The Integral Transform of k-Hypergeometric Series

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Abstract:

In this paper, we derive various integral transform, including Euler transform, Whittaker transform, Hankel transform of k-Hypergeometric series . Some results are expressed in terms of generalized Wright function. The transforms found here are likely to find useful in problem of Sciences, engineering and technology.

Keywords and Phrases: Euler transform, Whittaker transform, Hankel transform, Generalized Wright function, k-Hypergeometric Series.
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1 Introduction:

Throughout this work, N := {1, 2, 3,.. .} denotes the set of positive integers, N_0 = N ∪ {0}, Z– := {–1, –2, –3,.. .} denotes the set of negative integers, R+ denotes the set of positive real numbers, and C denotes the set of complex numbers.

In 1813, Gauss first summarized his investigations of hypergeometric functions, which has been of great significance in the mathematical modeling of physical phenomena and other applications. Recently, various developments and expansions of the hypergeometric functions have been proposed and discussed.

In [1], Diaz and Pariguan introduced an interesting extensions of the Gamma, Beta, Pochhammer, and hypergeometric functions as follows.

Definition 1.1 k-Gamma Function

$\Gamma_k(z)$ denotes the generalized k – gamma function [1] defined by

$$\Gamma_k(z) = \int_0^\infty e^{-\frac{t^k}{k}} t^{z-1} dt \quad (R(z) > 0; k \in R^+) \tag{1.1}$$

and

$$\Gamma_k(z) = \lim_{n\to\infty} \frac{n!k^n (nk)^{\frac{z}{k}-1}}{(z)_{n,k}}, \quad k \in R^+, z \in C \setminus kZ^- \tag{1.2}$$

which has the following relationship [8] :

$$\Gamma_k(z+k) = z\Gamma_k(z) \tag{1.3}$$

and

$$\Gamma_k(z) = k^{\frac{z}{k}-1}\Gamma\left(\frac{z}{k}\right) \tag{1.4}$$

$(z)_{n,k}$ is the k-Pochammer symbol introduced by Diaz and Pariguan [8] defined for complex $z \in C$ and $k \in R$ by

$$(z)_{n,k} = \begin{cases} 1 & if\, n = 0 \\ z(z+k)(z+2k).......(z+(n-1)k) & if\, n \in N \end{cases} = \frac{\Gamma_k(z+nk)}{\Gamma_k z} \tag{1.5}$$

For k=1, the generalized K-Wright function reduces to the generalized Wright function.

Definition 1.2 k- Beta Function

The k-Beta Function B^k (s,t) [2] is defined by

$$B^k(s,t) = \begin{cases} \frac{1}{k}\int_0^1 y^{\frac{s}{k}-1}(1-y)^{\frac{t}{k}-1}dy, & \left(k \in \Re^+, \min\{Re(s), Re(t) > 0\}\right) \\ \frac{\Gamma_k s\, \Gamma_k t}{\Gamma_k(s+t)}, & \left(k \in \Re^+, s,t \in C \setminus Z_0^-\right) \end{cases} \tag{1.6}$$

Clearly the case k = 1 in (1.6) reduces to known Beta Function

$$B(s,t) = \int_0^1 y^{s-1}(1-y)^{t-1}dy \tag{1.7}$$

Also, the relation between the k-beta function B^k (s, t) and the original beta function $B(s, t)$ is

$$B^k(s,t) = \frac{1}{k}B\left(\frac{s}{k}, \frac{t}{k}\right) \tag{1.8}$$

Definition 1.3 k- Hypergeometric Series

Let $k \in \Re^+$ and $\alpha_1, \alpha_2, y \in C$ and $\alpha_3 \in C \setminus Z_0^-$, then k – Hypergeometric series is defined by the form

$${}_2H_1^k\left[\begin{matrix}(\alpha_1;k),(\alpha_2;k) \\ (\alpha_3;k);\end{matrix}\right] = \sum_{n=0}^{\infty} \frac{(\alpha_1)_{n,k}(\alpha_2)_{n,k}}{(\alpha_3)_{n,k}} \frac{y^n}{n!} \tag{1.9}$$

where $(\alpha_1)_{n,k}$ is the k – Pochhammer symbol given in (1.5).

Indeed in their special case when k = 1 , Eq. (1.9) is reduced to the Gauss Hypergeometric function ${}_2H_1(.)$. The ${}_2H_1(.)$ is the special case of the generalized hypergeometric functions ${}_mH_n(.)$ of m numerator and n denominator parameters defined by [3].

