A Novel Algorithm for Solving L-R Intuitionistic Transportation Model

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ABSTRACT:

Transporting items from a certain source of supply at origin to a variety of requests at destinations at the lowest possible cost is a prominent application of chain-structured linear programming problems in an uncertain environment. As a result, the L-R intuitionistic transportation problem (LRIFTP) is established and a method for determining the algorithm is proposed, in which the amounts of expenses, disburses, and requirements are represented as L-R intuitionistic fuzzy numbers. Using deterministic linear programming, the L-R intuitionistic fuzzy numbers are transformed. The following are the article's contributions: 1) Using an accuracy function, the formulated LRIFTP is rearranged into a standard linear programming problem that is principally focused on the ordering of L-RTrIFN. 2) A contemporary algorithm provides an optimal cost and non-negative L-R intuitionistic fuzzy solution. 3) A numerical example of the optimal LRIFTP solution is shown using the proposed measure.

Keywords: *L-R trapezoidal intuitionistic fuzzy numbers (L-RTrIFN), Accuracy function, L-R Intuitionistic fuzzy transportation problem (LRIFTP).*

1. Introduction:

Operations research, management science, control theory, and other fields have all used the fuzzy set theory. The usage of fuzzy sets and fuzzy values is common in engineering applications. The distance minimize of two L-R intuitionistic fuzzy numbers, in which all restrictions are represented as L-R intuitionistic fuzzy numbers, was described in [1] as a new ranking function to answer the best solution of fully L-R intuitionistic fuzzy transportation problems. Vogel's method is used in [2] to solve a few numerical instances of transportation issues in triangular fuzzy numbers, intuitionistic fuzzy numbers, and triangular intuitionistic fuzzy numbers. The notion of IF sets is developed in [3].In several real-world

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Circumstances, this demonstrates how to define the ambiguous theory appropriately and include the defined ambiguous into the membership level. In IFS, a membership function describes the degree of acceptance, and a non-membership function depicts the degree of rejection. These factors make it anticipated that IFS will continue to function even in the face of ambiguity and uncertainty brought on by hazy information or knowledge. Three properties had the solid transportation. They serve as the beginning, end, and category of a good or method of transportation in an ambiguous setting. When an issue involves uncertainty, [4] studied interval solid shipping issue and fuzzy shipping models. The earliest kind emerges when the model is communicated as an interval on behalf of point assessments, and the second type appears when the identity of ambiguous data. In an interval scenario, known beneficial algorithms were used to tackle the common solid transportation problem, and in an uncertain case, it was possible to use a parametric approach to arrive at a fuzzy solution. Using the score function for sorting fuzzy numbers, Sudhgar and Ganesan solved a transportation problem in [21] without first turning it into a crisp one. That example demonstrated that the fuzzy optimal solution is not always obtained. In order to reduce transportation costs, a non-negative fuzzy transportation solution was provided in [5]. A two-step solution was put forth in [6] for addressing the transportation problem and the restrictions are described as non-negative triangular fuzzy numbers. The initial step is to handle fuzzy computing so that a fuzzy transportation problem can be converted to a LP problem with fuzzy expenditure and clear quantities. The next step is to suggest a brand-novel method of breaking down the developing problem into three clear, bounded shipping problems. The right-handed side variables and decision variables are considered as trapezoidal fuzzy numbers in a contemporary method that was proposed in [7] based on the bounded dual simplex method for solving the fuzzy variable linear programming problem to obtain the fuzzy optimal solution. In [8], a novel strategy was put out for addressing the fuzzy transportation issue, where the parameters are non-negative L-R flat fuzzy integers. As a result, the fuzzy transportation model is split into four transportation problems and addressed using conventional transportation simplex methods. Based on the identification of interval-valued fuzzy integers by the signed distance ranking approach, a fuzzy linear programming method for addressing interval-valued trapezoidal fuzzy transportation model was developed in [9]. An effective computational method for tackling IFTP was proposed by [10], which relies on classical shipping issues to produce improved workable solutions and optimum solutions with triangular intuitionistic fuzzy costs. This new technique provides a non-negative intuitionistic fuzzy optimum solution and optimal expenditure for the intuitionistic fuzzy transportation problem that was described in [11]. The intuitionistic fuzzy transportation problem was converted into a deterministic LPP by using standard LP models with an accuracy function, which considers supply, demand, and shipping costs as TrIFNs. The fuzzy transportation problem, in which the variables costs, supply, and requirements are represented by intuitionistic fuzzy integres, was investigated. Intuitionistic fuzzy zero point algorithm was introduced by [12] to obtain the optimal solution with the help of triangular intuitionistic fuzzy numbers. Three situations in particular—the sources, the destinations, and the type of product or shipping method used in an uncertain environment—require robust transportation. For resolving the data issue in an ambiguous path, [13] discusses two cases: interval solid shipment issue and fuzzy transportation model. The efficient approaches are used to obtain the conventional solid transportation approach for the interval way. The parametric process was utilized for fuzzy cases where it was conceivable to achieve a fuzzy solution to the original issue. A parametric evolutionary model was utilized by [14] to tackle the

