STOCHASTIC MODEL FOR TIME TO RECRUITMENT IN A SINGLE GRADE MANPOWER SYSTEM WITH TWO TYPES OF INTERDECISION TIMES WHEN THE BREAKDOWN THRESHOLD HAS THREE COMPONENTS

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Abstract

In this paper, the problem of time to recruitment is analyzed for a single grade manpower system in which attrition takes place due to two types of policy decisions where this classification is done according to intensity of attrition, it form an ordinary renewal process. Assuming (i) policy decisions and exits occur at different epochs (ii) wastage of manpower due to exits and wastage due to frequent breaks taken by the personnel working in the manpower system separately form a sequence of independent and identically distributed exponential random variables with different means and (iii) breakdown threshold for the cumulative wastage of manpower in the system has three components which are independent exponential random variables. A stochastic model is constructed and the variance of the time to recruitment is obtained using an univariate CUM policy of recruitment. Employing a different probabilistic analysis, analytical results in closed form for system characteristics are derived.

Keywords

Single grade manpower system, non-instantaneous exits, intensity of attrition, independent wastage of manpower, ordinary renewal process, breakdown threshold with three components, univariate CUM policy of recruitment and variance of time to recruitment.

Introduction

Wastage of personnel due to retirement, death and resignation is a common phenomenon in administrative as well as production oriented organizations. There are certain special problems associated with the organization engaged in sales and marketing. Frequent exits and recruitments are very common in such organizations. Whenever the organization announces revised polices regarding sales target, revision of wages, incentives and perquisites the exodus is possible. Reduction in the total strength of marketing personnel adversely affects the sales turnover of the organization. Frequent recruitments may also be expensive due to the cost of recruitments and training. As the wastage of manpower is unpredictable, a suitable recruitment policy has to be designed to overcome this wastage. The univariate recruitment policy, usually known as CUM policy of recruitment in the literature, is based on the replacement policy

associated with the shock model approach in reliability theory and is stated as follows: *Recruitment is made whenever the cumulative wastage of man hours exceeds its breakdown threshold.* Several researchers have studied the problem of time to recruitment for a single grade manpower system using shock model approach. In [1] and [2] the authors have discussed some manpower planning models for a single and multi-grade manpower system using Markovian and renewal theoretic approach. In [13] the authors have analyzed the problem of time to recruitment for the single grade manpower system which is subject to attrition with **instantaneous exits**, using CUM policy of recruitment when the wastage of manpower process and inter-decision time process are independent. In [11], the author has studied the problem by associating geometric process and order statistics for inter-decision times. In [9], the authors have analyzed the work in [12] with (i) exponential breakdown threshold and (ii) extended exponential threshold having shape parameter 2 using a bivariate CUM policy of recruitment when the inter-decision times form a geometric process. In [10], the authors have studied the problem of time recruitment by assuming that the attrition is generated by a geometric process of inter-decision times. Variance of the time to recruitment for a single grade manpower system with optional and mandatory thresholds is obtained in [16] when inter-decision times form an order statistics. In [17], the authors have obtained variance of time to recruitment when the breakdown threshold level for the cumulative wastage of manpower is the sum of the exponential breakdown threshold levels of wastage and backup resource for manpower using Laplace transform technique. Recently, in [15], the authors have studied the work in [17] when the inter-decision times form a geometric process using a different method employed in [10]. In [3] the authors have considered the single grade manpower system with **non-instantaneous exits** and obtained variance of the time to recruitment when the wastage of manpower, inter-decision times and exit times are independent and identically distributed continuous random variables according as the mandatory breakdown threshold is an exponential random variable or extended exponential random variable with shape parameter 2 or a continuous random variable with SCBZ property. In [6] and [7] the authors have extended the research work in [3] when the inter-decision times form (i) a geometric process and (ii) an order statistics respectively. In [4], [11] and [5] the authors have studied the work in [3], [6] and [7] using the method of [10]. In [8] the authors have discussed when the threshold has two components, The present paper extends the research work in [8] when the breakdown threshold level for the cumulative wastage in the manpower system is the sum of three components namely an exponential threshold for cumulative wastages due to exits, an exponential threshold for cumulative wastage due to frequent breaks of existing workers and an exponential threshold for cumulative wastage due to backup sources.