Definition 1.4 Fox – Wright Generalized Hypergeometric Function

In 1933, E. M. Wright defined a more interesting generalized hypergeometric function of one variable [4] and further generalizations of the series ${}_pF_q$ were given by Fox [5] and Wright [6,7,8] ;

$$ {}_p\psi_q(z) = {}_p\psi_q\left[\begin{matrix}(\alpha_1, A_1),\ldots,(\alpha_p, A_p)\\(\beta_1, B_1),\ldots,(\beta_q, B_q)\end{matrix}\middle| z\right] $$

$$ = \sum_{n=0}^{\infty}\frac{\Gamma(\alpha_1 + A_1 n)\Gamma(\alpha_2 + A_2 n)\ldots\Gamma(\alpha_p + A_p n)}{\Gamma(\beta_1 + B_1 n)\Gamma(\beta_2 + B_2 n)\ldots\Gamma(\beta_q + B_q n)}\frac{z^n}{n!} \tag{1.10} $$

where the coefficients $A_1,\ldots,A_p \in R^+$ and $B_1,\ldots,B_q \in R^+$ such that

$$ 1+\sum_{j=1}^{q}B_j - \sum_{i=1}^{p}A_i \geq 0 $$

for suitably bounded values of $|z|$. $\alpha_1,\alpha_2,\ldots,\alpha_p,\beta_1,\beta_2,\ldots,\beta_q$ are complex parameters.

The Fox - Wright function is a special case of the Fox – H function as [9]

$$ {}_p\psi_q\left[\begin{matrix}(\alpha_1, A_1),\ldots,(\alpha_p, A_p)\\(\beta_1, B_1),\ldots,(\beta_q, B_q)\end{matrix}\middle| z\right] = H^{1,p}_{p,q+1}\left[-z\middle|\begin{matrix}(1-\alpha_1, A_1),\ldots,(1-\alpha_p, A_p)\\(1-\beta_1, B_1),\ldots,(1-\beta_q, B_q)\end{matrix}\right] \tag{1.11} $$

Definition 1.5 The Euler Transform ([10], see also [11])

The Euler transform of a function f(z) is defined as

$$ B\{f(z); a,b\} = \int_0^1 z^{a-1}(1-z)^{b-1} f(z)dz \qquad a,b \in C, R(a) > 0, R(b) > 0 \tag{1.12} $$

Definition 1.6 The Whittaker Transform (Whittaker and Watson [12])

$$ \int_0^\infty e^{-\frac{t}{2}} t^{\zeta-1} W_{\alpha,\beta}(t)dt = \frac{\Gamma\left(\frac{1}{2}+\beta+\zeta\right)\Gamma\left(\frac{1}{2}-\beta+\zeta\right)}{\Gamma(1-\alpha+\zeta)} \tag{1.13} $$

Where $R(\beta \pm \zeta) > \frac{-1}{2}$ and $W_{\alpha,\beta}(t)$ is the Whittaker confluent Hypergeometric function

$$ W_{\beta,\zeta}(z) = \frac{\Gamma(-2\beta)}{\Gamma(1/2-\alpha-\beta)}M_{\alpha,\beta}(z) + \frac{\Gamma(2\beta)}{\Gamma(1/2+\alpha+\beta)}M_{\alpha,-\beta}(z) \tag{1.14} $$

where $M_{\alpha,\beta}(z)$ is given by

$$M_{\alpha,\beta}(z)=z^{1/2+\beta}e^{-1/2z}{}_1F_1\left(\frac{1}{2}+\beta-\alpha;2\beta+1;z\right) \tag{1.15}$$

Definition 1.7 The Hankel Transform [13]

The Hankel transform of $f(x)$, denoted by $g(p;v)$ is defined as

$$g(p;v)=\int_0^\infty (px)^{\frac{1}{2}}\ J_v(px)f(x)dx;\quad p>0 \tag{1.16}$$

The following formula can be used to solve the integral in equation (1.12) (see [9] p.56-57)

$$\int_0^\infty x^{\lambda-1}J_v(ax)dx=2^{\lambda-1}a^{-\lambda}\ \frac{\Gamma\left(\frac{\lambda+v}{2}\right)}{\Gamma\left(1+\frac{v-\lambda}{2}\right)}$$

