System Identification based Modelling and Level Controller Design for Process Control Application

Bansari Deb Majumder[1], Sanghamitra Layek[2]

[1]*Department of Electrical Engineering, Narula Institute of Technology, Kolkata*
[2]*Department of Electronics and Instrumentation Engineering, Narula Institute of Technology, Kolkata*

[1]*bansari.debmajumder@nit.ac.in*,
[2]*sanghamitra.layek@nit.ac.in*,

Abstract

The mathematical modeling and control of a process control application is the most complicated task because of non- linearity, uncertainties, and constraints related to the process variables. The paper provides the system identification-based modeling and a controller design principle for a process of Admittance type multifunctional sensor and instrumentation. In this method a mathematical model has been developed from the experimental input-output data. The transfer function of the process control application is estimated using the system identification principle. The generalized PID based controller is designed for the process control application. Zeigler-Nichols method has been used for tuning the PID Controller. Experimental results have been provided in this paper to validate the theoretical aspects

Keywords: System Identification; controller design; level measurement

1. Introduction

Process control applications are nonlinear, and the complexity of the application increases as the number of parameters increases. Mathematical modeling of process control applications is time-consuming and computationally expensive. Therefore, the system identification principle is used to frame the mathematical model of any application from experimental input and output data [1]. There are different methods of system identification techniques. Relay-feedback based method is one of the techniques of system identification, where relays are used to estimate the transfer function [2]. In [3], the authors have used the relay-feedback method to determine the transfer function of a real-time level control system set up. The relay with hysteresis band is used for excitation of the process and transfer function parameters are deduced from the sustained oscillations. In [4], an asymmetrical relay is used to induce sustained oscillations in the system, and the state-space model is estimated. The application of Parametric system identification method for controlling the temperature control of heat exchanger have been discussed in [5], where the authors used different time series models and prediction error methods to estimate the dynamics of the heat exchanger system. To teach the closed- loop scheme of system identification, a laboratory-based experiment has been discussed in [6]. A comparative analysis of different transfer function estimation schemes has been presented in [10].

This paper provides a detailed analysis of system identification and controller design aspect of a developed process plant. The real-time data is collected from the process, and the transfer function of the plant is estimated using two different approaches. The first method uses different linear time-series models and prediction errormethods, whereas the second approach uses different transfer function estimation approaches such as instrumental variable, N4SID, etc. Experimental results have been provided to validate the theoretical concepts.

This paper is organized as follows. The problem formulation is provided in section II. Section III discusses the system identification concept using the time-series model. Section IV presents the transfer function estimation concept. Section V provides a discussion on controller design and analysis. Section VI illustrates the simulation results, and Section VII provides the conclusion.

2. Problem Formulation

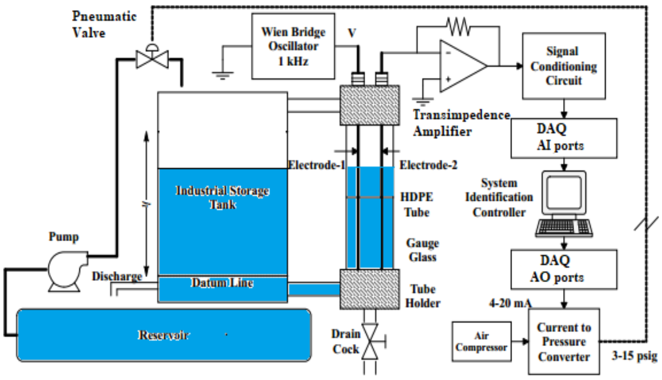

Fig1: Schematic diagram of the system

The schematic diagram of the process control application where the admittance type multifunctional sensor is used to measure the level and temperature [11] is shown in figure 1. The multifunctional data is then fed to the computer where necessary identification and controlling action takes place. The control signal from the computer is fed to the valve, which acts as a final control element via a current to pressure converter. Further, the transfer function needs to be formulated and controller design is implemented. Figure 2 shows the block diagram of a feedback control system considered in this case.

