Evidence of mean reversion and calculations in stock markets using GARCH model and Half-life method.

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Abstract

Stock markets have been a common interest among the wealth seeker community. They have always attracted individuals seeking fortunes. Moreover, they have been a leading source of capital for any country's economy. Stock markets are continuously monitored by the professional and individual communities over the world. Naturally, many people have attempted to understand the movement of stock price signals. There are two contradicting theories about the stock market namely Random Walk Hypothesis and Predictive Theory. Random Walk Hypothesis suggests that the movement of the stock signals are random and like a drunk person walking on a wall street having no control. The other theory which suggests that stock prices are dependent on the past data i.e., price today is dependent on yesterday price. There are various models which are proposed to understand this dependency such as vector autoregressive models, Auto Regressive Conditional Heteroskedasticity (ARCH) etc., The paper attempts to understand the mean reversion nature of the stocks from the developed and emerging nations using a Hybrid model consisting of Generalized Auto Regressive Conditional Heteroskedasticity (GARCH) and Artificial Neural Network.

Introduction

Stock markets over the globe are observed closely by many experts to understand and reveal the intricate patterns of movements. This has led to development of several theories to related to the movement of the stock price. One such theory is the Random Walk Hypothesis of the markets. The model was proposed in the paper (Godfrey, M. D., Granger, C. W., & Morgenstern, O. 1964.) where it is stated that the stock price signals move in a random way, and it is impossible to predict the next price of the signal. Random walk theory suggests that the stock markets are efficient and reflect all the available information. Therefore, it is impossible to develop a pricing mechanism which will predict the price of the stock to calculate the profits from the markets. The other group of researchers support the idea that the stock signals patterns can be predicted by observing the information generated by the stock signal over time. They say that the stock signals can be predicted by observing the historical data. There are various statistical models available to predict the stock market such as ARIMA, SARIMA, ARCH and GARCH models to predict the stock price from the historical data.

There are various statistical tools that are used for analyzing the stock markets various theorists have proposed few models after studying the whole stock market for several years such as Asset Pricing Model (APM) and Capital Asset Pricing Model (CAPM) etc. The capital asset Theory Proposed by the William Sharpe (1964) and John Lintner (1965) has been an important model for modelling the returns obtained in the stock markets. The equation used in Capital Asset Pricing Model is given by.

$$E(Ri) = Rf + \beta(Rm - Rf) \qquad (1)$$

Where E(Ri) is expected returns
Rf is Risk Free Rate
Rm is the Return offered
β is the risk parameter calculated for the individual asset.

Equation (1) is used to estimate the average returns an individual can expect for investing in the particular asset in discussion factoring in all the metrics such as risk free rate and beta is the factor that encompasses the risk involved in the return.

Stock markets over the globe are volatile and are influenced by various macro-economic variables such as inflation, economic cycles etc. The stock prices of the companies are more influenced by the company’s internal issues such as new product launches etc. There are certain shocks to the stock markets that are same throughout the globe such as the economic crisis of the 2008 and the pandemic named covid-19 in 2019 that made the stock market plunge into great depression. However, the markets around the world have demonstrated recovery at various paces that is one of the aims of the paper to understand the speed of recovery of the stock markets in emerging and developed countries.

Mean reversion is a one theory used in finance that states that asset prices tend to return to their mean level in the long run. This is used by investors to buy the stocks whose recent performance have differed from the long-term and hold until they revert to their mean price. This theory is used by analysts over the globe to understand the response of the stock markets to the shocks in the economy.

The paper aims to understand the mean reversion nature of the emerging and developed country economies by, considering the stock indices which reflect the top performing companies in the stock market. The paper also tries to understand the mean reversion in the emerging and developed countries. As the world has gone through unforeseen situations the governments have imposed unforeseen lockdowns on its residents and the operations of companies has come to a sudden halt. The paper also aims to understand the resilience of the stock markets during such situations. The paper aims to calculate the speed of mean reversion of the indices to see the overall mean reversion rate in the index by using half-life method proposed by Chaudhuri, K. et.al. The mean reversion rate is calculated using for both long term using GARCH(3,3) model.

