TO SOLVE TRAVELLING SALESMAN VIA ROBUST RANKING METHOD USING TETRADECOGONAL FUZZY NUMBER

S.Vimala[1] and V.Nagammal[2]

[1] *Assistant Professor, Department of Mathematics, Mother Teresa Women's University, Kodaikanal. Email: tvimss@gmail.com*

[2] *Research Scholar, Department of Mathematics, Mother Teresa Women's University, Kodaikanal. Email: singaraj79@gmail.com*

Abstract

Assignment difficulty is related to the problem of the traveling salesman. Finding the quickest route and reducing the overall amount of journey time are the objectives of the travelling salesman problem. This study uses fuzzy Tetra decagonal numbers to measure the traveling salesman problem. Tetra decagonal fuzzy numbers are ordered with respect to each other using two alternative ranking methods. By utilizing ranking approaches, the fuzzy traveling salesman problem is transformed into a clear traveling salesman problem. The Hungarian technique, which is mentioned with a numerical example, is used to determine the problem's optimal solution.

Keywords

Tetra decagonal Fuzzy Number, Robust ranking method, Average ranking method, Fuzzy travelling salesman problem.

1. Introduction

The challenge is to determine the shortest route that visits each city precisely once and loops back to the starting point, given a collection of cities and the distances between each pair of cities. Be aware of the variations between TSP and the Hamiltonian Cycle. The Hamiltonian cycle problem aims to determine if there is a tour that stops in each city precisely once. Because the graph is complete in this instance, we know that a Hamiltonian Tour exists. In fact, there are many such tours; the challenge is identifying the Hamiltonian Cycle with the lowest weight.

The fuzzy travelling salesman problem aims to find the order or sequence that the salesman should visit each city, so that the total distance travelled or cost or time of travelling is minimum, with the constraint that the salesman should visit each city once and return to the starting point. In 2016, Dr. Abha Singhal and Priyanka Pandey [1] gave an approach to solve the travelling salesman problems by dynamic programming algorithm. In 2011,2012, Amitkumar and Anila Gupta has worked on some fuzzy assignment problems and fuzzy travelling salesman problems were solved by using classical assignment method and Yager's ranking method [2] Assignment and Travelling Salesman Problems with Coefficients as LR Fuzzy Parameters [3]. In 2019 Fuzzy Travelling Salesman Problem Using Fuzzy Number was solved by Dr. Amit Kumar Rana using triangular fuzzy number [4, 5] . Zadeh found a new approach for travelling salesman problems in crisp and fuzzy environment have received great attention in recent years [5]. In 2004 A labeling algorithm for the fuzzy assignment problem, Fuzzy Sets and Systems, has proposed by Chi Jen Lin and Ue Pyng Wen, [6]. Dhanasekar. S, Hariharan .S and Sekar.P found a new approach for Classical Travelling Salesman Problem based approach to solve fuzzy TSP using Yager's ranking ,[7] in 2013.

An Approach for Solving Fuzzy Transportation Problem and One's Assignment method for solving Travelling Salesman Problem", has proposed by Hadi Basirzadeh [9],[10] in 2011and 2016."Hungarian method to solve Travelling Salesman Problem with fuzzy cost" has developed by Jagunath Nayak, Sudharsan Nanda and Srikumar Acharya [11] in 2017. "A new approach for solving Travelling Salesman Problem with fuzzy numbers using dynamic programming" was urbanized by Mythili.V, Kaliyappan. M, Hariharan .S and Dhanasekar, in 2018.

In this paper, we proposed a two ranking technique for ranking the Tetra decagonal fuzzy numbers. The idea is to transform a problem with fuzzy parameters to a crisp version in the TSP form and to solve it by ranking method. Other than the fuzzy assignment problem other applications of this method can be tried in project scheduling, sequencing, replacement problem, etc. Using this ranking the fuzzy assignment problem or fuzzy travelling salesman problem is converted to a crisp valued problem, which can be solved using Hungarian method. The optimal solution can be got either as a fuzzy number or as a crisp number. In section 2 consists of preliminaries and definition of a fuzzy set and fuzzy numbers. In section 3, we present some results on fuzzy tetra decagonal number and ranking techniques.

2. Preliminaries

Definition 2.1:
A fuzzy set is characterized by a membership function mapping element of a domain space or the universe of discourse X to the unit interval {0,1}.

(i.e) A = {x, μA(x); x ∈ X}. Here μA(x) = 1.

Definition 2.2:
A fuzzy set A of the universe of discourse X is called normal fuzzy set implying that there exist at least one x ∈ X Such that μA(x) = 1.

Definition 2.3:

The support of fuzzy set in the Universal set X is the set that contains all the elements of X that have an on zero membership grade in A.