fuzzy solid transportation problem. In order to provide a fuzzy solution to the problem, an arbitrary, linear, and non-linear objective function was assumed. By employing a parametric algorithm and connecting it to the actual problem, an auxiliary parametric solid transportation problem was created. [15] Proposed a novel method for resolving a special kind of fuzzy transportation model where the decision-maker was uncertain about the precise values of shipping costs but was certain about the distributions and requirements of the commodity and where the parameters were generalized trapezoidal fuzzy integers. For the fully intuitionistic fuzzy transportation issue in a specific instance where the constraints are taken as triangular intuitionistic fuzzy integers, a unique solution known as the PSK method was presented in an uncertain environment [16]. In order to obtain the best possible object value, a new multiplication operation on intuitionistic fuzzy integers was also introduced. In the supply, demand, and cost aspects of the transportation dilemma, there is uncertainty and hesitancy. A novel method for choosing the best outcome from intuitive fuzzy numbers using a new ranking function was proposed in [17]. According to level sets and recent developments, a general model for intuitionistic fuzzy linear programming problems was given in [18]. In this situation, the constraints of feasibility and optimality are represented by intuitionistic fuzzy relations. The weak and powerful dualism received special consideration. In [19], fuzzy sets were used to develop fuzzy quadratic programming issues, which are essentially fuzzy quadratic programming problems. To get a specific case of fuzzy quadratic programming problems with uncertain factors in the goal functions, [19] offered two novel fuzzy sets-based approaches. To solve transportation problems where the triangular intuitionistic fuzzyintegers are denoted as costs, [20] defined accuracy function and score functions for membership and non-membership functions of triangular intuitionistic fuzzy integers. They also suggested an algorithm for finding a feasible solution and an optimal solution using triangular intuitionistic fuzzy numbers using accuracy function. Intuitionistic fuzzy models were created by [21] in order to find the simplest possible answer and the best option. Accordingly, [21] took into account a shipping issue with erratic supply and demand, and the transportation model was created utilizing triangular intuitionistic fuzzy integers. The supply, requirements, and amounts are taken into account as trapezoidal intuitionistic fuzzy integers in the ranking procedure for intuitionistic fuzzy quantities that was established in [22] and proposed a number of intuitionistic fuzzy methods by utilizing the proposed ranking function to find the basic feasible solution and optimum solution. A transportation model's solution was transformed to crisp value in various writings. However, a paradigm for handling fuzzy transportation issues without turning them into sharp shipping issues was developed by [23].

Then, using Sudhagar score approaches, this result neared the fuzzy transportation cost with rank fuzzy numbers. The formula was presented by [24] for calculating the centroid of intuitionistic fuzzy quantities, and their properties were also covered. For a hybrid multi-basis category verdict maker, a contemporary interval-valued intuitionistic fuzzy mathematical programming approach was created [25]. The group decision making problem was obtained by using interval valued fuzzy preference relations, according to a novel intuitionistic fuzzy programming approach mentioned by [26].Today's institutions, industries, and businesses must identify the finest strategy to obtain the ideal solution in a highly competitive market. In a typical optimization situation, it is assumed that the decision-maker is aware of the precise values of the relevant variables. Due to the presence of irresistible components in the real-world circumstances, it is possible that not all transportation problem quantities are known with precision. This instance of fuzzy numbers being used to describe imprecise information was first presented by [27].