Literature Review:

Efficient Market Theory suggests that the stock markets are so efficient and consume all the information making it impossible to predict the next stock price movements i.e., it is impossible to outperform the markets at any given time. The evidence of this theory lies in the impossibility to outperform the market by many mutual funds managers and hedge mangers. The theory is like the random walk hypothesis both which states that it is impossible to predict the equity price of the company. (Mishra, 2017; Tripathy, 2017; Wang et al., 2015; Zakamulin, 2016) The equity price of the company consumes all the available in the market to be efficient and unpredictable.

The other researchers (Ahmed, R. R., Vveinhardt, J., Streimikiene, D., & Channar, Z. A. (2018)) claim that it is possible to predict the stock prices by studying the history of the stock prices. The theory states that stock markets have inherent trends (Jegadeesh, N. (1991) and Gropp, J. (2004).) such as increasing trend, decreasing trend and mean reversion trends etc. There are theories that have been supported by various researchers from time to time. Poshakwale et. al. and Mustafa et. al show evidence for the random walk behaviour (similar to efficient market hypothesis) in the emerging Indian stock markets. Chaudhuri, K. et. al have suggested evidence for the mean reversion in the stock markets which support the predictive nature of stock markets against the Efficient Market Hypothesis. The theories are debated by many researchers and there are stocks which follow both the theories.

The financial crisis of 2008 was arguably the greatest economic downturn in the history, many researchers (Murphy (2008)) argued that it was due to the financial policies of the institutions in the system. The markets took a while to recover from the financial crisis. However exhibited the recovery to previous levels which serve as an example to the mean reversion in the stock markets.

Since inception to until 1980s the focus of econometrics has always been on modelling the mean, median and mode of the data that are the actual values of the data However, from mid 1980s the focus of the researchers has shifted to model the volatility of the data i.e., the volatility effect on the mean values of the data. There are two types of the data collected for the statistical analysis such as cross-sectional data and time series data analysis. Cross sectional data is the data collected from different sectors at the same time instant whereas, Time series data refers to the data that is collected by observing the same variable across different time instants. There are multiple techniques available in literature to perform analysis on time series such as Auto Regression models, Auto Regressive Conditional Heteroskedasticity (ARCH) etc.

The stock markets are highly volatile the stock markets are affected by a various number of the factors such as economic state of the nation, political situation, consumer sentiments etc. This makes modelling the volatility of the stock markets very difficult and nearly impossible. With the surge of economic researchers there are various model that have gained importance to understand and predict the behaviour of the stock price signals.

The econometric models such as Auto Regressive Model are discussed by various researchers such as Shibata, R. The Auto Regressive models are used to predict the future values of the stocks using the past data of the stocks. However, Auto Regressive models have their own limitations such as fitting of the model etc. There are models such as Auto Regressive Moving Average which are used to predict the future values of the stocks. However they are not effective prediction without taking the volatility into consideration.

Auto Regressive Conditional. Heteroscedasticity (ARCH) (R.F. Engle (1980)) tries to model the conditional volatility of the stock prices. The conditional volatility is based on the idea that the volatility follows a pattern and is segmented into regions of high volatility and low volatility. R.F Engle et.al have proposed the ARCH model to model the conditional volatility in time series data. Then the Generalised ARCH models are used to predict the stock prices by many researchers from time to time.

Methodology

The main objective of the paper is to prove the mean reversion in the stock markets and calculate the mean reversion rate of the various stock indices across the globe. The stock markets indices data is collected from the yahoo finance website for a period of ten years from September 2012 to September 2023 and a data set of same set of stock during the period of covid from January 2020 to December 2022 to understand the mean reversion during that period in the stock markets. The stock data comprises of both the emerging and developed nations such as Germany (^GDAXI), Hong Kong (^HIS), US (^IXIC, ^RUT), South Korea (^KS11), UK stock exchange (^N100), JAPAN (^N225) and India (^NSEI), Indonesia

(^JKSE), China (SSE composite), Brazil (BVSP) across the world to understand the mean reversion rate difference among them.

The paper draws motivation from the Ahmed, R et. al. to understand the mean reversion in the stock markets among the emerging and developed country but we use the GARCH (3,3) model becuas to understand the long term mean reversion in the stock markets and GARCH (1,1) model to understand the covid-19 period stocks. The additional period data is chosen to under the This will help the investors to buy and sell the stock at the appropriate time to maximize returns from the stock markets.

