value of multiple transmit and receive antenna. Even when the SNR increases the channel capacity also increases by a factor of 4 Bits/Sec/Hz as shown in Figure 2 below.

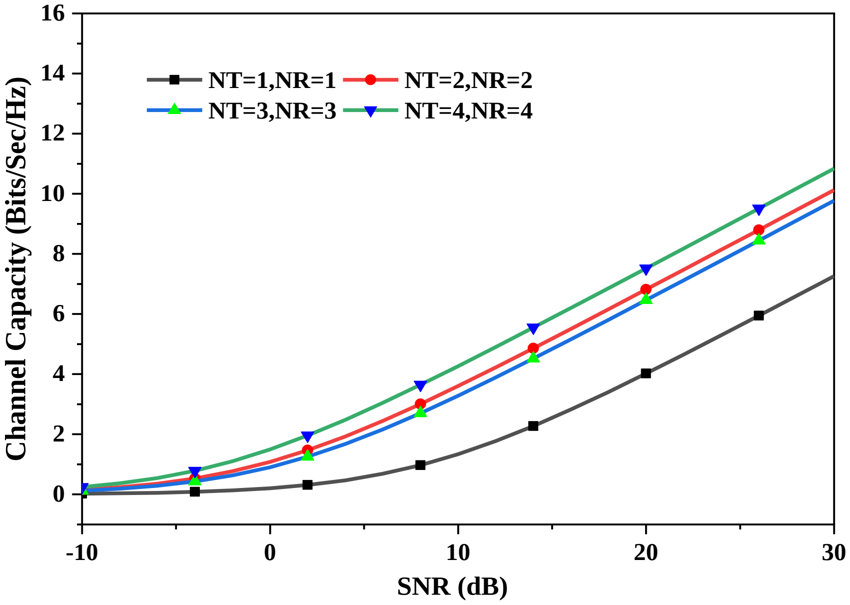

Figure 2: Channel Capacity of different MIMO schemes

2.1 Different Diversity Scheme

There are three transmit diversity schemes have been employed based on the Channel-State-Information (CSI). When the CSI is available at both the transmitter and receiver, it is called a closed-loop MIMO system. In open loop MIMO, CSI is available at the receiver but not at the transmitter. But when the CSI is not available at both the transmitter and receiver, it is called a blind MIMO system and should operate in a non-coherent mode.

2.2 Diversity-Multiplexing Trade-off

The rate gain is associated with the transmission data rate. The rate gain r can be calculated using the equation (1).

$$r = \lim_{SNR \to \infty} \frac{r\,(SNR)}{\log_2(SNR)} \ldots\ldots\ldots (1)$$

The diversity gain is associated with the probability of error in detection as expressed in equation (2). A transmission scheme is said to achieve diversity gain d if the probability of error P_e (SNR), satisfies the equation (2).

$$d = -\lim_{SNR \to \infty} \frac{\log_2\{P_e(SNR)\}}{\log_2(SNR)} \ldots\ldots\ldots (2)$$

For a given rate gain r, the optimal diversity gain d_{opt} (r), is the supreme diversity gain that can be accomplished by any MIMO system. If the fading block length is T such that,

$$T \geq N_T + N_R - 1 \ldots\ldots\ldots (3)$$

The d_{opt} (r), can be calculated using equation (4),

$$\mathrm{d_{opt}(r) = (N_T - r)(N_R - r), 0 \leq r \leq min(\, N_T - N_R)} \ldots\ldots\ldots (4)$$

3. Mathematical Modelling of Multipath Environment

In a multipath environment, the wireless signal reaches the mobile receiver from the base station (BS) through different paths, some of the links use Line-of-Sight (LOS) paths and some reach through different scattering components as shown in Figure 3.

