Anisotropic Viscous fluid String Cosmological Model in Saez-Ballester Theory of Gravitation

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Abstract:

The spatially homogeneous and anisotropic five-dimensional Bianchi type-III metric is considered together with the viscous fluid coupled to one-dimensional cosmic strings with the help of the special law of variation for Hubble's parameter proposed by Berman (1983) in the context of Saez-Ballester theory of gravitation. Finally, also discussed some physical properties.

Introduction:

The cosmological models are based on the General Theory of Relativity (GTR), a crucial theory in modern theoretical physics, which can explain physical phenomena. Experiments have been undertaken in a variety of fields of physics, and theories must be modified to account for observations. To compare theory with observations, a framework must be formed, which has been developed by numerous authors under a variety of circumstances. There are two main categories of gravitational theories involving a scalar field. The first category, proposed by Brans and Dicke (1961), considers a scalar field with dimension as the inverse of the gravitational constant G. This theory emphasizes the importance of introducing an additional scalar field φ, along with metric tensor $g_{\mu\nu}$ and ω as a coupling constant. The second category, proposed by Saez-Ballester (1986), describes a scalar field as dimensionless, addressing the missing mass in the Friedmann-Roberson Walker flat universe. Several authors have done their investigations in Saez- Ballester theory out of some are Rasouli et al. (2020), Lambat and Pund (2023), Vinutha et al. (2022), Singh et al. (2024), Naidu et al. (2021), Santhi et al. (2022), Khandro (2024).

The gravitation action is defined as

$$I = \int_{\Sigma} (L + GL_m)\sqrt{-g}\,dxdydzdt \tag{1}$$

The variation of gravitational action, based on the principle $\delta I = 0$ leads to the Saez- Ballester scalar-tensor theory field equations.

$$R_{ij} - \frac{1}{2} g_{ij} R - \omega\phi^n \left(\phi_{,i}\phi_{,j} - \frac{1}{2} g_{ij}\phi_{,k}\phi^{,k} \right) = -T_{ij} \tag{2}$$

Where the scalar field ϕ satisfies the equation,

$$2\phi^n \phi^{i}_{;i} + n\phi^{n-1}\phi_{,k}\phi^{,k} = 0 \tag{3}$$

Where Rij is the Ricci tensor, R is the Ricci scalar, w is a dimensionless constant, Tij is the energy momentum tensor of matter (The relativistic units, 8 π G = c = 1 are used here).

Bulk viscosity is a crucial factor in the early cosmological evolution of the Universe, influencing the description of the Universe's high entropy in the modern era. Its impact on cosmological models can be counteracted by gravitational contraction or expansion, levitating preliminary singularities, growing a bounded model, and modifying the impact of pressure and energy density during cosmological evolution.

Cosmic strings, stable objects found during a phase transition in the early Universe, are believed to cause density perturbations leading to the formation of galaxies. These strings have stress-energy and couple to the gravitational field, making it interesting to study their gravitational effects. Letelier's(1983) pioneering work in formulating the energy-momentum tensor for classical massive strings suggests they are formed by geometric strings with particles attached along their extension.

Vilenkin (1985), Gott (1985), and Garfinkle (1985) have all determined the gravitational effects of cosmic strings. Eventually, precise string cosmology solutions in different space-times have been examined by numerous writers, Ingle et. al. (2024), Baro and Singh (2023), Santhi et al. (2022), Vinutha et al. (2021), Borgade et al. (2021), Bhoyar and Ingole (2024).

Motivated from the studies outlined above, in this paper, we have investigated five dimensional Bianchi type –III string viscous fluid cosmological model in the Saez-Ballester theory of gravitation. The paper is organised as follows: In section 2: the paper presents a detailed analysis of metric and field equations, in section 3: their solutions considering physical assumptions, in section 4: the physical behavior of the models, and concludes with a summary of the findings in the final section.

2 Metric and field equations:

We consider the five dimensional Bianchi type –III metric as,

$$ds^2 = R^2\left(dx^2 + e^{-2bx}dy^2 + dz^2\right) + Q^2 dl^2 - dt^2, \tag{4}$$

where, the scale factors *P* and *Q* are the functions of *t* only, $b \neq 0$ is a constant. *l* is the fifth coordinate which is spacelike.