2 Integral Transforms of ${}_2H_1^{(p,k)}$:

Theorem 2.1: (Euler Transform)

$$B\{f(z);a,b\}=\int_0^1 z^{a-1}(1-z)^{b-1}f(z)dz$$

The Euler transform of k- hypergeometric series

$$B\left\{{}_2H_1^{(p,k)}\left[\begin{matrix}(\alpha_1;k),(\alpha_2;k)\\(\alpha_3;k)\end{matrix}\ \ z\right]m,n\right\}=\int_0^1 z^{m-1}(1-z)^{n-1}{}_2H_1^{(p,k)}\left[\begin{matrix}(\alpha_1;k),(\alpha_2;k)\\(\alpha_3;k)\end{matrix}\ \ z\right]dz$$

By using equation (1.9)

$$=\int_0^1 z^{m-1}(1-z)^{n-1}\sum_{l=0}^{\infty}\frac{(\alpha_1)_{l,k}(\alpha_2)_{l,k}}{(\alpha_3)_{l,k}}\frac{z^l}{(pl)!}dz$$

$$=\sum_{l=0}^{\infty}\frac{(\alpha_1)_{l,k}(\alpha_2)_{l,k}}{(\alpha_3)_{l,k}}\frac{1}{(pl)!}\int_0^1 z^{m+l-1}(1-z)^{n-1}dz$$

$$=\sum_{l=0}^{\infty}\frac{(\alpha_1)_{l,k}(\alpha_2)_{l,k}}{(\alpha_3)_{l,k}}\frac{1}{(pl)!}B(m+l,n)$$

By using equation (1.5)

$$=\sum_{l=0}^{\infty}\frac{\Gamma_k(\alpha_1+lk)\Gamma_k(\alpha_2+lk)\Gamma_k\alpha_3}{\Gamma_k\alpha_1\Gamma_k\alpha_2\Gamma_k(\alpha_3+lk)}\frac{1}{(pl)!}\frac{\Gamma(m+l)\Gamma n}{\Gamma(m+l+n)}$$

By using equation (1.4) and multiply and divide the above equation with $l!$

$$=\frac{\Gamma\frac{\alpha_3}{k}\ \Gamma n}{\Gamma\frac{\alpha_1}{k}\ \Gamma\frac{\alpha_2}{k}}\sum_{l=0}^{\infty}\frac{\Gamma\left(\frac{\alpha_1}{k}+l\right)\Gamma\left(\frac{\alpha_2}{k}+l\right)\Gamma(m+l)\Gamma(l+1)}{\Gamma\left(\frac{\alpha_3}{k}+l\right)\Gamma(pl+1)\Gamma(m+n+l)}\ \frac{k^l}{l!}$$

$$B\left\{ {}_2H_1^{(p,k)}\left[\begin{matrix}(\alpha_1;k),(\alpha_2;k)\\(\alpha_3;k)\end{matrix}\quad z\right]m,n\right\}=\frac{\Gamma\frac{\alpha_3}{k}\ \Gamma n}{\Gamma\frac{\alpha_1}{k}\ \Gamma\frac{\alpha_2}{k}}\ {}_4\psi_3\left[\left.\begin{matrix}(\frac{\alpha_1}{k},1);(\frac{\alpha_2}{k},1);(m,1);(1,1)\\\left(\frac{\alpha_3}{k},1\right);(1,p);(m+n,1)\end{matrix}\right|k\right]$$

Corollary

Use the relation between ψ function and Fox - H function, we get

$$B\left\{ {}_2H_1^{(p,k)}\left[\begin{matrix}(\alpha_1;k),(\alpha_2;k)\\(\alpha_3;k)\end{matrix}\quad z\right]m,n\right\}=\frac{\Gamma\frac{\alpha_3}{k}\ \Gamma n}{\Gamma\frac{\alpha_1}{k}\ \Gamma\frac{\alpha_2}{k}}H_{4,4}^{1,4}\left[-k\left|\begin{matrix}(1-\frac{\alpha_1}{k},1);(1-\frac{\alpha_2}{k},1);(1-m,1);(0,1)\\\left(1-\frac{\alpha_3}{k},1\right);(0,p);(1-m-n,1)\end{matrix}\right.\right]$$

Theorem 2.2: (Whittaker Transform)

$$\int_0^\infty e^{-\frac{t}{2}}t^{\zeta-1}W_{\alpha,\beta}(t)dt=\frac{\Gamma\left(\frac{1}{2}+\beta+\zeta\right)\Gamma\left(\frac{1}{2}-\beta+\zeta\right)}{\Gamma(1-\alpha+\zeta)}$$