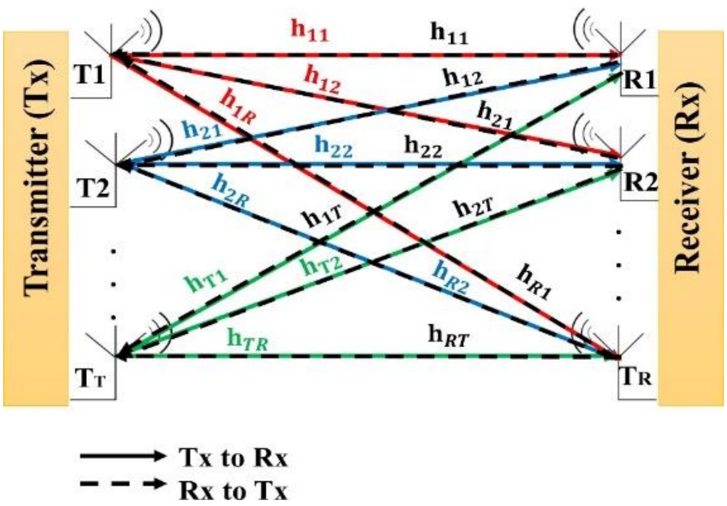

Figure 3: Multi-antenna propagation scenario

As a result, the mobile receiver leads to the superposition of those multiple signals, i.e., LOS and NLOS. As, in the multiple scenarios, there are L components, therefore considering the kth path which can be characterized by a delay of τk and attenuation of ak. The superimposed signal of the multiple components at the receiver can be modelled as the sum of individual path responses as dissipated in equation (5).

$$\mathrm{h(t)} = \sum_{\mathrm{k=0}}^{\mathrm{L-1}} \mathrm{a_k \delta(t - \tau_k)} \ldots\ldots\ldots (5)$$

For the kth path, the received signal can be denoted by equation (6)

$$\mathrm{y_p(t) = Re} \left\{ \left(\sum_{\mathrm{k=0}}^{\mathrm{L-1}} \mathrm{a_k\, S(t - \tau_k) e^{-j2\pi F_c \tau_k}} \right) \mathrm{e^{j2\pi F_c t}} \right\} \ldots\ldots\ldots (6)$$

$$\sum_{\mathrm{k=0}}^{\mathrm{L-1}} \mathrm{a_k\, S(t - \tau_k) e^{-j2\pi F_c\, \tau_k}}) \text{ is received baseband signal}$$

Therefore, the received baseband signal becomes,

$$y(t) = (\sum_{k=0}^{L-1} a_k e^{-j2\pi F_c \tau_k})\ S(t) \ldots\ldots\ldots\ (7)$$

As,

$$h = \sum_{k=0}^{L-1} a_k e^{-j2\pi F_c \tau_k}$$

therefore, equation (7) can be simplified as,

$$y(t) = h \times S(t) \ldots\ldots\ldots\ (8)$$

As the channel coefficient (h) varies depending on the various channel attenuation factors a_k and delay τ_k, therefore, the channel coefficient is called the Fading-Channel Coefficient. This fading process causes the received power to vary, which is a key barrier to wireless communication.

3.1 Fading Channel Distribution Model

The fading channel coefficient can further be rewritten as,

$$h = \sum_{k=0}^{L-1} a_k \cos{(2\pi F_c \tau_k)} - j \sum_{k=0}^{L-1} a_k \sin{(2\pi F_c \tau_k)} \ldots\ldots (9)$$

Where,

$$x = \sum_{k=0}^{L-1} a_k \cos(2\pi F_c \tau_k) \text{ and}$$

$$y = -\sum_{k=0}^{L-1} a_k \sin{(2\pi F_c \tau_k)}$$

Since x and y are the sum of many random components involved in attenuation and delay, they are also random in nature. By the Central Limit Theorem (CLT), x and y can be assumed to be the Gaussian Distribution in nature, and x and y are independent Gaussian-Random Variable, with zero mean and variance to 1/2.

3.2 Characterization of Fading Channel Coefficient

The marginal distribution of the amplitude may also be called as Rayleigh Distribution (RD) and the channel coefficient h is also termed as Rayleigh Fading Channel Coefficient. Then the amplitude of the marginal RD of the equation may also be written as follows,

$$F_A(a) = \int_{-\pi}^{\pi} F_{A,\varphi}(a, \varphi)\, d\varphi = 2ae^{-a^2} \ldots\ldots\ldots (10)$$

And,

$$F_{A,\varphi}(a, \varphi) = \frac{a}{\pi} e^{-a^2}$$

The distribution of the phase can be written as,

$$F_{\varphi}(\varphi) = \int_0^{\infty} F_{A,\varphi}(a, \varphi)\, da = \frac{1}{2\pi} \text{ for } -\pi < \varphi \leq \pi \dots\dots\dots (11)$$