From the aforementioned information regarding this paper, the main strategies of this piece are 1) The ordering of L-R trapezoidal intuitionistic fuzzy numbers using an accuracy function determines how a formulated LRIFTP is converted into a classical linear programming model. 2) We created a novel method that produces an optimal non-negative L-R intuitionistic fuzzy solution at the lowest possible cost. 3) A numerical example of the optimal LRIFTP solution is shown using the proposed measure.

The organization of this work's structure is as follows. Some essential fuzzy set theory and L-R intuitionistic fuzzy set theory concepts are explored in Section 2. The L-R intuitionistic fuzzy transportation issue and the fuzzy transportation problem with fuzzy status in crisp status are developed in Section 3. A unique approach for finding the intuitionistic fuzzy optimum solution of the completely LRIFTP is described in Section 4. The numerical example and result produced from the suggested method are explained in Sec. 5. Conclusions are discussed in section 6.

2. Basic Fundamentals

This section uses definitions and attributes of L-R intuitionistic fuzzy numbers from [27] and some basic principles of intuitionistic fuzzy numbers from [26].

Definition: 1. A fuzzy set $\tilde{P}$ in T is described by a set of ordered pairs $\tilde{P}=\{(t,\mu_{\tilde{P}}(t));t\in T\}$ where T is universe set and $\mu_{\tilde{P}}(t)\in[0,1]$ denoted as the membership degree of t in $\tilde{P}$, and it is known as membership function of $\tilde{P}$.

Definition: 2. A fuzzy set $\tilde{P}^I$ be an intuitionistic fuzzy set in T is described by a set of ordered triple $\tilde{P}=\{(t,\mu_{\tilde{P}^I}(t),\upsilon_{\tilde{P}^I}(t));t\in T\}$ where T is universe set and the functions $\mu_{\tilde{P}^I}(t):T\rightarrow[0,1]$ and $\upsilon_{\tilde{P}^I}(t):T\rightarrow[0,1]$ respectively denoted as the membership degree and non-membership degree of t in $\tilde{P}$ such that for every element $t\in T$, $0\leq\mu_{\tilde{P}^I}+\upsilon_{\tilde{P}^I}(t)\leq 1$.

Definition: 3. A set $\widetilde{P}=\left\{\left(t,\mu_{\widetilde{P}^I}(t),\upsilon_{\widetilde{P}^I}(t)\right);t\in t\right\}$ is known as intuitionistic fuzzy normal if there is any $t_0\in T$ such that $\mu_{\widetilde{P}^I}(t_0)=1$ (so $\upsilon_{\widetilde{P}^I}(t_0)=0$).

Definition: 4. Let $\widetilde{P}^I$ is a Trapezoidal intuitionistic fuzzy integer is a precise intuitionistic fuzzy number with the membership function and non-membership function is explained as below:

$$\mu_{\widetilde{P}^I}(t)=\begin{cases}\dfrac{t-p_1}{p_2-p_1} & p_1<t\le p_2\\ 1 & p_2\le t\le p_3\\ \dfrac{p_4-t}{p_4-p_3} & p_3\le t<p_4\\ 0 & Otherwise\end{cases} \quad and \quad \upsilon_{\widetilde{P}^I}(t)=\begin{cases}\dfrac{p_2^{'}-t}{p_2^{'}-p_1^{'}} & p_1^{'}<t\le p_2^{'}\\ 0 & p_1^{'}\le t\le p_3^{'}\\ \dfrac{t-p_3^{'}}{p_4-p_3} & p_3^{'}\le t<p_4^{'}\\ 1 & Otherwise\end{cases}$$

Where $p_1^{'}\le p_1\le p_2^{'}\le p_2\le p_3\le p_3^{'}\le p_4\le p_4^{'}$.

This TrIFN is expressed as $\widetilde{P}^I=\left(p_{1,}\,p_2,p_3,p_4;p_1^{'},p_2^{'},p_3^{'},p_4^{'}\right)$.

Remark 1: If $p_2^{'}=p_2=p_3=p_3^{'}$, then TrIFN $\widetilde{P}^I=\left(p_{1,}\,p_2,p_3,p_4;p_1^{'},p_2^{'},p_3^{'},p_4^{'}\right)$ expressed as a triangular intuitionistic fuzzy number (TIFN). A TIFN is denoted by $\widetilde{P}^I=\left(p_{1,}\,p_2,p_3;p_1^{'},p_2\;p_3^{'}\right)$.