MODEL DESCRIPTION AND ANALYSIS

Consider an organization taking policy decisions at random epochs in $(0,\infty)$ and at every decision making epoch a random number of persons quit the organization. There is an associated wastage of manpower, if a person quits. It is assumed that the loss of manpower is linear and cumulative. For i=1,2,3…, let X_i be independent and identically distributed exponential random variables representing the amount of depletion of manpower (wastage of man hours) at the ith exit epoch with probability distribution function M(.), and mean $\frac{1}{\alpha}$ (α>0).

Let S_i be the cumulative wastage of manpower in the first i exit epochs and m_i be its probability density function. The policy decisions which produce depletion of manpower are classified into two types depending upon the intensity of attrition. It is assumed that the first type of policy decisions has high attrition rate λ_1 ($\lambda_1 > 0$) and the second type has low attrition rate λ_2 ($\lambda_2 > 0$). Let a_1 and (1-a_1) be the proportion of decisions with high and low attrition rate respectively. Let A_n be the time between (n-1)th and nth policy decisions, forming a sequence of independent hyper exponential random variable. Let B_i be the time between (i-1)th and ith exit epochs, forming a sequence of independent and identically distributed random variables with probability distribution function G(.) and density function g(.). Let D_{i+1} be the waiting time upto (i+1)th exit epoch. Let Y be the breakdown threshold for the cumulative wastage of manpower in the organization with probability density function h(.). It is assumed that sum of the exponential breakdown threshold level Y_1 for cumulative wastage due to exits with mean $\frac{1}{\theta_1}$ ($\theta_1 > 0$) and exponential threshold Y_2 for cumulative wastage due to frequent breaks of existing workers with mean $\frac{1}{\theta_2}$ ($\theta_2 > 0$) and Y_3 backup sources with mean $\frac{1}{\theta_3}$ ($\theta_3 > 0$). Let q (q≠0) be the probability that every policy decision has exit of personnel. Let $\chi(I)$ be the indicator function of the event I. Let T be the random variable denoting the time to recruitment with mean E(T) and variance V(T).

Deviating from the conventional method of using Laplace transform, a different probabilistic analysis is employed here.

By the recruitment policy, recruitment is done whenever the cumulative wastage exceeds the threshold $Y_1+Y_2+Y_3$. When the first decision is taken, recruitment would not have been done for B_1 units of time. If the wastage $X_1(=S_1)$ in the first exit epoch is greater than $Y_1+Y_2+Y_3$ then recruitment is done and in this case $T=B_1=D_1$.

However, if $S_1 \le Y_1+Y_2+Y_3$, the non-recruitment period will continue till the arrival of next exit epoch. If the cumulative sum S_2 of wastage in the first two exit epochs exceeds $Y_1+Y_2+Y_3$, then recruitment is done and $T=B_1+B_2=D_2$.

If $S_2 \le Y_1+Y_2+Y_3$, then the non-recruitment period will continue till the arrival of next exit epoch. Depending on $S_3 > Y_1+Y_2+Y_3$ or $S_3 \le Y_1+Y_2+Y_3$, recruitment is done or the non-recruitment period continues and so on.

$$\text{ie. } T = \begin{cases} D_1, & if\ S_1 > Y1 + Y2 + Y3 \\ D_2, & if\ S_1 \le Y1 + Y2 + Y3 < S_2 \\ \quad . \\ \quad . \\ D_{i+1}, & if\ S_i \le Y1 + Y2 + Y3 < S_{i+1} \\ \quad . \\ \quad . \end{cases}$$

In terms of indicator function of an event, we can write T as

$$T = \sum_{i=0}^{\infty} D_{i+1}\chi\ (S_i \le Y_1 + Y_2 + Y_3 < S_{i+1})$$
$$(1)$$