The Whittaker transform of k-Hypergeometric series

$$W\left\{ {}_2H_1^{(p,k)}\left[\begin{matrix}(\alpha_1;k),(\alpha_2;k)\\(\alpha_3;k)\end{matrix}\quad t\right]\right\}=\int_0^\infty t^{\rho-1}e^{-\frac{\eta t}{2}}\mathrm{W}_{\lambda,\mu}(\eta t)\left\{ {}_2H_1^{(p,k)}\left[\begin{matrix}(\alpha_1;k),(\alpha_2;k)\\(\alpha_3;k)\end{matrix}\quad t\right]\right\}dt$$

$$W\left\{ {}_2H_1^{(p,k)}\left[\begin{matrix}(\alpha_1;k),(\alpha_2;k)\\(\alpha_3;k)\end{matrix}\quad t\right]\right\}=\int_0^\infty t^{\rho-1}e^{-\frac{\eta t}{2}}\mathrm{W}_{\lambda,\mu}(\eta t)\sum_{l=0}^{\infty}\frac{(\alpha_1)_{l,k}(\alpha_2)_{l,k}}{(\alpha_3)_{l,k}}\frac{t^l}{(pl)!}dt$$

$$=\sum_{l=0}^{\infty}\frac{(\alpha_1)_{l,k}(\alpha_2)_{l,k}}{(\alpha_3)_{l,k}}\frac{1}{(pl)!}\int_0^\infty t^{l+\rho-1}e^{-\frac{\eta t}{2}}\mathrm{W}_{\lambda,\mu}(\eta t)dt$$

Put $\eta t=\delta$ and $dt=\frac{d\delta}{\eta}$, we get

$$W\left\{ {}_2H_1^{(p,k)}\left[\begin{matrix}(\alpha_1;k),(\alpha_2;k)\\(\alpha_3;k)\end{matrix}\quad t\right]\right\}=\sum_{l=0}^{\infty}\frac{(\alpha_1)_{l,k}(\alpha_2)_{l,k}}{(\alpha_3)_{l,k}}\frac{1}{(pl)!}\int_0^\infty\left(\frac{\delta}{\eta}\right)^{l+\rho-1}e^{-\frac{\delta}{2}}\mathrm{W}_{\lambda,\mu}(\delta)\frac{d\delta}{\eta}$$

$$=\sum_{l=0}^{\infty}\frac{(\alpha_1)_{l,k}(\alpha_2)_{l,k}}{(\alpha_3)_{l,k}}\frac{1}{(pl)!}\frac{1}{\eta^{\rho+l}}\int_0^\infty\delta^{l+\rho-1}e^{-\frac{\delta}{2}}\mathrm{W}_{\lambda,\mu}(\delta)d\delta$$

By definition of Whittaker transform

$$=\sum_{l=0}^{\infty}\frac{(\alpha_1)_{l,k}(\alpha_2)_{l,k}}{(\alpha_3)_{l,k}}\frac{1}{(pl)!}\frac{1}{\eta^{\rho+l}}\frac{\Gamma\left(\frac{1}{2}+\mu+\rho+l\right)\Gamma\left(\frac{1}{2}-\mu+\rho+l\right)}{\Gamma(1-\lambda+\rho+l)}$$

By using equation (1.5)

$$=\sum_{l=0}^{\infty}\frac{\Gamma_k(\alpha_1+lk)\Gamma_k(\alpha_2+lk)\Gamma_k\alpha_3}{\Gamma_k\alpha_1\Gamma_k\alpha_2\Gamma_k(\alpha_3+lk)}\frac{1}{(pl)!}\frac{1}{\eta^{\rho+l}}\frac{\Gamma\left(\frac{1}{2}+\mu+\rho+l\right)\Gamma\left(\frac{1}{2}-\mu+\rho+l\right)}{\Gamma(1-\lambda+\rho+l)}$$