(i.e.) Supp (A˜) = {x ∈ X/μA˜(x) > 0}.

Definition 2.4:

Given a fuzzy set A defined on X and any number α ∈ [0,1] the α −cut, αA is the crisp set αA = {x ∈ X/A(x) ≥ α,α ∈ [0,1].

Definition 2.5

A fuzzy set A defined on the set of real numbers R is said to be fuzzy number if its membership function μA(x) : R → [0,1] has the following properties.

(i) A must be a normal and convex fuzzy set.
(ii) α A must be a closed interval for every α ∈ (0,1].
(iii) The support of A must be bounded.

FORMULATION OF FUZZY TRAVELLING SALESMAN PROBLEM

Cities are visited in such a way that the total travelling time (cost) is minimum. The mathematical formulation of fuzzy travelling salesman problem is as follows.

Minimize Z* = $\sum_{i=1}^{n} \sum_{j-1}^{n} \check{c}$ij Xij

Subject to constraints

$$\sum_{i=0}^{n} Xij = 1,\ \ j = 1,2 \ldots n \ and \ j \neq i$$

$$\sum_{j=1}^{n} Xij = 1, \qquad i = 1,2 \ldots . n \ and \ i \neq j$$

$$x_{ij} = \begin{cases} 1 & if\ he\ goes\ from\ city\ i\ to\ city\ j \\ 0 & otherwise \end{cases}$$

Where $C_{ij}$the fuzzy travelling is cost (time) form i city to j city.

The fuzzy travelling salesman problem can be displayed in the following tabular form

City	1	2	3	……..	N
1	∞	$\check{C}$12	$\check{C}$13	……..	$\check{C}$1n
2	$\check{C}$21	∞	$\check{C}$23	……..	$\check{C}$2n
3	$\check{C}$31	$\check{C}$32	∞	……..	$\check{C}$3n
…..	……	…….	…….	∞	…….
N	$\check{C}$n1	$\check{C}$n2	$\check{C}$n3	……..	∞

NUMERICAL EXAMPLE:

Consider the following tetra decagonal fuzzy travelling salesman problem which consists of four persons and four cities. The cost matrix $\widetilde{c_{ij}}$ those elements are tetra decagonal fuzzy numbers. The objective of travelling salesman problem is to find the shortest possible route and the total travelling time is minimized.
Ranking of Tetra decagonal fuzzy number using robust ranking method:
In order to find the optimum value of the give Tetra decagonal fuzzy cost given in table 1, first we convert the fuzzy cost into the crisp cost using the proposed ranking method as shown in table 2.

TETRA DECAGONAL FUZZY NUMBER

A fuzzy number $\tilde{A}$ is a Tetra decagonal fuzzy number defined by

$$\widetilde{A} = (a_{1,} a_2, a_3, a_{4,} a_{5,} a_6, a_7, a_8, a_{9,} a_{10}, a_{11,} a_{12}, a_{13,} a_{14,})$$

$(a_{1,} a_2, a_3, a_{4,} a_{5,} a_6, a_7, a_8, a_{9,} a_{10}, a_{11,} a_{12}, a_{13,} a_{14,})$ where real numbers and its membership function are given by

Arithmetic operations on Tetra decagonal Fuzzy number:

In this section, Arithmetic operations on Tetra decagonal Fuzzy numbers are presented. Let,

$\widetilde{A_{TRDFN}}$=$(a_{1,} a_2, a_3, a_{4,} a_{5,} a_6, a_7, a_8, a_{9,} a_{10}, a_{11,} a_{12}, a_{13,} a_{14,})$ and

$\widetilde{B_{\mathrm{TR}DFN}}$=$(b_{1,} b_2, b_3, b_{4,} b_{5,} b_6, b_7, b_8, b_9, b_{10}, b_{11,} b_{12}, b_{13}, b_{14})$

Be two the Tetra decagonal fuzzy numbers. Then the addition and subtraction of Tetra decagonal fuzzy numbers can be performed as follows

$$\widetilde{A_{TRDFN}} + \widetilde{B_{TRDFN}} = [a_1 + b_1, a_2 + b_{,2}\, a_3 + b_3, a_4 + b_{4,} a_5 + b_5, a_6 + b_6, a_7 + b_7, a_8 + b_8, a_9 + b_9, a_{10} + b_{10,} a_{11+} b_{11,} a_{12} + b_{12}, a_{13} + b_{13}, a_{14} + b_{14,}]$$

$$\widetilde{A_{TRDFN}} - \widetilde{B_{TRDFN}} = [a_1 - b_{14}, a_2 - b_{,13}\, , a_3 - b_{12}, a_4 - b_{11,} a_5 - b_{10}, a_6 - b_9, a_7 - b_8, a_8 - b_7, a_9 - b_6, a_{10} - b_{5,} a_{11} - b_{4,} a_{12} - b_{3,} a_{13} - 2, a_{14} - b_{1,}]$$

ROBUST RANKING METHOD:

The Robust ranking technique for Tetra decagonal fuzzy number is defined as follows.