Taking expectation on both sides of (1) and using the result (for any event L)
E($X \chi(L)$) = E(X)P(L), we get

$$E(T) = \sum_{i=0}^{\infty} E(D_{i+1})P(s_i \le Y_1 + Y_2 + Y_3 \le s_{i+1})$$

(2)

From (1) and from the definition of D_{i+1}, we get

$$E(T) = \sum_{i=0}^{\infty} E\left(\sum_{k=1}^{i+1} B_k\right)P(s_i \le Y_1 + Y_2 + Y_3 \le s_{i+1})$$

(3)

Since the inter-exit times B_i's are identically distributed, we write
$E(B_i)=E(B)$, i=1,2,… . Therefore from (3) we get

$$E(T) = \sum_{i=0}^{\infty}(i + 1)E(B)P(s_i \le Y_1 + Y_2 + Y_3 \le s_{i+1})$$

(4)

By using law of total probability we get

$$P(S_i \le Y_1 + Y_2 + Y_3 < S_{i+1}) = P(0 \le (Y_1 + Y_2 + Y_3) - S_i < X_{i+1})$$

$$= \int_0^{\infty} \int_0^t P(0 \le (Y_1 + Y_2 + Y_3) - S_i < X_{i+1} / S_i = x, Y_1 + Y_2 + Y_3$$

$$= t)\big(e_{S_i, Y_1+Y_2+Y_3}(x,t)\big)dxdt$$

$$= \int_0^{\infty} \int_0^t P(X_{i+1} > t - x)\left(e_{S_i, Y_1+Y_2+Y_3}(x,t)\right)dxdt$$

$$= \int_0^{\infty} \int_0^t \tilde{M}(t - x)m_i(x)h(t)dxdt$$

(5)

By hypothesis, $\tilde{M}(t - x) = e^{-\alpha(t-x)}$ and $m_i(x) = \left\{\dfrac{\alpha^i x^{i-1} e^{-\alpha x}}{\lceil i}\right\}$.

Therefore

$$P(S_i \le Y_1 + Y_2 + Y_3 < S_{i+1}) = \int_0^{\infty}\left[\int_0^t e^{-\alpha(t-x)}\left\{\frac{\alpha^i x^{i-1} e^{-\alpha x}}{\lceil i}\right\}h(t)dxdt\right]$$

$$= \int_0^{\infty} \frac{e^{-\alpha t}\alpha^i}{\alpha^i(i - 1)!}\left[\int_0^t x^{i-1}dx\right]h(t)dt$$

$$= \int_0^{\infty} \frac{e^{-\alpha t}\alpha^i t^i}{i!}h(t)dt$$

(6)

Since, for i=1,2, 3 $Y_i \sim \exp(\theta_i)$ and $Y_i's$ are independent, the probability density function h(t) of

$Y_1 + Y_2 + Y_3$ is found to be $\dfrac{\theta_1\theta_2}{\theta_1 - \theta_2}\left[e^{-\theta_2 t} - e^{-\theta_1 t} + e^{-(\theta_1+\theta_3)t} - e^{-(\theta_2+\theta_3)t}\right]$

Therefore

$$P(S_i \le Y_1 + Y_2 + Y_3 < S_{i+1}) = \int_0^{\infty} \frac{e^{-\alpha t}\alpha^i t^i}{i!} \frac{\theta_1\theta_2}{\theta_1 - \theta_2}\left[e^{-\theta_2 t} - e^{-\theta_1 t} + e^{-(\theta_1+\theta_3)t} - e^{-(\theta_2+\theta_3)t}\right]dt$$

(7)

$$P(S_i \leq Y_1 + Y_2 + Y_3 < S_{i+1}) = \frac{\theta_1\theta_2\alpha^i}{(\theta_1 - \theta_2)i!}\left[\frac{1}{(\alpha+\theta_2)^{i+1}} - \frac{1}{(\alpha+\theta_1)^{i+1}} + \frac{1}{(\alpha+\theta_1+\theta_3)^{i+1}} - \frac{1}{(\alpha+\theta_2+\theta_3)^{i+1}}\right] \quad (8)$$

from (4) and (8) and on simplification it can be shown that

$$E(T) = E(B)\sum_{i=0}^{\infty}(i+1)\frac{\theta_1\theta_2\alpha^i}{(\theta_1-\theta_2)i!}\left[\frac{1}{(\alpha+\theta_2)^{i+1}} - \frac{1}{(\alpha+\theta_1)^{i+1}} + \frac{1}{(\alpha+\theta_1+\theta_3)^{i+1}} - \frac{1}{(\alpha+\theta_2+\theta_3)^{i+1}}\right]$$
(9)

We now find $E(B)$ for the present model.