The returns of the equations are modeled using the equation.

$$r_t = \ln(\frac{p_t}{p_{t-1}})$$

Where r_t is the returns and p_t and p_{t-1} are the closing prices of the stock prices.

The equation for generalized GARCH (p, q) model is given by the following equation.

$$\sigma_t^2 = \omega + \sum_{i=0}^{p} \alpha i \, r_{t-1}^2 + \sum_{i=0}^{q} \beta i \, \sigma_{t-1}^2$$

The mean reversion of the equation according to GARCH (1,1) is given by.

$$r_t^2 - \sigma_t^2 = (\alpha_1 + \beta_1)(r_{t-1}^2 - \sigma^2) + \mu_t - \beta_1 \mu_{t-1}$$

The half-life of the mean reversion for GARCH (1,1) is given by

$$L_{half} = \ln\left(\frac{1}{2}\right) / \ln(\alpha_1 + \beta_1)$$

Most researchers have reported that the $(\alpha_1 + \beta_1) \approx 1$ similarly author have considered the GARCH (p, q) have the following properties $\sum_{i=1}^{p} \alpha i + \sum_{i=1}^{q} \beta i$ ~ 1 the equation becomes.

$$L_{half} = \ln\left(\frac{1}{2}\right) / \ln\left(\sum_{i=1}^{p} \alpha i + \sum_{i=1}^{q} \beta i\right)$$

The returns are tested for stationarity using the Augmented Dickey Fuller (ADF) Test and the results are tabulated in **Table 1**:

Table 1: Results of ADF test.

Index Name	**t Statistic**	**P Value**	**Critical Value**		
			1 % Level	**2% Level**	**3% level**
^GDAXI	-17.814196	0	-3.433	-2.863	-2.567
^HSI	-49.051918	0	-3.433	-2.863	-2.567
^IXIC	-16.484172	0	-3.433	-2.863	-2.567
^JKSE	-27.959356	0	-3.433	-2.863	-2.567
^KS11	-10.733572	0	-3.433	-2.863	-2.567
^N100	-10.490947	0	-3.433	-2.863	-2.567
^N225	-28.8062	0	-3.433	-2.863	-2.567
^NSEI	-13.581755	0	-3.433	-2.863	-2.567
SSE Composite	-9.116813	0	-3.433	-2.863	-2.567
^BSVP	-11.75078	0	-3.457	-2.873	-2.573
^NYA	-15.473615	0	-3.457	-2.873	-2.573
^RUT	-16.341355	0	-3.457	-2.873	-2.573

The results indicate that the null hypothesis can be safely rejected, and the returns do not have the unit root test, and the returns data is stationary therefore the equations of half-life and GARCH can be applied to the returns data. The ADF results have not been shown for the covid-19 dataset as the data is already included in the other dataset and thus the GARCH model can be applied on the covid-19 dataset.

Mean reversion rate of the stocks based on the sum of ARCH and GARCH coefficients is given in the following Table 2.

Index Name	$\sum_{i=0}^{p} \alpha i$	$\sum_{i=0}^{q} \beta i$	**A + B**	**Ranking**
^GDAXI	0.2606	0.6655	0.9261	4
^HSI	0.1244	0.8479	0.9723	8
^IXIC	0.3614	0.5753	0.9367	6
^JKSE	0.2166	0.687	0.9036	1
^KS11	0.1899856	0.7421	0.9320856	5
^N100	0.3865	0.522	0.9085	3
^N225	0.2467	0.6611	0.9078	2
^NSEI	0.1938	0.7622	0.956	7

SSE Composite	0.1612	0.830600002	0.991800002	12
^BSVP	4.32E-10	9.76E-01	0.975900001	9
^NYA	0.002331492	0.9737	0.976031492	10
^RUT	0.00831	9.76E-01	9.84E-01	11

Table 2: Mean reversion ranking using sum of ARCH and GARCH components.

The sum of ARCH and GARCH components is less than 1 which indicates mean reversion in the stock markets in both emerging and developed nations.