By using equation (1.4) and multiply and divide by $l!$

$$=\frac{\Gamma\frac{\alpha_3}{k}}{\eta^{\rho}\Gamma\frac{\alpha_1}{k}\ \Gamma\frac{\alpha_2}{k}}\sum_{l=0}^{\infty}\frac{\Gamma\left(\frac{\alpha_1}{k}+l\right)\Gamma\left(\frac{\alpha_2}{k}+l\right)\Gamma\left(\frac{1}{2}+\mu+\rho+l\right)\Gamma\left(\frac{1}{2}-\mu+\rho+l\right)\Gamma(l+1)}{\Gamma\left(\frac{\alpha_3}{k}+l\right)\Gamma(pl+1)\Gamma(1-\lambda+\rho+l)}\ \frac{\left(\frac{k}{\eta}\right)^l}{l!}$$

$$W\left\{{}_2H_1^{(p,k)}\left[\begin{matrix}(\alpha_1;k),(\alpha_2;k)\\(\alpha_3;k)\end{matrix}\ t\right]\right\}=\frac{\Gamma\frac{\alpha_3}{k}}{\eta^{\rho}\Gamma\frac{\alpha_1}{k}\ \Gamma\frac{\alpha_2}{k}}\times$$

$${}_5\psi_3\left[\left.\begin{matrix}(\frac{\alpha_1}{k},1);(\frac{\alpha_2}{k},1);\left(\frac{1}{2}+\mu+\rho,1\right);\left(\frac{1}{2}-\mu+\rho,1\right);(1,1)\\\left(\frac{\alpha_3}{k},1\right);(1,p);(1-\lambda+\rho,1)\end{matrix}\right|\frac{k}{\eta}\right]$$

Corollary

Use the relation between ψ function and Fox-H function

$$W\left\{{}_2H_1^{(p,k)}\left[\begin{matrix}(\alpha_1;k),(\alpha_2;k)\\(\alpha_3;k);\end{matrix}\ t\right]\right\}=\frac{\Gamma\frac{\alpha_3}{k}}{\eta^{\rho}\Gamma\frac{\alpha_1}{k}\ \Gamma\frac{\alpha_2}{k}}\times$$

$$H_{5,4}^{1,5}\left[-\frac{k}{\eta}\left|\begin{matrix}(1-\frac{\alpha_1}{k},1);(1-\frac{\alpha_2}{k},1);\left(\frac{1}{2}-\mu-\rho,1\right);\left(\frac{1}{2}+\mu-\rho,1\right);(0,1)\\\left(1-\frac{\alpha_3}{k},1\right);(0,p);(\lambda-\rho,1)\end{matrix}\right.\right]$$

Theorem 2.3: (Hankel Transform)

The Hankel transform of $f(x)$, denoted by $g(p;v)$ is defined as

$$g(p;v)=\int_0^{\infty}(px)^{\frac{1}{2}}\ J_v(px)f(x)dx;\quad p>0$$

The Hankel transform of k-Hypergeometric series

$$H\left\{{}_2H_1^{(p,k)}\left[\begin{matrix}(\alpha_1;k),(\alpha_2;k)\\(\alpha_3;k)\end{matrix}\ x\right]\right\}=\int_0^{\infty}x^{\tau-1}(qx)^{\frac{1}{2}}J_{\upsilon}(qx)\left\{{}_2H_1^{(p,k)}\left[\begin{matrix}(\alpha_1;k),(\alpha_2;k)\\(\alpha_3;k)\end{matrix}\ x\right]\right\}dx$$

$$=\int_0^\infty x^{\tau-1}(qx)^{\frac{1}{2}}J_\upsilon(qx)\left\{\sum_{l=0}^{\infty}\frac{(\alpha_1)_{l,k}(\alpha_2)_{l,k}}{(\alpha_3)_{l,k}}\frac{x^l}{(pl)!}\right\}dx$$

$$=\sum_{l=0}^{\infty}\frac{(\alpha_1)_{l,k}(\alpha_2)_{l,k}}{(\alpha_3)_{l,k}}\frac{q^{1/2}}{(pl)!}\int_0^\infty x^{\frac{1}{2}+\tau+l-1}J_\upsilon(qx)dx$$

Using the following formula to solve the integral

$$\int_0^\infty x^{\lambda-1}J_\nu(ax)dx=2^{\lambda-1}a^{-\lambda}\frac{\Gamma\left(\frac{\lambda+\nu}{2}\right)}{\Gamma\left(1+\frac{\nu-\lambda}{2}\right)}$$