The distribution function G(.) of the inter-exit times B_i, i=1, 2, … satisfy the relation

$$G(x) = q\sum_{n=1}^{\infty}(1-q)^{n-1}F_n(x)$$
(10)

Therefore from (10) we get

$$\begin{aligned}
E(B) &= \int_0^\infty x\, dG(x)\\
&= q\sum_{n=1}^{\infty}(1-q)^{n-1}\int_0^\infty x f_n(x)dx\\
&= q\sum_{n=1}^{\infty}(1-q)^{n-1}E(C_n)\\
&= qE(A)\sum_{n=1}^{\infty}n(1-q)^{n-1}\\
&= q\frac{a_1\lambda_2 + a_2\lambda_1}{\lambda_1\lambda_2}\sum_{n=1}^{\infty}n(1-q)^{n-1}
\end{aligned}$$

$$ie.\ E(B) = \frac{a_1\lambda_2 + a_2\lambda_1}{\lambda_1\lambda_2 q}$$
(11)

Using (11) in (9), we get

$$E(T) = \frac{a_1\lambda_2 + a_2\lambda_1}{\lambda_1\lambda_2 q}\left[\frac{(\alpha(\theta_1-\theta_2)+\theta_1\theta_2}{\theta_1\theta_2} - \frac{\theta_1\theta_2\alpha}{(\theta_1+\theta_3)(\theta_2+\theta_3)}\right]$$

$$E(T) = N_1 E(B)$$
(12)

where $N_1 = \frac{(\alpha(\theta_1-\theta_2)+\theta_1\theta_2}{\theta_1\theta_2} - \frac{\theta_1\theta_2\alpha}{(\theta_1+\theta_3)(\theta_2+\theta_3)}$, $E(B) = \left[\frac{a_1\lambda_2 + a_2\lambda_1}{\lambda_1\lambda_2 q}\right]$
(13)

We now evaluate $E(T^2)$ for the present model.

Squaring both sides on (2), we get

$$E(T^2) = \sum_{i=0}^{\infty} E(D_{i+1}^2) P(S_i \leq Y_1 + Y_2 + Y_3 < S_{i+1}).$$
$$\text{(14)}$$

Now

$$E(D_{i+1}^2) = V(D_{i+1}) + [E(D_{i+1})]^2$$

$$ie. E(D_{i+1}^2) = (i+1)V(B) + [(i+1)E(B)]^2 \qquad .$$
$$\text{(15)}$$

Using (15) in (14), we get

$$E(T^2) = \sum_{i=0}^{\infty}(i+1)V(B) [P(S_i \leq Y_1 + Y_2 + Y_3 < S_{i+1})] + \sum_{i=0}^{\infty}(i+1)^2 E^2(B) [P(S_i \leq Y_1 + Y_2 + Y_3 < S_{i+1})] \qquad \text{(16)}$$

We now evaluate E(B^2) for the present model

From (10) we get

$$E(B^2) = \int_0^{\infty} x^2 dG(x) = \int_0^{\infty} x^2 \left[q \sum_{n=1}^{\infty} (1-q)^{n-1} f_n(x) \right] dx$$

$$ie. E(B^2) = q \sum_{n=1}^{\infty} (1-q)^{n-1} \int_0^{\infty} x^2 f_n(x) dx.$$
$$\text{(17)}$$

We now evaluate $\int_0^{\infty} x^2 f_n(x) dx$ in (17).