The energy momentum tensor for viscous fluid coupled with one dimensional string is given as

$$T_{ij} = \left(\rho + \bar{p}\right)v_i v_j + \bar{p} g_{ij} - \lambda x_i x_j, \tag{5}$$

where, ρ denotes the energy density, λ denotes the string tension, $\bar{p}$ is the total pressure, v^j represents the five velocity vector of fluid flow and x^i represents space like unit vector that represents the direction of string.

In a comoving coordinate system, we have,

where, $v^i = (0,0,0,0,1)$ and $x^i = (0,0,0,\frac{1}{Q},0)$ (6)

In commoving co-ordinates, the direction of string x^i and velocity vector v^i satisfy the following conditions

$$v^i v_i = -x^i x_i = -1,\ v^i x_i = 0. \tag{7}$$

Using (2) - (7) the field equations are obtained as

$$2\frac{\ddot{R}}{R}+\frac{\ddot{Q}}{Q}+\frac{\dot{R}^2}{R^2}+2\frac{\dot{R}\dot{Q}}{RQ}-\frac{\omega}{2}\phi^n\dot{\phi}^2=-\bar{p} \tag{8}$$

$$2\frac{\ddot{R}}{R}+\frac{\ddot{Q}}{Q}+\frac{\dot{R}^2}{R^2}+2\frac{\dot{R}\dot{Q}}{RQ}-\frac{b^2}{R^2}\frac{\omega}{2}\phi^n\dot{\phi}^2=-\bar{p} \tag{9}$$

$$3\frac{\ddot{R}}{R}+3\frac{\dot{R}^2}{R^2}-\frac{b^2}{R^2}+\frac{\omega}{2}\phi^n\dot{\phi}^2=-\bar{p}+\lambda \tag{10}$$

$$3\frac{\dot{R}\dot{Q}}{RQ}+3\frac{\dot{R}^2}{R^2}-\frac{b^2}{R^2}+\frac{\omega}{2}\phi^n\dot{\phi}^2=\rho \tag{11}$$

$$\ddot{\phi}+\dot{\phi}\left(\frac{\dot{R}}{R}+\frac{\dot{Q}}{Q}\right)+\frac{\dot{\phi}^2}{\phi}\frac{n}{2}=\frac{1}{3}\mu(4\bar{p}+\lambda-\rho) \tag{12}$$

Furthermore, for the line element (4), we define the following kinematical space-time quantities of physical significance in cosmology:

The spatial volume : $V=a^4=R^3Q$ (13)

The mean Hubble's parameter : $4H=\theta=3\frac{\dot{R}}{R}+\frac{\dot{Q}}{Q}$ (14)

The expansion scalar : $\theta=v^i_{;i}=3\frac{\dot{R}}{R}+\frac{\dot{Q}}{Q}$ (15)

The deceleration parameter : $q=-1+\frac{d}{dt}\left(\frac{1}{H}\right)$ (16)

3 Solution of the field equations:

As there are five equations (8) - (12) and six unknowns; $R,Q,\omega,\bar{p},\lambda,$ and ρ, so we need one extra condition to get the exact solution of the field equation. Since the field equations are highly non-linear therefore, we assume,

$$4\bar{p}+\lambda-\rho=0 \tag{17}$$

Also, let us take the deceleration parameter as a constant i.e.

$$q=-\frac{a\ddot{a}}{\dot{a}^2}=n-1 \tag{18}$$

From equation (13) and (18) we get,

$$R=\left[n(ct+d)\right]^{\frac{4}{\alpha}} \tag{19}$$

$$Q=\left[n(ct+d)\right]^{\frac{4k}{\alpha}} \tag{20}$$

where, $\alpha=n(3+k)$.

