On Open and Closed Sets in Fuzzy Bitopological Space

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Abstract:

The purpose of this paper is to define and study a new concept of fuzzy $(1,2)^*$-β^*open and fuzzy $(1,2)^*$-β^* closed sets in fuzzy bitopological space. Also, some basic concepts and properties of them are investigated. Some theorems and examples for $(1,2)^*$-β^*open and $(1,2)^*$-β^* closed sets are introduced.

Keyword:
$(1,2)^*$-β^*open sets, $(1,2)^*$-β^* closed sets, $(1,2)^*$-β^* closure, fuzzy bitopological space.

Introduction:

Zadeh [4] was introduced the fuzzy sets and Chang [1] was initiated the fuzzy topology. Khalik [3] was study on certain types of fuzzy separation axioms in fuzzy topological space on fuzzy sets. The concept of bitopological spaces was introduced by J. C. Kelly [5]. Kandil [6] introduced the fuzzy bitopological spaces as a natural generalization of Chang's fuzzy topological spaces. The concept of β^*-open set and β^*-closed set in fuzzy topological space are introduced by R. Thangappan [8].

In this paper, we introduce and study a new class of fuzzy sets in a fuzzy bitopological space (X,τ_1,τ_2), namely $(1,2)^*$-β^* open sets and closed sets and its properties are studied.

2. Preliminaries:

Throughout this paper (X, τ_1,τ_2) or simply X denote the fuzzy bitopological spaces (briefly, FBTS). For a subset A of a space X, the closure, interior and complement of A are denoted by cl(A), int (A) and A^c respectively. We recall some basic definitions that are used in the sequel.

Definition 2.1[1]:

A fuzzy topology (briefly FT) on X is a family τ of FSs in X satisfying the following axioms.
1. 0~, 1~ $\in \tau$
2. $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$
3. $\cup G_i \in \tau$ for any family $\{G_i / i \in J\} \subseteq \tau$.

In this state the pair (X,τ) is called a fuzzy topological space (briefly FTS) and any FS in τ is known as a fuzzy open set (briefly FOS) in X. The complement of a fuzzy open set is called a fuzzy closed set (briefly FCS) in X.

Definition 2.2[6]:

Let τ_1 and τ_2 be two fuzzy topologies on a non-empty set X. The triple (X,τ_1,τ_2) is called a fuzzy bitopological spaces (briefly FBTS), every member of $\tau_{1,2}$ is called $\tau_{1,2}$- fuzzy open sets
($\tau_{1,2}$-FOS) and the complement of $\tau_{1,2}$-FOS is $\tau_{1,2}$- fuzzy closed sets ($\tau_{1,2}$-FCS).

Definition 2.3[9]:

A Fuzzy Set A = <X, μ_A > in an FBTS (X,τ_1,τ_2) is said to be an

- $(1,2)^*$-fuzzy semi-open set ($(1,2)^*$-FSOS) if A $\subseteq \tau_{1,2}$- $cl(\tau_{1,2}$- $int(A))$
- $(1,2)^*$-fuzzy semi-closed set ($(1,2)^*$-FSCS) if $\tau_{1,2}$-$int(\tau_{1,2}$- $cl(A)) \subseteq A$
- $(1,2)^*$-fuzzy α-open set ($(1,2)^*$-FαOS) if A $\subseteq \tau_{1,2}$- $int(\tau_{1,2}$- $cl(\tau_{1,2}$- $int(A))$
- $(1,2)^*$-fuzzy α-closed set ($(1,2)^*$-FαCS) if $\tau_{1,2}$-$cl(\tau_{1,2}$-$int(\tau_{1,2}$-cl(A)) $\subseteq$ A
- $(1,2)^*$-fuzzy regular open set ($(1,2)^*$-FROS) if A = $\tau_{1,2}$- $int(\tau_{1,2}$- $cl(A))$
- $(1,2)^*$-fuzzy regular closed set ($(1,2)^*$-FRCS) if A = $\tau_{1,2}$- $cl(\tau_{1,2}$- $int(A))$
- $(1,2)^*$-fuzzy e-open if $A \leq \tau_{1,2}$- $cl(int_\delta(A)) \vee \tau_{1,2}$-$int(cl_\delta(A))$
- $(1,2)^*$-fuzzy e-closed if $A \geq \tau_{1,2}$- $cl(int_\delta(A)) \wedge \tau_{1,2}$-$int(cl_\delta(A))$