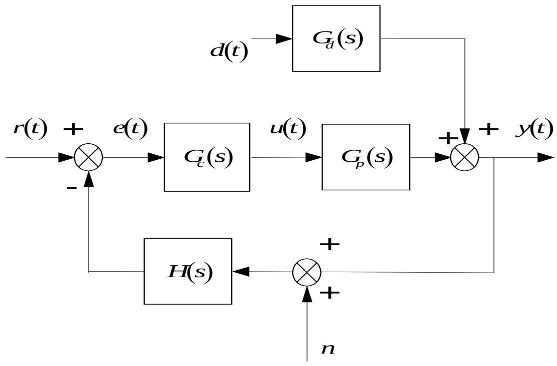

Fig 2: Block diagram of the feedback control system

The transfer function is estimated using input and output data of the process. For the level control system, the admittance type level sensor has been used. The dynamics of the admittance type level sensor has been considered as a unity. The disturbance variable and measurement noise are neglected for the case. The controller is a simple PID controller, and the Ziegler-Nichols tuning method is used to tune the controller parameters.

3. System Identification Using Time Series Models

The block diagram of the system identification process is shown in Figure 3, where the process dynamics and process parameters are unknown. Only the input stimulus and output response are known. From time-varying input and output signals, the dynamics of the process needs to be ascertained. There are broadly two types of system identification method such as

- Parametric system identification
- Non-parametric system identification

Parametric system identification is a time-domain based approach, whereas non-parametric system identification is a frequency domain approach. As process control applications are a time-dependent system, so this paper considers the parametric system identification approach to estimate the dynamics of the plant. Figure 4 presents the flow chart of the parametric system identification scheme.

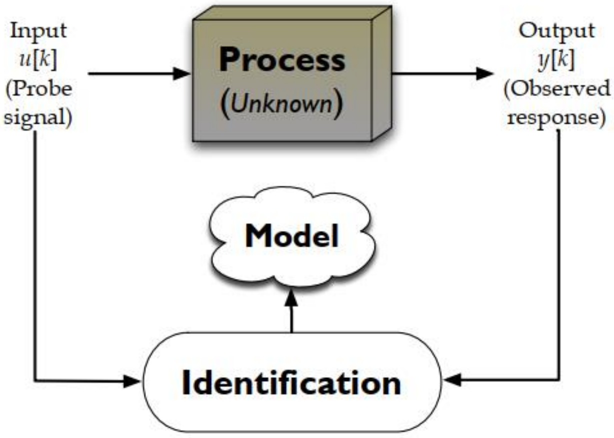

Fig 3: Block diagram of system identification schemes

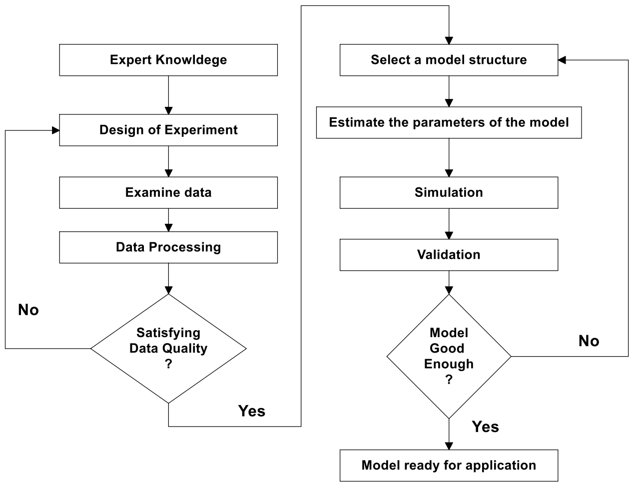

Fig 4: Flow chart of parametric system identification schemes

The first step of parametric system identification is to collect input and output data of the plant at a regular interval with uniform sampling time. Once the data is collected, the data is verified for its authenticity and genuineness. If the acquired data meet the set quality standards, then different time series polynomial models are selected, and the parameters of the models are estimated using different estimation algorithm. The selection of a proper model structure is based on expert knowledge. Once the parameter is determined, the estimated model is validated. If the validation process fails, then the same procedure needs to be repeated several times until the desired output is achieved