Remark 2: A Trapezoidal Intuitionistic fuzzy number $\widetilde{P}^I=\left(p_{1,}\,p_2,p_3,p_4;p_1^{'},p_2^{'},p_3^{'},p_4^{'}\right)$ represents a real number p if $p_1=p_2=p_3=p_4=p_1^{'}=p_2^{'}=p_3^{'}=p_4^{'}=p$.

Definition: 5. A fuzzy number $\widetilde{P}_{LR}=(m,n,\alpha,\beta)$ is known as LR – type fuzzy number and its membership function is shown in below

$$\mu_{\widetilde{P}_{LR}}=\begin{cases}L\left(\dfrac{m-t}{\alpha}\right) & if\ t\le m\\ 1 & if\ m\le t\le n,\\ R\left(\dfrac{t-n}{\beta}\right) & if\ x\ge n\end{cases}$$

where $\alpha>0$, $\beta>0$. A function represented by L, R: $\{0,+\infty)\to[0,1]$ is non-increasing, non-constant reference function of fuzzy number with $L(0)-R(0)-1$ and m,n are real numbers.

Definition: 6. If $\widetilde{P}_{LR}=(m,n,\alpha,\beta)$ is an L-R fuzzy number, then the crisp set $(\widetilde{P}_{LR})_\lambda=\left\{t\in T/\mu_{\widetilde{P}_{LR}}(t)\ge\lambda\right\}$ is called α - cut of $\widetilde{P}_{LR}$ and $(\widetilde{P}_{LR})_\lambda=\left[m-\alpha L^{-1}(\lambda),n+\beta R^{-1}(\lambda)\right]$ $,\alpha\in[0,1]$.

Definition: 7. (LR-IFN). If $\tilde{P}_{LR}{}^{I}$ be an L-R intuitionistic fuzzy number and its membership and non-membership functions are expressed as follows:

Membership function of the structure is Non- membership function of the structure is

$$\mu_{\tilde{P}^I_{LR}}(t) = \begin{cases} \dfrac{t-(m-\alpha)}{\alpha} & m-\alpha < t \le m \\ 1 & m \le t \le n \\ \dfrac{(\beta+n)-t}{\beta} & n \le t \le n+\beta \\ 0 & otherwise \end{cases} \qquad \upsilon_{\tilde{P}^I_{LR}}(t) = \begin{cases} \dfrac{m'-t}{\alpha'} & (m-\alpha)' < t \le m' \\ 0 & m' \le t \le n' \\ \dfrac{t-n'}{\beta} & n' \le t \le (n+\beta)' \\ 1 & otherwise \end{cases}$$

Definition: 8. (α - cut of LR-IFN). If $\tilde{P}^I_{LR} = (m, n, \alpha, \beta; m', n', \alpha', \beta')$ is known as an LR-type IFN, then the crisp set $(\tilde{P}_{LR})_\lambda = \{(t, \mu_{\tilde{P}^I_{LR}}(t), \upsilon_{\tilde{P}^I_{LR}}(t)) / \mu_{\tilde{P}^I_{LR}}(t) \ge \lambda \; and \; \upsilon_{\tilde{P}^I_{LR}}(t) \le 1-\alpha\}$, $\forall \lambda \in [0,1]$ is called α -cut of LR-IFN and $\tilde{P}^I_{LR}$ = $\{[\mu^L_{\tilde{P}^I_{LR}}(\lambda), \mu^U_{\tilde{P}^I_{LR}}(\lambda)], [\upsilon^L_{\tilde{P}^I_{LR}}(\lambda), \upsilon^U_{\tilde{P}^I_{LR}}(\lambda)]\}$ where $\mu^L_{\tilde{P}^I_{LR}}(\lambda) = (m-\alpha)+\alpha\lambda$, $\mu^U_{\tilde{P}^I_{LR}}(\lambda) = (\beta+n)-\beta\lambda$, and $\upsilon^L_{\tilde{P}^I_{LR}}(\lambda) = m-\alpha\lambda$, $\upsilon^U_{\tilde{P}^I_{LR}}(\lambda) = n+\beta\lambda$ $\forall \lambda \in [0,1]$.