3. Fuzzy $(1,2)^*$-β^* Open Sets in Fuzzy Bitopological Space

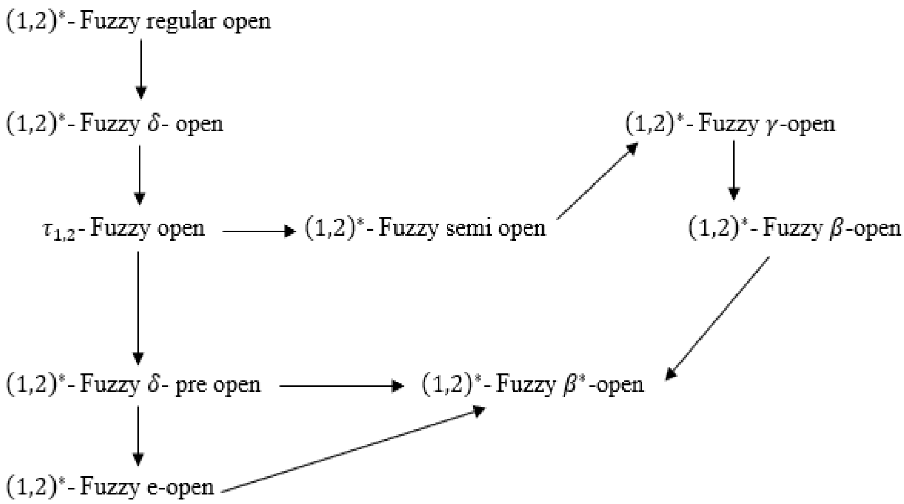
Definition 3.1:

A fuzzy subset A of a fuzzy bitopological space (X, τ_1, τ_2) is said to be $(1,2)^*$- β^*open set if $A \leq \tau_{1,2}$-cl ($\tau_{1,2}$-int ($\tau_{1,2}$-cl (A))) $\vee$ $\tau_{1,2}$-int ($\tau_{1,2}$-cl_δ (A)).

Example 3.2:

Let X = {x, y, z} and the fuzzy bitopology τ_1 = { 0, 1, $< x_{0.4}, y_{0.1}, z_{0.1} >$, $< x_{0.8}, y_{0.9}, z_{0.9} >$}, τ_2= { 0, 1, $< x_{0.2}, y_{0.1}, z_{0.1} >$, $< x_{0.6}, y_{0.9}, z_{0.9} >$, $< x_{0.6}, y_{0.5}, z_{0.7} >$} and τ_1^c = { 0, 1, $< x_{0.6}, y_{0.9}, z_{0.9} >$, $< x_{0.2}, y_{0.1}, z_{0.1} >$}, τ_2^c= { 0, 1, $< x_{0.8}, y_{0.9}, z_{0.9} >$, $< x_{0.4}, y_{0.1}, z_{0.1} >$, $< x_{0.4}, y_{0.5}, z_{0.3} >$}. Let $A = < x_{0.8}, y_{0.9}, z_{0.9} >$ then $\tau_{1,2}$-cl ($\tau_{1,2}$-int ($\tau_{1,2}$-cl (A))) $\vee$
$\tau_{1,2}$-int ($\tau_{1,2}$-cl_δ (A)) = $< x_{0.8}, y_{0.9}, z_{0.9} >$. Hence A is $(1,2)^*$- β^*open set.

Remark 3.3:



Result 3.4:

(i) Fuzzy $(1,2)^*$- β^*open is fuzzy $(1,2)^*$- δ preopen if $\tau_{1,2}$-cl ($\tau_{1,2}$-int ($\tau_{1,2}$-cl (A))) = 0.

(ii) Fuzzy $(1,2)^*$- β^*open is $(1,2)^*$- β open if $\tau_{1,2}$-int ($cl_\delta(A)$) = 0.

Proposition 3.5:

If A is fuzzy $(1,2)^*$- δ preopen and B is fuzzy $(1,2)^*$- β open then $A \vee B$ is fuzzy $(1,2)^*$- β^*open.

Proof. Obvious from Definition 3.1

Proposition 3.6:

Let (X, τ_1, τ_2) be a fuzzy bitopological space. Then the union of any two fuzzy $(1,2)^*$- β^*open sets is an $(1,2)^*$- β^*open set.