The generalized representation of LTI model can be written as

$$y(k) = G(q,\theta)u(k) + H(q,\theta)\varepsilon(k) \qquad (1)$$

where

$$
\begin{aligned}
G(q,\theta) &= \sum_{k=0}^{\infty} g(k) q^{-k} = \frac{B(q)}{A(q)} \\
H(q,\theta) &= \sum_{k=0}^{\infty} h(k) q^{-k} = \frac{1}{A(q)}
\end{aligned} \qquad (2)
$$

Where $G(q,\theta)$ is the plant model, $H(q,\theta)$ is the disturbance model, $u(k)$ represents the input, $y(k)$ represents the output, $\varepsilon(k)$ represents the white noise with zero mean, q^{-1} is the backward shift operator.

Equation 1 is rearranged and simplified and shown in equation 3.

$$
A(q)\, y(k) = \frac{B(q)}{F(q)} u(k) + \frac{C(q)}{D(q)} \varepsilon(k) \qquad (3)
$$

There are significant models, such as the Auto-Regressive eXogeneous (ARX) model, Auto Regressive Moving Average eXogeneous (ARMAX), Output Error (OE), and Box Jenkin (BJ) models used for system identification.

Models	Equation
ARX	$y(k) = \frac{B(q)}{A(q)} q^{-n_k} u(k) + \frac{1}{A(q)} \varepsilon(k)$
ARMAX	$y(k) = \frac{B(q)}{A(q)} q^{-n_k} u(k) + \frac{C(q)}{A(q)} \varepsilon(k)$
OE	$y(k) = \frac{B(q)}{F(q)} q^{-n_k} u(k) + \varepsilon(k)$
BJ	$y(k) = \frac{B(q)}{F(q)} q^{-n_k} u(k) + \frac{C(q)}{D(q)} \varepsilon(k)$

Where

$$
\begin{cases}
A(q) = 1 + a_1 q^{-1} + .. + a_{n_a} q^{-n_a} \\
B(q) = b_1 q^{-1} + .. + b_{n_b} q^{-n_b} \\
C(q) = 1 + c_1 q^{-1} + .. + c_{n_c} q^{-n_c} \\
D(q) = 1 + d_1 q^{-1} + .. + d_{n_d} q^{-n_d} \\
F(q) = 1 + f_1 q^{-1} + .. + f_{n_f} q^{-n_f}
\end{cases}
$$

3.1 Prediction Error Method

The prediction error method is used to estimate the parameters of the models mentioned above.

The prediction error can be represented as

$$e(k,\theta) = y(k) - \hat{y}(k \mid k-1,\theta) \tag{4}$$

The one step ahead prediction can be described as

$$\hat{y}(k \mid k-1,\theta) = H^{-1}(q,\theta)G(q,\theta)u(k) + \left(1 - H^{-1}(q,\theta)\right)y(k) \tag{5}$$

3.2 Model Validation

For model validation, final prediction error (FPE) can be represented as

$$FPE = \left| \frac{1}{N}\sum_{1}^{N} e(k,\theta)\left(e(k,\theta)\right)^{T} \right| \left(\frac{1+\frac{d_m}{N}}{1-\frac{d_m}{N}} \right) \tag{6}$$

Where d_m is the number of estimated parameters, N is the number of values in the predicted dataset.

For model validation, FIT (%) is used which can be represented as

$$FIT = 100\left(1 - \frac{\sqrt{\sum_{k=1}^{N}\left(y(k) - \hat{y}(k)\right)^{2}}}{\sqrt{\sum_{k=1}^{N}\left(y(k) - \bar{y}\right)^{2}}} \right) \tag{7}$$

The maximum fitness percentage between estimated and experimental results provides the criteria for choosing the appropriate model.