We have

$$H\left\{{}_2H_1^{(p,k)}\left[\begin{matrix}(\alpha_1;k),(\alpha_2;k)\\(\alpha_3;k)\end{matrix}\;x\right]\right\}=\sum_{l=0}^{\infty}\frac{(\alpha_1)_{l,k}(\alpha_2)_{l,k}}{(\alpha_3)_{l,k}}\frac{q^{1/2}}{(pl)!}\times2^{\frac{1}{2}+\tau+l-1}\times q^{-\frac{1}{2}-\tau-l}\times\frac{\Gamma\left(\frac{\frac{1}{2}+\tau+l+\nu}{2}\right)}{\Gamma\left(1+\frac{\nu-\frac{1}{2}-\tau-l}{2}\right)}$$

$$=\sum_{l=0}^{\infty}\frac{\Gamma_k(\alpha_1+lk)\Gamma_k(\alpha_2+lk)\Gamma_k\alpha_3}{\Gamma_k\alpha_1\Gamma_k\alpha_2\Gamma_k(\alpha_3+lk)}\frac{q^{1/2}}{(pl)!}\times2^{\frac{1}{2}+\tau+l-1}\times q^{-\frac{1}{2}-\tau-l}\times\frac{\Gamma\left(\frac{\frac{1}{2}+\tau+l+\nu}{2}\right)}{\Gamma\left(1+\frac{\nu-\frac{1}{2}-\tau-l}{2}\right)}$$

By using equation (1.4) and multiply and divide by $l!$

$$=\frac{2^{\tau-\frac{1}{2}}\Gamma\frac{\alpha_3}{k}}{q^\tau\Gamma\frac{\alpha_1}{k}\;\Gamma\frac{\alpha_2}{k}}\sum_{l=0}^{\infty}\frac{\Gamma\left(\frac{\alpha_1}{k}+l\right)\Gamma\left(\frac{\alpha_2}{k}+l\right)\Gamma\left(\frac{\frac{1}{2}+\tau+\nu}{2}+\frac{l}{2}\right)\Gamma(l+1)}{\Gamma\left(\frac{\alpha_3}{k}+l\right)\Gamma(pl+1)\Gamma\left(\frac{\frac{3}{2}+\nu-\tau}{2}-\frac{l}{2}\right)}\;\frac{\left(\frac{2k}{q}\right)^l}{l!}$$

$$H\left\{ {}_2H_1^{(p,k)}\left[\begin{matrix}(\alpha_1;k),(\alpha_2;k)\\(\alpha_3;k)\end{matrix}\; t\right]\right\} = \frac{2^{\tau-\frac{1}{2}}\Gamma\frac{\alpha_3}{k}}{q^{\tau}\Gamma\frac{\alpha_1}{k}\;\Gamma\frac{\alpha_2}{k}}\; {}_4\psi_3\left[\left.\begin{matrix}(\frac{\alpha_1}{k},1);(\frac{\alpha_2}{k},1);\left(\frac{\frac{1}{2}+\tau+\nu}{2},\frac{1}{2}\right);(1,1)\\\left(\frac{\alpha_3}{k},1\right);(1,p);\left(\frac{\frac{3}{2}+\nu-\tau}{2},\frac{-1}{2}\right)\end{matrix}\right| \frac{2k}{q}\right]$$

Corollary

By using relation between ψ function and Fox H function

$$H\left\{ {}_2H_1^{(p,k)}\left[\begin{matrix}(\alpha_1;k),(\alpha_2;k)\\(\alpha_3;k)\end{matrix}\; t\right]\right\} = \frac{2^{\tau-\frac{1}{2}}\Gamma\frac{\alpha_3}{k}}{q^{\tau}\Gamma\frac{\alpha_1}{k}\;\Gamma\frac{\alpha_2}{k}} H_{4,4}^{1,4}\left[-\frac{2k}{q}\left|\begin{matrix}(1-\frac{\alpha_1}{k},1);(1-\frac{\alpha_2}{k},1);\left(1-\frac{\frac{1}{2}+\tau+\nu}{2},\frac{1}{2}\right);(0,1)\\\left(1-\frac{\alpha_3}{k},1\right);(0,p);\left(1-\frac{\frac{3}{2}+\nu-\tau}{2},\frac{-1}{2}\right)\end{matrix}\right.\right]$$

3 Concluding Remarks:

The unique insights obtained from this research can be further refined in numerous original and well-known integral transformations that are useful in applied mathematics, science, engineering, and bioengineering, among other domains. Moreover, several extensions of the main results are considered. Several integral transformations are obtained from the current work and can be computed using the k-Hypergeometric series in terms of the Fox-Wright function.

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