If $\widetilde{a}$ is a fuzzy number then the Robust Ranking is defined by

$\Re(\tilde{a}) = \int_0^1 (0.5)\ (a_\propto^L, a_\propto^U)\ d \propto$
Where
$(a_\propto^L, a_\propto^U)$ = [(b-a) α+a, d-(d-c) α, (f-e) α+e, h-(h-g) α, (j-i) α+i, l-(l-k) α,(n-m)α+m]

AVERAGE RANKING METHOD:

The Average ranking technique for Tetra decagonal fuzzy number is defined as follows. If $\tilde{a}$ is a fuzzy number then the Average Ranking is defined by

$$R(\tilde{a}) = \frac{(a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 + a_9 + a_{10} + a_{11} + a_{12} + a_{13} + a_{14})}{14}$$

TETRADECAGONAL FUZZY NUMBER

The membership function of Tetra decagonal fuzzy number

$\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14})$

Where $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14})$ are real numbers, is given by

$$\begin{cases} 0 & x \le a_1 \\ k_1\left(\dfrac{x-a_1}{a_2-a_1}\right), & a_1 \le x \le a_2 \\ k_1, & a_2 \le x \le a_3 \\ k_1 + (1-k_1)\dfrac{x-a_3}{a_4-a_3}, & a_3 \le x \le a_4 \\ k_2, & a_4 \le x \le a_5 \\ k_2 + (1-k_2)\dfrac{x-a_5}{a_6-a_5}, & a_5 \le x \le a_6 \\ 1, & a_6 \le x \le a_7 \\ k_2 + (1-k_2)\dfrac{a_8-x}{a_8-a_7}, & a_7 \le x \le a_8 \\ k_2, & a_8 \le x \le a_9 \\ k_1 + (1-k_1)\dfrac{a_{10}-x}{a_{10}-a_9}, & a_9 \le x \le a_{10} \\ k_1, & a_{10} \le x \le a_{11} \\ k_1\left(\dfrac{a_{12}-x}{a_{12}-a_{11}}\right), & a_{11} \le x \le a_{12} \end{cases}$$

K_3, $a_{12} \le x \le a_{13}$

0 $a_{14} \le x$

To gain better understanding about Travelling Salesman Problem

Table: 1 Tetra decagonal fuzzy number travelling salesman problem

	A	B	C	D
A	∞	0,1,2,3,4,5,6,7,8,9,10, 11,12,13	1,3,5,7,9,11,13,15,17, 19,21,23,25,27	1,3,4,5,7,9,10,12, 14,15,16,18,19,20
B	2,4,6,8,10,12,14,16, 18,19,22,24,25,27	∞	1,4,7,10,13,16,19,22, 25,28,31,34,35,37	0,4,6,9,11,12,13,14,15, 16,19,21,22,23
C	1,2,3,4,7,10,13,15, 16,17,22,26,27,29	2,4,6,8,9,13,15,16,18, 19,22,25,27,30	∞	2,4,8,9,11,13,16,19,20, 22,24,26,28,29
D	1,2,3,4,8,9,10,12,13, 15,17,18,20,22	0,2,3,5,6,7,9,10,13,15, 16,18,22,24	3,4,7,10,11,13,15,16, 18,21,24,26,29,30	∞

Robust Ranking Method:

The Robust Ranking technique for dodecagonal fuzzy numbers defined as follows. If $\breve{a}$ is a fuzzy numbers,

Then the Robust Ranking is defined by,

R ($\breve{a}$) = $\int_0^1 (0.5)(a_\propto^L, a_\propto^U)\, d \propto$

Where

$(a_\propto^L, a_\propto^U)$ = [(b-a) α+a, d-(d-c) α, (f-e) α+e, h-(h-g) α, (j-i) α+i, l-(l-k) α,(n-m) α+m]

Ranking of tetra decagonal fuzzy number using robust ranking method

In order to find the optimum value of the given tetra decagonal fuzzy cost given in table x first we convert the fuzzy cost into the crisp cost using the proposed ranking method as shown in table 2

R ($\breve{a}$) = $\int_0^1 (0.5)(a_\propto^L, a_\propto^U)\, d \propto$

$a_\alpha^L \;\; a_\alpha^U$= {(b-a)α+a, d-(d-c)α, (f-e) α+e, h-(h-g) α(j-L) α+L, l-(l-k) α,(n-m) α+m }

R (0,1,2,3,4,5,6,7,8,9,10,11,12,13)

= {(1-0) α+0, 3-(3-2) α, (5-4) α+4, 7-(7-6) α, (9-8) α+8, 11-(11-10) α(13-12) α +13}
= { α, 3- α, α+4, 7- α, α+8, 11- α, α +13}
= 46

R ($\breve{a}$) =$\int_0^1 (0.5)(46)\, d\alpha$

= 2.3

Table: 2

Crisp travelling salesman problem of the corresponding Tetradecagonal Fuzzy travelling salesman problem.