4. Transfer Function Estimation

4.1 Instrumental Variable Method

Eq.(3) can be rewritten as

$$A\left(q^{-1}\right)y(k) = B\left(q^{-1}\right)u(k) + \varepsilon(k) \tag{8}$$

The linear regression model of Eq(8) can be represented as

$$y(k)=\varphi^T(k)\theta+\varepsilon(k) \tag{9}$$

where

$$\varphi^T(k)=\left[-y(k-1) \quad .. \quad -y(k-n_a) \quad u(k-1) \quad .. \quad u(t-n_b)\right]$$

$$\theta=\left[a_1 \quad .. \quad a_{n_a} \quad b_1 \quad .. \quad b_{n_b}\right]^T$$

The least-square estimated parameter can be represented as

$$\hat{\theta}=\left[\frac{1}{N}\sum_{t=1}^{N}\varphi(t)\varphi^T(t)\right]^{-1}\left[\frac{1}{N}\sum_{t=1}^{N}\varphi(t)y(t)\right] \tag{10}$$

The instrumental variable estimated parameter can be described as

$$\hat{\theta}=\left[\frac{1}{N}\sum_{t=1}^{N}z(t)\varphi^T(t)\right]^{-1}\hat{\theta}=\left[\frac{1}{N}\sum_{t=1}^{N}z(t)y(t)\right] \tag{11}$$

The instrumental variable method is a generalization of the least square method [7].

4.2 N4SID

N4SID is a subspace state-space estimation algorithm used for system identification. It is a non-iterative and stable algorithm because it uses singular value decomposition. N4SID has no difference between zero and non-zero state [8].

4.3 GPMF

Generalized Poisson moment functional (GPMF) based subspace method is one of the widely used continuous-time system identification methods. In a linear system identification method, a derivative of input and output samples is required. GPMF finds out the time derivative of input and output using the Poisson moment function [9].

Let us consider a continuous-time LTI system

$$\dot{x}(t)=Ax(t)+Bu(t)$$
$$y(t)=Cx(t)+Du(t) \tag{12}$$

The state-space transformation in Eq(12) can be transformed into matrix form as

$$Y_{0|r-1}^{C}=O_r^C X_0^C+\psi_r^C U_{0|r-1}^C \tag{13}$$

$$Y_{0|r-1}^{C}=\begin{bmatrix} y(t_1) & y(t_2) & .. & y(t_N) \\ \dot{y}(t_1) & \dot{y}(t_2) & .. & \dot{y}(t_N) \\ : & : & : & : \\ y^{r-1}(t_1) & y^{r-1}(t_1) & .. & y^{r-1}(t_N) \end{bmatrix}\in R^{n\times N}$$

Where $(.)^{r-1}:=\dfrac{d^{r-1}(.)}{dt^{r-1}}$

Observability matrix can be defined as

$$O_r^C = \begin{bmatrix} C^T & (CA)^T & .. & (CA^{r-1})^T \end{bmatrix}^T \in R^{pr \times n} \quad (14)$$

Block Toeplitz system matrix can be defined as

$$\psi_r^C = \begin{bmatrix} D & 0 & .. & 0 \\ CB & D & .. & : \\ : & : & : & : \\ CA^{r-2}B & CA^{r-3}B & CB & D \end{bmatrix} \in R^{pr \times mr} \quad (15)$$

5. Controller Design

Conventional PID controller is described as

$$u(t) = K_p \left(e(t) + \frac{1}{T_i} \int_0^t e(\tau)\, d\tau + T_d \frac{de(t)}{dt} \right) \quad (16)$$

where u(t) is the controller output, e(t) is the error signal, K_p is the proportional gain, T_i is the integral time, and T_d is the derivative time, respectively.