4. Estimation of Bit Error Rate

There are many matrices to evaluate the performance of a wireless communication system. One of the most convenient matrices is the Bit Error Rate (BER) which is also called the Average Bit Error Rate, is often expressed by the theory of probability and lies between 0 and 0.5. The maximum possible BER can be 0.5. In the Binary Phase Shift Keying (BPSK), the symbols 0 and 1 are presented as,

$$0 \rightarrow \sqrt{P} \text{ and } 1 \rightarrow -\sqrt{P}$$

Therefore, there is an 180^0 phase-shifting between the symbols and the average power (P) in BPSK. Under the assumption of noise in the channel, the received signal can be expressed as,

$$y = hx + n$$

Then the received power can be calculated as,

$$\text{Received Power} = |h|^2 P = a^2 P \dots\dots\dots (12)$$

So, the SNR due to fading,

$$SNR_F = \frac{a^2 P}{\sigma^2} = a^2 \, SNR \dots\dots\dots (13)$$

The calculated BER of the BPSK modulation is,

$$BER = Q\sqrt{SNR_F} = Q\sqrt{a^2 \, SNR} \dots\dots\dots (14)$$

Whose solution becomes,

$$\text{Average BER} = \frac{1}{2}\left(1 - \sqrt{\frac{SNR}{2 + SNR}}\right) \dots\dots\dots (15)$$

This is the average BER of a Rayleigh Fading Channel. For the Rayleigh Fading Channel, the Bit Error Rate is a decaying exponential function. Figure 4 shows that the increasing SNR decreases the BER. However, the SNR cannot be increased as much as required due to the limitations of the large power-handing capability.

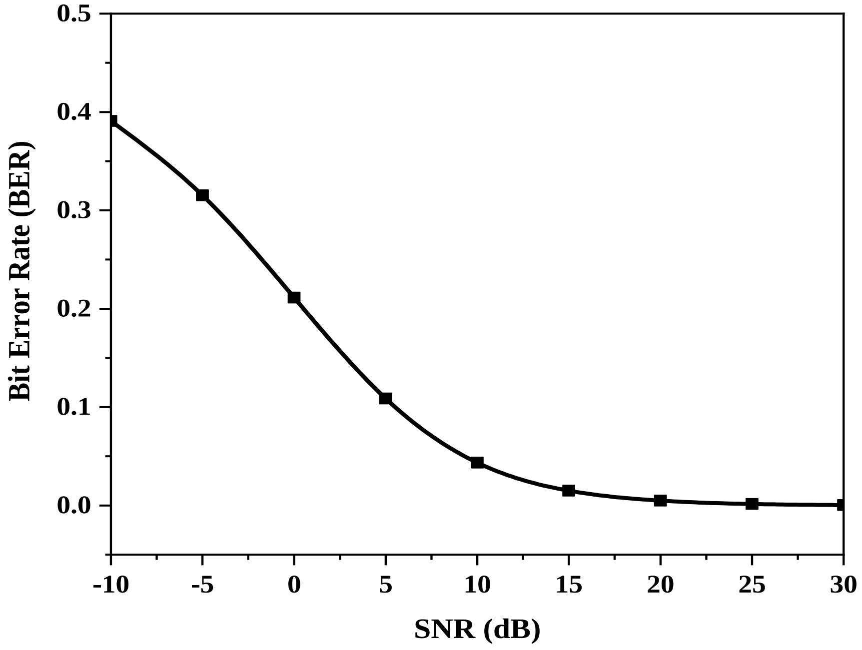

Figure 4: SNR vs BER plot

5. Estimation of Power Profile in Multipath Scenario

The power of the kth multipath component can be given as,

$$\varphi(t) = \sum_{k=0}^{L-1} |a_k|^2 \, \delta\,(t - \tau_i)$$

$$\text{if } |a_k|^2 = g_k$$

$$\varphi(t) = \sum_{k=0}^{L-1} g_k \, \delta\,(t - \tau_k)$$

there is a power a_i or the gain g_i which is arriving with a delay of τ_i which gives the power profile of the wireless channel. For L=4 receiving antennas of a multipath channel,

$$\varphi(t) = \sum_{k=0}^{3} g_k \, \delta\,(t - \tau_k)$$

therefore, the gain $g_0, g_1, g_2, and\ g_3$ are the gain at different delay $\tau_0, \tau_1, \tau_2, and\ \tau_3$ respectively. So, the maximum delay spread may be given as,

$$\sigma_\tau^{\text{Max}} = \tau_3 - \tau_0$$

In general terms, the maximum delay spread is represented by,

$$\sigma_\tau^{\mathrm{Max}} = \tau_{L-1} - \tau_0$$

This maximum delay spread (MDS) is another metric to characterize the delay spread of the wireless communication channel but this MDS is not very appropriate, as in the power profile there might be very small power contents at the far distance are negligible and called the spurious components. In such scenarios, instead of MDS an alternative metric called RMS delay spread becomes very useful. The fraction of power at the kth path,

$$b_k = \frac{g_k}{\sum_{j=0}^{L-1} g_k}$$

Then the average delay,

$$\bar{\tau} = b_0\tau_0 + b_1\tau_1 + \ldots\ldots + b_{L-1}\tau_{L-1} \ldots\ldots\ldots (16)$$