Proof:

Let A_1, A_2 be two fuzzy $(1,2)^*$- β^*open sets,

$A_1 \leq \tau_{1,2}$-cl ($\tau_{1,2}$-int ($\tau_{1,2}$-cl (A_1))) $\vee$ $\tau_{1,2}$-int ($\tau_{1,2}$-cl_δ (A_1)). and $A_2 \leq \tau_{1,2}$-cl ($\tau_{1,2}$-int ($\tau_{1,2}$-cl (A_2))) $\vee$ $\tau_{1,2}$-int ($\tau_{1,2}$-cl_δ (A_2)). (by Definition 3.1)

Then we have,

$A_1 \vee A_2 \leq \tau_{1,2}$-cl ($\tau_{1,2}$-int ($\tau_{1,2}$-cl ($A_1$))) $\vee$ $\tau_{1,2}$-int ($\tau_{1,2}$-cl_δ (A_1))

$\vee$ $\tau_{1,2}$-cl ($\tau_{1,2}$-int ($\tau_{1,2}$-cl (A_2))) $\vee$ $\tau_{1,2}$-int ($\tau_{1,2}$-cl_δ (A_2)).

$A_1 \vee A_2 \leq \tau_{1,2}$-cl ($\tau_{1,2}$-int ($\tau_{1,2}$-cl ($A_1 \vee A_2$))) $\vee$ $\tau_{1,2}$-int ($\tau_{1,2}$-cl_δ ($A_1 \vee A_2$)
Since, the arbitrary union of fuzzy $(1,2)^*$- β^*open sets is fuzzy $(1,2)^*$- β^*open set.

Theorem 3.7:

Let (X, τ_1, τ_2) be a fuzzy bitopological space and let $\{A_\alpha\}_{\alpha\in\mathcal{F}}$ be the collection of fuzzy $(1,2)^*$- β^* open sets in fuzzy bitopological space X, then $\bigvee_{\alpha\in\mathcal{F}}(A_\alpha)$ is fuzzy $(1,2)^*$- β^* open set.

Proof:

Let F be the collection of fuzzy $(1,2)^*$- β^*open sets in fuzzy bitopological space (X, τ_1, τ_2). For each $\alpha \in \mathcal{F}$, $A_\alpha \leq \tau_{1,2}$-cl ($\tau_{1,2}$-int ($\tau_{1,2}$-cl (A_α))) $\vee$ $\tau_{1,2}$-int ($\tau_{1,2}$-cl_δ (A_α)).
Thus, $\bigvee_{\alpha\in\mathcal{F}}(A_\alpha) \leq \bigvee_{\alpha\in\mathcal{F}} \tau_{1,2} - \text{cl}\,(\tau_{1,2} - \text{int}\,(\tau_{1,2} - \text{cl}\,(A_\alpha))) \vee \tau_{1,2} - \text{int}\,(\tau_{1,2} - cl_\delta\,(A_\alpha))$

$\bigvee_{\alpha\in\mathcal{F}}(A_\alpha) \leq \tau_{1,2}$-cl ($\tau_{1,2}$-int ($\tau_{1,2}$-cl ($\bigvee_{\alpha\in\mathcal{F}}$) ($A_\alpha$))) $\vee$ $\tau_{1,2}$-int ($\tau_{1,2}$-cl_δ ($\bigvee_{\alpha\in\mathcal{F}}$) ($A_\alpha$)).
Since, the arbitrary union of fuzzy $(1,2)^*$- β^*open sets is fuzzy $(1,2)^*$- β^*open set.

Theorem 3.8:

Let (X, τ_1, τ_2) and (Y, σ_1, σ_2) be any two fuzzy bitopological spaces such that X is product related to Y. Then the product $A_1 \times A_2$ of a fuzzy $(1,2)^*$- β^* open set A_1 of X and fuzzy $(1,2)^*$- β^* open set A_2 of Y is fuzzy $(1,2)^*$- β^*open set of the fuzzy product space X ×Y.

Proof:

Let A_1, A_2 are two fuzzy $(1,2)^*$- β^*open sets of X and Y respectively,

From Definition 3.1, $A_1 \leq \tau_{1,2}$-cl ($\tau_{1,2}$-int ($\tau_{1,2}$-cl (A_1))) $\vee$ $\tau_{1,2}$-int ($\tau_{1,2}$-cl_δ (A_1)) and $A_2 \leq \tau_{1,2}$-cl ($\tau_{1,2}$-int ($\tau_{1,2}$-cl (A_2))) $\vee$ $\tau_{1,2}$-int ($\tau_{1,2}$-cl_δ (A_2)). Then we have,

$A_1 \times A_2 \leq \tau_{1,2}$-cl ($\tau_{1,2}$-int ($\tau_{1,2}$-cl ($A_1$))) $\vee$ $\tau_{1,2}$-int ($\tau_{1,2}$-cl_δ (A_1))

$\times$ $\tau_{1,2}$-cl ($\tau_{1,2}$-int ($\tau_{1,2}$-cl (A_2))) $\vee$ $\tau_{1,2}$-int ($\tau_{1,2}$-cl_δ (A_2)).