Therefore, the metric takes the form as,

$$ds^2=\left[n(ct+d)\right]^{\frac{8}{\alpha}}\left(dx^2+e^{-2bx}dy^2+dz^2\right)+\left[n(ct+d)\right]^{\frac{4k}{\alpha}}dl^2-dt^2 \tag{21}$$

4 Physical aspect of the model

For the model (21) the expressions for the spatial volume, the mean Hubble's parameter, the expansion scalar, the deceleration parameter, the mean anisotropy parameter and the shear scalar are given by,

$$V = a^4 = \left[n(ct+d)\right]^{\frac{4(k+3)}{\alpha}} \tag{22}$$

$$H = \frac{(k+3)}{\alpha\left[n(ct+d)\right]} \tag{23}$$

$$\theta = \frac{4(k+3)}{\alpha\left[n(ct+d)\right]} \tag{24}$$

$$\sigma^2 = \frac{8(k+3)^2}{3\alpha^2\left[n(ct+d)\right]^2} \tag{25}$$

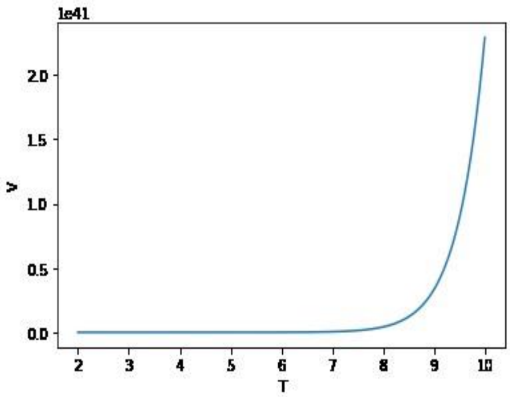

Fig. 1 : Spatial volume vs Time

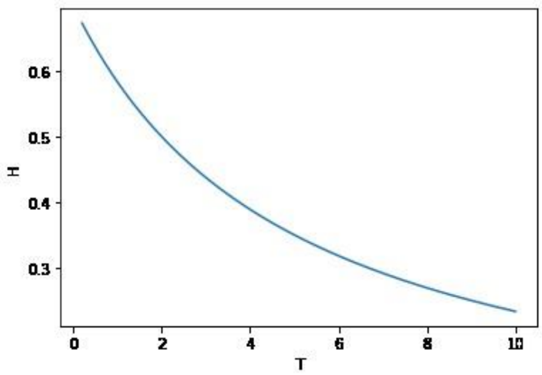

Fig.2 : Hubble parameter vs Time

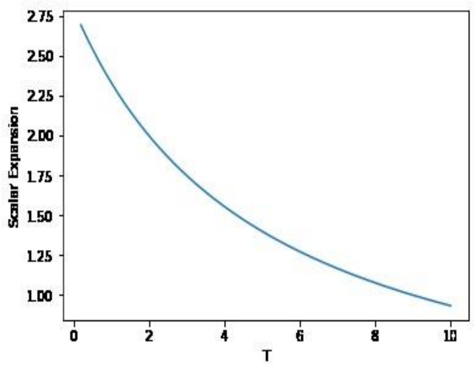

Fig. 3: Expansion scalar vs Time

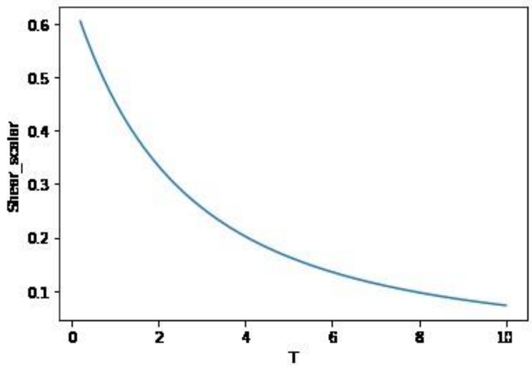

Fig.4 : Shear scalar vs Time

The energy density and the total pressure is given by,

$$\bar{p} = \frac{2l_1 + l_2}{(n(ct+d))^2} + \frac{16n^2c^2(1+2k)}{\alpha^2(n(ct+d))^2} - \frac{\omega}{2}\frac{c_1^2}{(n(ct+d))^{\frac{8(3+k)}{\alpha}}} \tag{26}$$

$$\rho = \frac{48n^2c^2(1+k)}{\alpha^2(n(ct+d))^2} - \frac{a^2}{(n(ct+d))^{\frac{8}{\alpha}}} + \frac{\omega c_1^2}{2(n(ct+d))^{\frac{8(3+k)}{\alpha}}} \tag{27}$$