$$\int_0^{\infty} x^2 f_n(x) dx = V(C_n) + [E(C_n)]^2$$

$$= nV(A) + n^2[E(A)]^2$$

$$= n \left\{ \frac{2[a_1\lambda_2^2 + a_2\lambda_1^2]}{(\lambda_1\lambda_2)^2} - [E(A)]^2 \right\} + n^2[E(A)]^2$$

$$ie. \int_0^{\infty} x^2 f_n(x) dx = \frac{2n[a_1\lambda_2^2 + a_2\lambda_1^2]}{(\lambda_1\lambda_2)^2} - n[E(A)]^2 + n^2[E(A)]^2.$$
$$\text{(18)}$$

From (17) and (18), we get

$$E(B^2) = \frac{2q\left[a_1\lambda_2^2 + a_2\lambda_1^2\right]}{(\lambda_1\lambda_2)^2} \sum_{n=1}^{\infty} n(1-q)^{n-1} - q\left[E(A)\right]^2 \sum_{n=1}^{\infty} n(1-q)^{n-1} + q\left[E(A)\right]^2 \sum_{n=1}^{\infty} n^2(1-q)^{n-1}$$

$$= \frac{2q[a_1\lambda_2^2 + a_2\lambda_1^2]}{(\lambda_1\lambda_2)^2 q^2} - \frac{q[E(A)]^2}{q^2} + \frac{q[E(A)]^2(2-q)}{q^3}$$

$$= \frac{2[a_1\lambda_2^2 + a_2\lambda_1^2]}{(\lambda_1\lambda_2)^2 q} - \frac{\left[a_1\lambda_2 + a_2\lambda_1\right]^2}{(\lambda_1\lambda_2)^2 q} + \frac{\left[a_1\lambda_2 + a_2\lambda_1\right]^2 (2-q)}{(\lambda_1\lambda_2)^2 q^2}$$

$$ie.\,E\left(B^2\right) = \frac{2q\left[a_1\lambda_2^2 + a_2\lambda_1^2\right] + 2(1-q)\left[a_1\lambda_2 + a_2\lambda_1\right]^2}{\left(\lambda_1\lambda_2 q\right)^2}.$$

(19)

From (8), (11) and (19), we get
$$E(T^2) = E(B^2)N_1 - [N_1 - (N_2 + N_3)]E^2(B) \tag{20}$$

Where,

$$N_1 = \left[\frac{(\alpha(\theta_1-\theta_2)+\theta_1\theta_2}{\theta_1\theta_2} - \frac{\theta_1\theta_2\alpha}{(\theta_1+\theta_3)(\theta_2+\theta_3)}\right]$$

$$N_2 = \frac{2\alpha^2(\theta_1^2+\theta_2^2+\theta_1\theta_2)+3\alpha\theta_1\theta_2(\theta_1+\theta_2)+\theta_1^2\theta_2^2}{\theta_1^2\theta_2^2}$$

$$N_3 =$$
$$\frac{\theta_1\theta_2[(\theta_1^5-\theta_2^5)+(\theta_1^4-\theta_2^4)[3\alpha+5\theta_3]+(\theta_1^3-\theta_2^3)[2\alpha^2+12\alpha\theta_3+10\theta_3^2]+(\theta_1^2-\theta_2^2)[6\alpha^2\theta_3+6\alpha\theta_3^2+10\theta_3]+(\theta_1-\theta_2)[6\alpha^2\theta_3^2+12\alpha\theta_3^3+5\theta_3^4]]}{(\theta_1+\theta_3)^3(\theta_2+\theta_3)^3}$$

From (12) and (20) we get

$$V(T) = E(B^2)N_1 - E^2(B)[N_1{}^2 + N_1 - (N_2 + N_3)]$$

Where $E(B^2)$, $E^2(B)$, N_1, N_2 and N_3 represents from (11), (19) and (20) respectively.

CONCLUSION

The models discussed in this paper improve the earlier relevant research work in the context of admitting the realistic assumption of non-instantaneous exits in the system and taking into account wastage due to policy decisions, frequent breaks and backup sources for this system. They will be useful in the process of planning recruitment when the system has the above cited provision. The suitability of distributions assumed in the present work can be tested by data analysis.

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