$A_1 \times A_2 \leq \tau_{1,2}$-cl ($\tau_{1,2}$-int ($\tau_{1,2}$-cl ($A_1 \times A_2$))) $\vee$ $\tau_{1,2}$-int ($\tau_{1,2}$-cl_δ ($A_1 \times A_2$) is fuzzy $(1,2)^*$- β^* open in the fuzzy product space X ×Y.

4. Fuzzy $(1,2)^*$-β^* Closed sets in Fuzzy Bitopological Space.

Definition 4.1:

A fuzzy subset A of a fuzzy bitopological space (X, τ_1, τ_2) is said to be $(1,2)^*$- β^*closed set if $A \geq \tau_{1,2}$-int ($\tau_{1,2}$-cl ($\tau_{1,2}$-int (A))) $\wedge$ $\tau_{1,2}$-cl ($\tau_{1,2}$-int_δ (A)).

Example 4.2:

Let X = {x, y, z} and the fuzzy bitopology τ_1 = { 0, 1, $< x_{0.6}, y_{0.9}, z_{0.9} >, < x_{0.2}, y_{0.1}, z_{0.1} >$}, τ_2= { 0, 1, $< x_{0.8}, y_{0.9}, z_{0.9} >, < x_{0.4}, y_{0.1}, z_{0.1} >, < x_{0.4}, y_{0.5}, z_{0.3} >$} and τ_1^c = { 0, 1, $< x_{0.4}, y_{0.1}, z_{0.1} >, < x_{0.8}, y_{0.9}, z_{0.9} >$}, τ_2^c= { 0, 1,$< x_{0.2}, y_{0.1}, z_{0.1} >, < x_{0.6}, y_{0.9}, z_{0.9} >, < x_{0.6}, y_{0.5}, z_{0.7} >$}. Let $A = < x_{0.8}, y_{0.9}, z_{0.9} >$ then $\tau_{1,2}$-int ($\tau_{1,2}$-cl ($\tau_{1,2}$-int (A))) $\wedge$ $\tau_{1,2}$-cl ($\tau_{1,2}$-int_δ (A)) = $< x_{0.8}, y_{0.9}, z_{0.9} >$. Hence A is $(1,2)^*$- β^*closed set.

Result 4.3:

(i) Fuzzy $(1,2)^*$- β^*closed is fuzzy $(1,2)^*$- δ semi open if $\tau_{1,2}$-int ($\tau_{1,2}$-cl ($\tau_{1,2}$-int (A))) = 0.

(ii) Fuzzy $(1,2)^*$- β^*closed is $(1,2)^*$- α open if $\tau_{1,2}$-cl ($\tau_{1,2}$-int_δ (A)) = 0.

Proposition 4.4:

If A is fuzzy $(1,2)^*$- δ semi open and B is fuzzy $(1,2)^*$- α open then $A \vee B$ is fuzzy $(1,2)^*$- β^*closed.

Proof. Obvious from Definition 4.1

Proposition 4.5:

Let (X, τ_1, τ_2) be a fuzzy bitopological space. Then the intersection of any two fuzzy $(1,2)^*$- β^*closed sets is an $(1,2)^*$- β^*closed set.

Proof:

Let A_1, A_2 be two fuzzy $(1,2)^*$- β^*open sets,

$A_1 \geq \tau_{1,2}$-int ($\tau_{1,2}$-cl ($\tau_{1,2}$-int (A_1))) $\wedge$ $\tau_{1,2}$-cl ($\tau_{1,2}$-int_δ (A_1)). and

$A_2 \geq \tau_{1,2}$-int ($\tau_{1,2}$-cl ($\tau_{1,2}$-int (A_2))) $\wedge$ $\tau_{1,2}$-cl ($\tau_{1,2}$-int_δ (A_2)). (by Definition 4.1)

Then we have,

$A_1 \wedge A_2 \geq \tau_{1,2}$-int ($\tau_{1,2}$-cl ($\tau_{1,2}$-int ($A_1$))) $\wedge$ $\tau_{1,2}$-cl ($\tau_{1,2}$-int_δ (A_1))

$\wedge$ $\tau_{1,2}$-int ($\tau_{1,2}$-cl ($\tau_{1,2}$-int (A_2))) $\wedge$ $\tau_{1,2}$-cl ($\tau_{1,2}$-int_δ (A_2)).