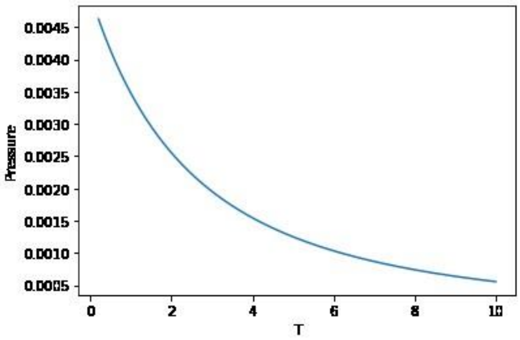

Fig.5: Variations of Pressure vs time

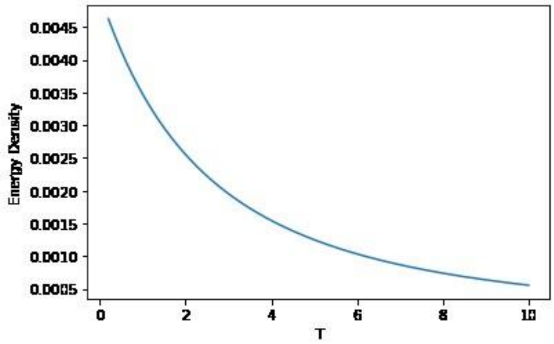

Fig.6: Variations of Energy density vs time

The string tension density and the scalar field is given by,

$$\lambda = \frac{l_1 - l_2}{(n(ct+d))^2} + \frac{32n^2c^2(1-k)}{\alpha^2(n(ct+d))^2} \tag{28}$$

$$\phi = \left[c_1\alpha \frac{(n+2)}{2} \frac{1}{(-(4(3+k)+\alpha)} \frac{1}{(n(ct+d))^{\frac{4(3+k)}{\alpha}}} \right]^{\frac{2}{n+2}} \tag{29}$$

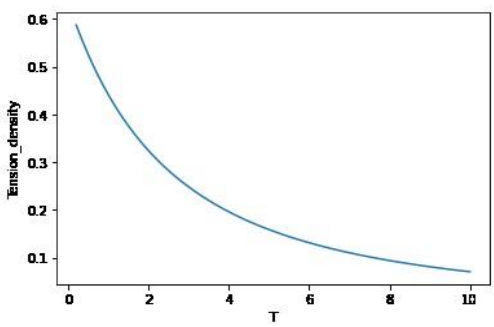

Fig. 7: Variations of Tension density vs Time

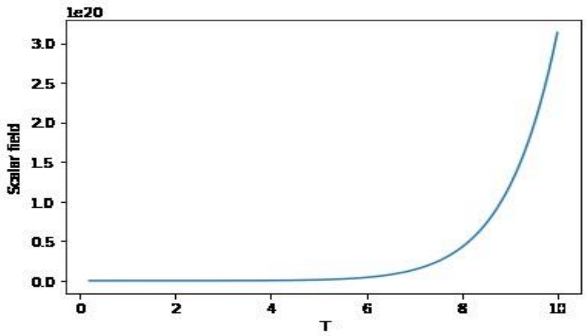

Fig.8: Variations of Scalar field vs Time

It may be observed that at an initial moment, when $T = 0$ the spatial volume is zero. Furthermore, the expansion scalar θ, the shear scalar σ^2 and the mean Hubble's parameter H tends to ∞ as $T \to \infty$. Since $\lim_{t\to\infty}\left(\sigma/\theta\right) \neq 0$ and hence the model does not approach isotropy for large values of T. The scalar field φ increases indefinitely as time $T \to \infty$ and is free from an initial singularity while the energy density diverges also the string tension density decreases as time increases.

Conclusion:

This research examines five-dimensional Bianchi type-III string with bulk viscous fluid cosmological models using the Saez-Ballester theory. The models is free from an initial singularities, expanding, shearing, and non-rotating. As T increases, the scalar expansion θ, the mean Hubble's parameter H and shear scalar decrease, eventually vanishing when T →∞.

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