Definition: 9. An accuracy function of Trapezoidal LRIFN $\tilde{P}^I_{LR} = (m, n, \alpha, \beta; m', n', \alpha', \beta')$is defined as shown in below

$$H(\tilde{P}^I_{LR}) = \frac{A(\mu_{\tilde{P}^I_{LR}}) + A(\upsilon_{\tilde{P}^I_{LR}})}{2}$$

i.e, $$H(\tilde{P}^I_{LR}) = \frac{(2m+2n-\alpha+\beta+2m'+2n'-\alpha'+\beta')}{8} \qquad (1)$$

3. Transportation Model

The linear programming formulations of the transportation model in the crisp state and the LR-intuitionistic fuzzy state are provided in light of this segment.

3.1 Fuzzy transportation model in crisp state

The main goal of a transportation model is to find ways to move goods at the lowest possible cost while still satisfying demand at final destinations and supply at source locations. The transportation model is explained mathematically as follows:

$$\text{Min} \quad \sum_{i=1}^{p}\sum_{j=1}^{q} c_{ij}\, t_{ij}$$

$$\text{s.t.} \quad \sum_{j=1}^{q} t_{ij} = a_i \qquad i = 1,2,3,\ldots p,$$

$$\sum_{i=1}^{p} t_{ij} = b_j \qquad j = 1,2,3,\ldots q,$$

$$t_{ij} \geq 0 \qquad i = 1,2,3,\ldots p, \quad j = 1,2,3,\ldots q. \qquad (2)$$

Where p and q is the whole number of origins and targets, a_i be the distribution of the material at the i^th origin, b_j is the requirements of the material at the j^th target, c_{ij} is the expenditure of transportation for an amount of the material from the i^th origin to the j^th target, and the t_{ij} be the amount of the material that should be shipped from the i^th origin to the j^th target to lower the overall shipping cost.

3.2 Transportation model in LR- Intuitionistic fuzzy state

The traditional transportation models take into account the true costs of supply, demand, and transportation. These are frequently ambiguous and imprecise when used in practical applications. In these situations, fuzzy sets can be used to identify the corresponding model-explaining components.

However, it might not be appropriate in situations where discussing indefinitely is required. In these situations, the transportation model's LR-Intuitionistic fuzzy numbers are utilized to handle the unknown restrictions. A LR-intuitionistic fuzzy transportation problem (LRIFTP) is therefore designated as the solution to the ensuing transportation model.

The arithmetical formulation of the LR-intuitionistic fuzzy transportation model is as follows:

$$\text{Min } \tilde{Z}_{LR}^{I} = \sum_{i=1}^{p}\sum_{j=1}^{q} \tilde{c}_{ij}^{I} \otimes \tilde{t}_{ij}^{I}$$

$$\text{s.t. } \sum_{j=1}^{q} \tilde{t}_{ij}^{I} = \tilde{a}_{i}^{I} \qquad i = 1,2,3,\ldots p,$$

$$\sum_{i=1}^{p} \tilde{t}_{ij}^{I} = \tilde{b}_{j}^{I} \qquad j = 1,2,3,\ldots q,$$

$$\tilde{t}_{ij}^{I} \geq 0 \qquad i = 1,2,3,\ldots p, \quad j = 1,2,3,\ldots q. \qquad (3)$$

Where $\tilde{a}_i^I = (a_{i,1}, a_{i,2}, a_{i,3}, a_{i,4}; a'_{i,1}, a'_{i,2}, a'_{i,3}, a'_{i,4})$ is LR-intutitionistic fuzzy supply of the product at i^th source, $\tilde{b}_j^I = (b_{j,1}, b_{j,2}, b_{j,3}, b_{j,4}; b'_{j,1}, b'_{j,2}, b'_{j,3}, b'_{j,4})$ is LR-intutitionistic fuzzy demand of the product at the j^th destination, $\tilde{c}_{ij}^I = (c_{ij,1}, c_{ij,2}, c_{ij,3}, c_{ij,4}; c'_{ij,1}, c'_{ij,2}, c'_{ij,3}, c'_{ij,4})$ is LR - intuititonistic fuzzy transportation cost for a unit part of the product from i^th source to j^th destination and $\tilde{t}_{ij}^I = (t_{ij,1}, t_{ij,2}, t_{ij,3}, t_{ij,4}; t'_{ij,1}, t'_{ij,2}, t'_{ij,3}, t'_{ij,4})$ be LR- intuitionistic fuzzy elements of the product that should be exported from i^th source to j^th destination to reduce or decrease the total LR-intuitionistic fuzzy shipping cost.