$A_1 \wedge A_2 \geq \tau_{1,2}$-int ($\tau_{1,2}$-cl ($\tau_{1,2}$-int ($A_1 \wedge A_2$))) $\wedge$ $\tau_{1,2}$-cl ($\tau_{1,2}$-int_δ ($A_1 \wedge A_2$)

Therefore $A_1 \wedge A_2$ is fuzzy $(1,2)^*$- β^*closed set.

Theorem 4.6:

Let (X, τ_1, τ_2) be a fuzzy bitopological space and let $\{A_\alpha\}_{\alpha \in \mathcal{F}}$ be the collection of fuzzy $(1,2)^*$- β^* closed sets in fuzzy bitopological space X, then $\wedge_{\alpha \in F}(A_\alpha)$ is fuzzy $(1,2)^*$- β^* closed set.

Proof:

Let $\mathcal{F}$ be the collection of fuzzy $(1,2)^*$- β^*closed sets in fuzzy bitopological space (X, τ_1, τ_2). For each $\alpha \in \mathcal{F}$, $A_\alpha \geq \tau_{1,2}$-int ($\tau_{1,2}$-cl ($\tau_{1,2}$-int (A_α))) $\wedge$ $\tau_{1,2}$-cl ($\tau_{1,2}$-int_δ (A_α)).

Thus, $\wedge_{\alpha \in F}(A_\alpha) \geq \quad \wedge_{\alpha \in F}(\tau_{1,2} - \text{int}\,(\tau_{1,2} - \text{cl}\,(\tau_{1,2} - \text{int}\,(A_\alpha))) \wedge \tau_{1,2} - \text{cl}\,(\tau_{1,2} -$

int_δ $(A_\alpha)))$
$\wedge_{\alpha\in F}(A_\alpha) \geq \tau_{1,2}$-int ($\tau_{1,2}$-cl ($\tau_{1,2}$-int ($\wedge_{\alpha\in F}(A_\alpha)$))) $\wedge$ $\tau_{1,2}$-cl ($\tau_{1,2}$-int_δ ($\wedge_{\alpha\in F}$ (A_α))).
Since, the arbitrary union of fuzzy $(1,2)^*$- β^*closed sets is fuzzy $(1,2)^*$- β^*closed set.

Theorem 4.7:

Let (X, τ_1, τ_2) and (Y, σ_1, σ_2) be any two fuzzy bitopological spaces such that X is product related to Y. Then the product $A_1 \times A_2$ of a fuzzy $(1,2)^*$- β^* closed set A_1 of X and fuzzy $(1,2)^*$- β^* closed set A_2 of Y is fuzzy $(1,2)^*$- β^*closed set of the fuzzy product space X ×Y.

Proof:

Let A_1, A_2 are two fuzzy $(1,2)^*$- β^*closed sets of X and Y respectively,
From Definition 4.1, $A_1 \geq \tau_{1,2}$-int ($\tau_{1,2}$-cl ($\tau_{1,2}$-int (A_1))) $\wedge$ $\tau_{1,2}$-cl ($\tau_{1,2}$-int_δ (A_1)) and $A_2 \geq \tau_{1,2}$-int ($\tau_{1,2}$-cl ($\tau_{1,2}$-int (A_2))) $\wedge$ $\tau_{1,2}$-cl ($\tau_{1,2}$-int_δ (A_2)). Then we have,
$A_1 \times A_2 \geq \tau_{1,2}$-int ($\tau_{1,2}$-cl ($\tau_{1,2}$-int (A_1))) $\wedge$ $\tau_{1,2}$-cl ($\tau_{1,2}$-int_δ (A_1))
$\times$ $\tau_{1,2}$-int ($\tau_{1,2}$-cl ($\tau_{1,2}$-int (A_2))) $\wedge$ $\tau_{1,2}$-cl ($\tau_{1,2}$-int_δ (A_2)).
$A_1 \times A_2 \geq \tau_{1,2}$-int ($\tau_{1,2}$-cl ($\tau_{1,2}$-int $(A_1 \times A_2)$)) $\wedge$ $\tau_{1,2}$-cl ($\tau_{1,2}$-int_δ $(A_1 \times A_2)$ is fuzzy $(1,2)^*$- β^* open in the fuzzy product space X ×Y.

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