So,

$$\bar{\tau} = \sum_{k=0}^{L-1} b_k\tau_k = \sum_{k=0}^{L-1} \frac{g_k\tau_k}{\sum_{j=0}^{L-1} g_k}$$

Therefore, the average delay,

$$\bar{\tau} = \frac{\sum_{i=0}^{L-1} g_k\tau_k}{\sum_{i=0}^{L-1} g_k}$$

So, the RMS delay spread is given by the deviation of the multipath power profile,

$$\sigma_\tau^2 = b_0(\tau_0 - \bar{\tau})^2 + b_1(\tau_1 - \bar{\tau})^2 + \ldots\ldots + b_{L-1}(\tau_{L-1} - \bar{\tau})^2$$

Then,

$$\sigma_\tau = \left(\sum_{k=0}^{L-1} b_k(\tau_k - \bar{\tau})^2\right)^{\frac{1}{2}}$$

Therefore, the RMS delay spread, in terms of gain,

$$\sigma_\tau = \left(\frac{\sum_{k=0}^{L-1} g_k(\tau_k - \bar{\tau})^2}{\sum_{k=0}^{L-1} g_k}\right)^{\frac{1}{2}}$$

And as, $g_k = |a_k|^2$

Therefore, the RMS delay spread of the multipath delay profile,

$$\sigma_\tau = \left(\frac{\sum_{k=0}^{L-1}|a_k|^2(\tau_k - \bar{\tau})^2}{\sum_{k=0}^{L-1}|a_k|^2}\right)^{\frac{1}{2}} \dots\dots\dots (17)$$

Table 1. Average Delay Spread and RMS Delay Spread

Table 1 below shows the different values of τ_k and g_k for L=4 multipath components.

τ_k (µs)	g_k	dB Gain	$a = \sqrt{g}$
0	0.01	-20	0.1
1	0.1	-10	0.316
2	1	0	1
3	0.1	-10	0.316

Using the equation (16),

$$\bar{\tau} = \frac{0.01 \times 0 + 0.1 \times 1 + 1 \times 2 + 0.1 \times 3}{0.01 + 0.1 + 1 + 0.1} = 1.98\ \mu\text{s}$$

Now, the RMS delay spread using the equation (17),

$$\sigma_\tau = \left(\frac{\sum_{k=0}^{L-1}|a_i|^2(\tau_k - \bar{\tau})^2}{\sum_{k=0}^{L-1}|a_k|^2}\right)^{\frac{1}{2}}$$

$$\sigma_\tau = 0.445\ \mu\text{s}$$

Therefore,

$$\sigma_\tau \ll \sigma_\tau^{\text{Max}}$$

So, it is convenient to use RMS delay spread instead of Maximum Delay Spread.

6. Channel estimation for MIMO system

MIMO is extremely key technology in 3G, 4G, and 5G communication systems but more specifically in 5G, massive MIMO is prime technology to increase the data rate over the wireless channel [17-19].

Let us consider NT-transmit and NR-receive antennas are there in the transmitter and receiver respectively. Therefore, the mathematical model for MIMO can be represented as,

$$\bar{\mathbf{y}} = \mathrm{H}\bar{\mathbf{x}} + \bar{\mathbf{w}}$$

Where H is $r \times t$ channel matrix with r number of rows and t number of columns.

$$H = \begin{bmatrix} h11 & h12 & h1t \\ h12 & h22 & h2t \\ hr1 & hr2 & hrt \end{bmatrix}$$

For a 4×4 *MIMO system,*

The transmit vector=$\begin{bmatrix} x1 \\ x2 \\ x3 \\ x4 \end{bmatrix}$, and received vector=$\begin{bmatrix} y1 \\ y2 \\ y3 \\ y4 \end{bmatrix}$

Therefore, the system model,

$$\begin{bmatrix} y1 \\ y2 \\ y3 \\ y4 \end{bmatrix} = \begin{bmatrix} h11 & h12 & h13 & h14 \\ h12 & h22 & h23 & h24 \\ h13 & h23 & h33 & h34 \\ h14 & h24 & h34 & h44 \end{bmatrix} \begin{bmatrix} x1 \\ x2 \\ x3 \\ x4 \end{bmatrix} + \begin{bmatrix} w1 \\ w2 \\ w3 \\ w4 \end{bmatrix}$$

$$\text{Let. } H = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 0 & -2 & 1 \\ 1 & 0 & 0 & -3 \end{bmatrix}$$