The speed of mean reversion given by the half-life method is given in the following Table 3:

Index Name	$\sum_{i=0}^{p} \alpha i$	$\sum_{i=0}^{q} \beta i$	**Ln (A + B)**	**Half-life**
^GDAXI	0.2606	0.6655	-0.076773059	9.02852109
^HSI	0.1244	0.8479	-0.02809088	24.67516776
^IXIC	0.3614	0.5753	-0.065392219	10.5998419
^JKSE	0.2166	0.687	-0.101368494	6.837895588
^KS11	0.1899856	0.7421	-0.070330623	9.85555297
^N100	0.3865	0.522	-0.095960391	7.223263393
^N225	0.2467	0.6611	-0.096731189	7.165705167
^NSEI	0.1938	0.7622	-0.044997366	15.40417236
SSE Composite	0.1612	0.830600002	-0.008233803	84.18311379
^BSVP	4.32E-10	9.76E-01	-0.024395156	28.41331213
^NYA	0.002331492	0.9737	-0.024260427	28.57110402
^RUT	0.00831	9.76E-01	-0.015712802	44.11353105

Table 3: Speed of Mean reversion

The speed of the mean reversion in the equation as shown in table 3 is highest among the developed nations than emerging nations.

The Mean Reversion days using half-life is tabulated in the following table 4.

Index Name	Days to Mean Revert	Ranking
^GDAXI	9	4
^HSI	25	8
^IXIC	11	6
^JKSE	7	1
^KS11	8	5
^N100	7	3
^N225	7	2
^NSEI	15	7
SSE Composite	84	12
^BSVP	28	9
^NYA	29	10
^RUT	44	11

Table 4: Mean Reversion days

Similarly, half-life for the stock closing price during January 2020 to December 2022 is given by using GARCH (1,1)

Index Name	alpha	Beta	Half life	Ranking
^GDAXI	0.1641	0.7913	15.19220769	7
^HIS	0.0898	0.8848	26.94119939	10
^IXIC	0	0.9868	52.16380892	12
^JKSE	0.1824	0.7192	6.691622879	2
^KS11	0.2345	0.6865	8.42268833	3
^N100	0.203	0.7471	13.54119486	8
^N225	0	0.9466	12.63054093	6
^NSEI	0.1248	0.8504	27.60145895	11
SSE Composite	0.1926	0.6623	4.421405561	1
^BSVP	0.1221	0.8188	11.37828653	4
^NYA	0.2306	0.7268	15.92197244	9
^RUT	0.1626	0.7811	11.96175228	5

Table 5: Mean reversion in the stocks during the Covid-19 period.

The mean reversion during the covid 19 time is shown in Table 5 indicate that the average half-life recovery of the country is around 25 days.

Conclusion

The results of the paper indicate the presence of mean reversion in the stock markets using GARCH (3,3) (lag of 3 days in both ARCH and GARCH components) has been used in the calculation of the stock markets. The paper also proves the existence of mean reversion in the stock markets as supported by many researchers. The paper also calculates the speed of mean reversion in the stock markets among the developed and emerging nations and concludes that the mean reversion among the developed countries on an average is faster than the emerging countries than developed countries but there may be exceptions like Indonesian (^JKSE) due to other factors. This also concludes that developed nations are less prone to the volatility than the emerging markets. The recovery during the covid-19 period is also fast in developed countries however, the emerging countries have also showed good recovery during this phase. Predicting mean reversion in the stock markets can help the investors to maximize their gains the half-life time of the ^N225 is 7 days. i.e., an investor can initiate a buy signal at any day and can wait for 14 days to sell the stocks for maximizing the gains.

Future Work

The work can be further extended by using advanced method to optimize the p, q value in GARCH (p, q) or use other volatility models to predict the volatility in the stock markets. The researchers can also study the effect of the other Macro Economic Variables on the stock prices using models such as M-GARCH etc. This will help in getting better accurate results in mean reversion and calculating speed of mean reversion. Artificial Intelligence is an emerging field in many research areas. Artificial Neural Networks are efficient in handling non-linear problems such as classification tasks etc. ANN can be used to model the volatility as they are efficient in handling nonlinear data. The stock data is more nonlinear thus there is a scope for the application of ANN in volatility prediction.

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