The transfer function of an ideal PID controller is described as

$$G_c(s) = K_p \left(1 + \frac{1}{T_i s} + T_d s \right) \quad (17)$$

The error indices of the closed-loop transfer function can be represented as

$$\begin{cases} IAE = \int_0^\infty e(t)\, dt \\ ISE = \int_0^\infty e^2(t)\, dt \\ ITAE = \int_0^\infty t e(t)\, dt \\ ISTE = \int_0^\infty t^2 e(t)\, dt \\ IST^2E = \int_0^\infty t^2 e^2(t)\, dt \end{cases} \quad (18)$$

For a closed-loop system, the sensitivity and complementary sensitivity can be defined as

$$\begin{cases} S(s) = \dfrac{1}{1 + G_p(s) G_c(s)} = \dfrac{1}{1 + L(s)} \\ T(s) = 1 - S(s) = \dfrac{L(s)}{1 + L(s)} \end{cases} \quad (19)$$

6. Results

For simulation purposes, the experimental data are collected from the process plant at 1-sec interval. The process plant consists of an admittance type level sensor. A small number of samples were initially collected, and the water level of the system in (cm) is measured (Table I).

Number of Samples	Level (cm)
1	5.05
2	6.04
3	7.10
4	8.00
5	9.20
6	10.15
7	11.95
8	13.05
9	14.96
10	15.95
11	17.10
12	18.00
13	19.20
14	20.10

Table I: Experimental Data

Figure 5 shows the experimental results and estimated results using ARX, ARMAX, OE, and BJ models.

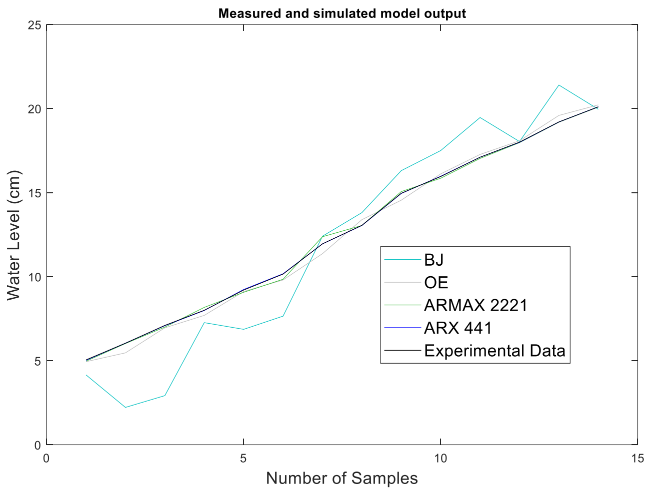

Fig 5: Comparative analysis of model validation scheme of different models

Figure 6 shows the comparative study of validation results using different model structures. From Figure 5 and corresponding Table II, it is clear that the ARX model provides the highest FIT% for the data.

	FPE	FIT%
ARX	0.07628	98.47
ARMAX	0.7467	90.78
OE	3.429	71.81
BJ	28.87	69.57

Table II: Summarized Results of Parametric System Identification

In another investigation, the authors have provided a comparative analysis of different parameter estimation schemes for transfer function estimation. Four different schemes have been compared. Table III and Table IV shows the performance analysis of different system identification scheme and estimated transfer function, respectively.

	FPE	FIT%
IV	0.07628	98.47
SVF	0.7467	90.78
N4SID	3.429	71.81
GPMF	28.87	69.57

TableI III: Performance Analysis of System Identification Schemes

IV	$G(s)=\dfrac{4.299s+0.02311}{s^2+0.8648s+0.0002166}$
SVF	$G(s)=\dfrac{273.9s+49.54}{s^2+54.84s+10.12}$
N4SID	$G(s)=\dfrac{3.122s+8.612}{s^2+0.5675s+1.738}$
GPMF	$G(s)=\dfrac{273.9s+49.54}{s^2+54.84s+10.12}$

Table IV: Estimated Transfer Function

Figure 6 shows the validation results of the transfer function estimation scheme, where the N4SID scheme provides the best possible estimation results.