Then, the computation of U, S, and V gives,

$$U = \begin{bmatrix} -0.2887 & 0.4082 & -0.5000 & -0.7071 \\ -0.2887 & 0.4082 & -0.5000 & 0.7071 \\ -0.2887 & -0.8165 & -0.5000 & -0.0000 \\ 0.8660 & -0.0000 & -0.5000 & 0.0000 \end{bmatrix}$$

$$S = \begin{bmatrix} 3.4641 & 0 & 0 & 0 \\ 0 & 2.4495 & 0 & 0 \\ 0 & 0 & 2.0000 & 0 \\ 0 & 0 & 0 & 1.4142 \end{bmatrix}$$

$$V = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}$$

$$\text{where, } \sigma_1 = 3.4641, \sigma_2 = 2.4495, \sigma_3 = 2.0000, \sigma_4 = 1.4142$$

Further,

$$\sigma_1 > \sigma_2 > \sigma_3 > \sigma_4$$

The channel capacity of a MIMO system with NT-transmit and NR-received antennas can increase by a factor of min (NT, NR) without additional transmit power or spectral bandwidth over the conventional SISO system for the Rayleigh fading channel. The diversity technique improves transmission reliability whereas the spatial multiplexing maximizes the transmission rate as of MIMO channel capacity.

By using the SVD, the MIMO fading channel with the channel matrix H can be represented by decoupled parallel Gaussian sub-channels. Thus, the capacities of sub-channels add up, giving the overall instantaneous capacity for uniform or equal power allocation. The mean MIMO capacity for the Ergodic fading channel is

$$\langle \mathrm{C} \rangle = \mathrm{E}\{\mathrm{W} \log_2 \det\left(\mathrm{I} + \frac{\mathrm{PQ}}{t\sigma^2}\right)\}$$

$$\text{Where, Q is the Wishart Matrix} = \begin{cases} \mathrm{HH^H}, & \mathrm{r} < \mathrm{t} \\ \mathrm{H^H H}, & \mathrm{r} \geq \mathrm{t} \end{cases}$$

Using the singular value decomposition to find the singular values of the channel for,

$$\sigma_1 = 3.4641, \sigma_2 = 2.4495, \sigma_3 = 2.0000, \text{and } \sigma_4 = 1.4142$$

So, the singular values are

$$\sqrt{\lambda_1} = 3.46, \sqrt{\lambda_2} = 2.44, \sqrt{\lambda_3} = 2, and \sqrt{\lambda_4} = 1.41$$

$$\text{or}, \lambda_1 = 12, \lambda_2 = 6, \lambda_3 = 4, \lambda_4 = 2$$

For optimal power distribution,

$$\gamma_i = \frac{P}{\sigma^2} \times \lambda_i$$

For, L=4 antenna elements, the SNR=16 dB for the average BER=10^{-6},

$$\gamma_1 = 477.72, \gamma_2 = 238.86, \gamma_3 = 159.24, \gamma_4 = 79.62$$

Considering the power is distributed to the parallel channels, the power constraint becomes,

$$\sum_{i=0}^{L-1} \frac{1}{\gamma_0} - \frac{1}{\gamma_i} = 1$$

Therefore, solving the above equation becomes,

$$\frac{4}{\gamma_0} = 1 + \frac{1}{\gamma_1} + \frac{1}{\gamma_2} + \frac{1}{\gamma_3} + \frac{1}{\gamma_4} = 1.025$$

$$\gamma_0 = 3.90 \cong 4$$

Then, the capacity of the first channel becomes,

$$\mathrm{C} = \log_2\left(\frac{\gamma_1}{\gamma_0}\right) = 6.90\ \mathrm{bits/sec/Hz}$$

The capacity of the second channel becomes,

$$\mathrm{C} = \log_2\left(\frac{\gamma_2}{\gamma_0}\right) = 5.90\ \mathrm{bits/sec/Hz}$$

The capacity of the third channel becomes,

$$\mathrm{C} = \log_2\left(\frac{\gamma_3}{\gamma_0}\right) = 5.3150\ \mathrm{bits/sec/Hz}$$

And the capacity of the fourth channel becomes,

$$\mathrm{C} = \log_2\left(\frac{\gamma_4}{\gamma_0}\right) = 4.3150\ \mathrm{bits/sec/Hz}$$

So, the maximum capacity of the channel is 6.9 $bits/sec/Hz$.

Conclusion

This research article surveys the beginning of MIMO wireless communication with the different fading scenarios, channel estimation, and application of the Singular-Value-Decomposition (SVD) method to find out the channel capacity in a 4x4 MIMO antenna. It is seen that the channel capacity increases with the increment of antenna arrays.

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