Write the initial cost matrix and reduce it.

Rules:

To reduce a matrix perform the row reduction and column reduction of the matrix separately.

A row or a column is said to be reduced if it contains at least one entry 0 in it.

	A	**B**	**C**	**D**
A	∞	2.3	48.5	38
B	51.5	∞	70	46
C	48	52.5	∞	57.5
D	38	37	51.5	∞

Row Reduction

Consider the rows of above matrix one by one.

If the row already contains an entry ‘0’ then, there is no need to reduce the row.

If the row does not contains an entry ‘0’, then

- Reduce that particular row.
- Select the least value element from that row.
- Subtract that element from each element of that row.
- This will create an entry ‘0’ in that row, thus reducing that row.

Following this, we have

- Reduce the elements of row-1 by 2.3
- Reduce the elements of row-2 by 46
- Reduce the elements of row-3 by 48
- Reduce the elements of row-4 by 37

The optimal travelling schedule is given by

A $\longrightarrow$ D $\longrightarrow$ C $\longrightarrow$ B $\longrightarrow$ A

Using the optimal travelling schedule cost (time) is given by

R (1,3,4,5,7,9,10,12,14,15,16,18) + R (1,4,7,10,13,16,19,22,25,28,31,34) +R (2,4,6,8,9,13,15,16,18,20,21,26) + R (1,2,3,4,8,9,10,12,13,15,17,18) =

R (5+13+20+27+37+47+54+62+70+78+85+96)

The crisp travelling cost (time) = 38+70+52.5+38

= 198.5

Ranking of Tetra decagonal Fuzzy Number using Average method:

In order to fine the optimum value of the given Tetra decagonal fuzzy cost given in table 1 first we convert the fuzzy cost into the crisp cost by using average ranking method.

R $(a^{\wedge})$ = $(a_1+a_2+a_3+a_4+a_5+a_6+a_7+a_8+a_9+a_{10}+a_{11}+a_{12}+a_{13}+a_{14})$

$= \frac{0+1+2+3+4+5+6+7+8+9+10+11+12+13}{14}$

$=\frac{91}{14} = 6.5$

Table: 2 Crisp travelling salesman problem of the corresponding Tetra decagonal fuzzy travelling salesman problem

	A	**B**	**C**	**D**
A	∞	6.5	14	10.9
B	14.7	∞	20.1	13.2
C	13.7	15.2	∞	16.5
D	11	11.2	16.2	∞

By applying the Hungarian method we find the optimal travelling schedule and the optimum travelling schedule.

Row Reduction:

Consider the rows of above matrix one by one.

If the row already contains an entry ‘0’ then, there is no need to reduce tht row.

If the row does not contains an entry ‘0’, then

- Reduce that particular row.
- Select the least value element from that row.
- Subtract that element from each element of that row.
- This will create an entry ‘0’ in that row, thus reducing that row.

Following this, we have

- Reduce the elements of row-1 by 5.5.
- Reduce the elements of row-2 by 11.7.
- Reduce the elements of row-3 by 11.3.
- Reduce the elements of row-4 by 9.3.

Performing this, we obtain the following row-reduced matrix

∞	0 $\boxed{0}$	7.5	4.4
1.5	∞	6.7	0 $\boxed{0}$
0 $\boxed{0}$	1.5	∞	2.8
0.7	$\boxed{0}$	5.2	∞

RESULT AND DISCUSSION

The solution is attained by using two ranking technique is listed in table 3.

Table 3:

Method	Optimal solution
Robust ranking method	198.5
Average ranking method	66.5

From the above results, we conclude that the optimum solution of the problem is obtained with the help of the average ranking method is minimum than that of Robust ranking method.

Conclusion

In difficult decision-making situations, fuzzy number ranking is crucial. Decisions are made by decision makers in problem-solving situations based on the ranking of fuzzy numbers. For ranking the tetra decagonal fuzzy numbers, we discussed two alternate ranking approaches in this study. It is best to use the average ranking method to arrive at the traveling salesman problem answer. Robust ranking approach is inferior to the average ranking method. The provided example demonstrates the accuracy and efficiency of the way these ranking methods operate.

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