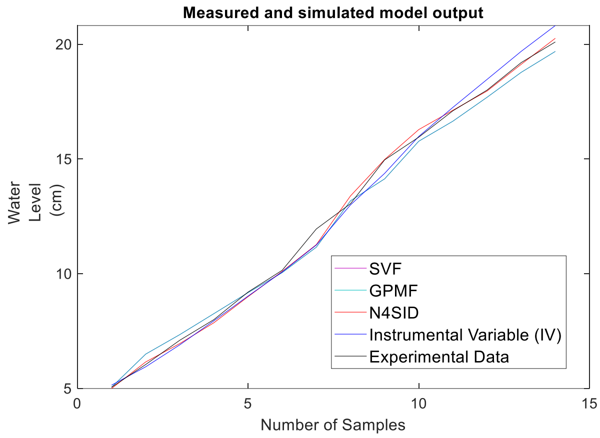

Table V: Comparative analysis of validation results of system identification schemes

6.1 Controller Design

After the system identification is completed, the transfer function of the process has been ascertained. The controller design for the level control application has been carried out in this section. The open-loop frequency response of the system has been shown in Figure 7.

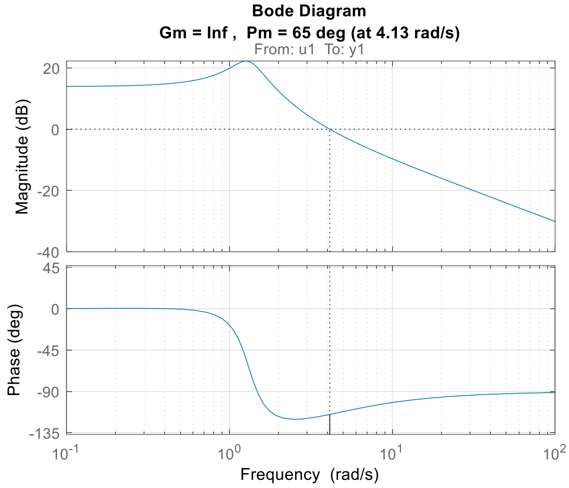

Fig 6:The frequency response of estimated transfer function

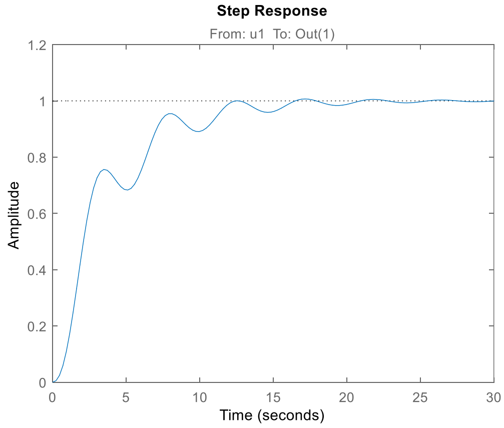

Fig 8:Unit step response of the controlled model determined by the N4SID estimation technique (Set-point regulation)

	FPE	FIT%
IV	0.07628	98.47
SVF	0.7467	90.78
N4SID	3.429	71.81
GPMF	28.87	69.57

Table VI: Performance Analysis PID Controller

PID controller is used to controlling the level control system, and the Ziegler-Nichols based tuning method is adopted to find the parameters of the controller [12]. From experimental results in Figure 8, the error indices, and the transient response parameters of the controlled system are found to be satisfactory, as shown in Table VI.

7. Conclusion

This paper provides an overview of different system identification principles as well as the controller design principle for a level control system in a process plant application. The time series model and transfer function estimation concepts have been discussed in this paper. Simulation results have been provided to validate the theoretical concepts. The said admittance type level sensor can be used to measure the liquid level in the boiler continuously and the said controller can be used to control the boiler drum level control. A miniaturized steam generation unit need to de developed incorporating the said sensor and designed control scheme.

Nomenclature

ARX	Autoregressive eXogeneous Input
ARMAX	Autoregressive moving average eXogeneous Input
OE	Output Error
BJ	Box Jenkins
IV	Instrumental variable
SVF	State variable filter
GPMF	Generalized Poisson moment functional
PID	Proportional-Integral-Derivative
IAE	Integral Absolute Error
ITAE	Integral Time Absolute Error
ISE	Integral Square Error
ISTE	Integral Square Time Error
IST^2E	Integral Square Time Square Error
MSE	Mean Square Error
FPE	Final Prediction Error

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