Remark: If $\sum_{i=1}^{p} \tilde{a}_i^I = \sum_{j=1}^{q} \tilde{b}_j^I$ is balanced LR-intuitionistic fuzzy transportation problem otherwise unbalanced LR-intuitionistic fuzzy transportation problem.

In this, the LR-intuitionistic objective function $\tilde{Z}_{LR}^{I} = \sum_{i=1}^{p}\sum_{j=1}^{q} \tilde{c}_{ij}^{I} \otimes \tilde{t}_{ij}^{I}$ can be revised as follows in that supply, demand and transportation costs of LR-IFTP are all represented by non-negative LR- intuitionistic fuzzy numbers.

$$\tilde{Z}^{I}_{LR} = \left(\sum_{i=1}^{p}\sum_{j=1}^{q} c_{ij,1}\, t_{ij,1}, \sum_{i=1}^{p}\sum_{j=1}^{q} c_{ij,2}\, t_{ij,2}, \sum_{i=1}^{p}\sum_{j=1}^{q} c_{ij,3}\, t_{ij,3}, \sum_{i=1}^{p}\sum_{j=1}^{q} c_{ij,4}\, t_{ij,4}; \right.$$
$$\left. \sum_{i=1}^{p}\sum_{j=1}^{q} c'_{ij,1}\, t'_{ij,1}, \sum_{i=1}^{p}\sum_{j=1}^{q} c'_{ij,2}\, t'_{ij,2}, \sum_{i=1}^{p}\sum_{j=1}^{q} c'_{ij,3}\, t'_{ij,3}, \sum_{i=1}^{p}\sum_{j=1}^{q} c'_{ij,4}\, t'_{ij,4}, \right)$$

Eight elements make up the resulting LR-intuitionistic fuzzy objective function, which is multiobjective in nature. The weighted sum approach is used to convert it into a single objective function and give each element the same weight $\frac{1}{8}$. Here is how the single objective function is expressed:

$$\sum_{i=1}^{p}\sum_{j=1}^{q} H\,(c_{ij,1}\, t_{ij,1}, c_{ij,2}\, t_{ij,2}, c_{ij,3}\, t_{ij,3}, c_{ij,4}\, t_{ij,4}; c'_{ij,1}\, t'_{ij,1}, c'_{ij,2}\, t'_{ij,2}, c'_{ij,3}\, t'_{ij,3}, c'_{ij,4}\, t'_{ij,4})$$
$$=$$
$$\frac{1}{8}\sum_{i=1}^{p}\sum_{j=1}^{q} (c_{ij,1}\, t_{ij,1} + c_{ij,2}\, t_{ij,2} + c_{ij,3}\, t_{ij,3} + c_{ij,4}\, t_{ij,4} + c'_{ij,1}\, t'_{ij,1}, + c'_{ij,2}\, t'_{ij,2} + c'_{ij,3}\, t'_{ij,3} + c'_{ij,4}\, t'_{ij,4})$$

4. Proposed Method

The following new algorithm is provided in this segment to get the LR- intuitionistic fuzzy optimal solution of the LR- IFTP (3):

$$\text{Min } H\left(\tilde{Z}^{I}_{LR}\right) = H\left(\sum_{i=1}^{p}\sum_{j=1}^{q} \tilde{c}^{I}_{ij} \otimes \tilde{t}^{I}_{ij}\right)$$
$$\text{s.t. Constraints of LR-IFTP (3)} \qquad (4)$$

To specify the ambiguous and hesitating in the choice variables in (4), we take into consideration non-negative trapezoidal intuitionistic fuzzy elements. As a result, Eqn(4) can be rewritten as follows:

Min

$$H\left(\sum_{i=1}^{p}\sum_{j=1}^{q}\left(c_{ij,1}, c_{ij,2}, c_{ij,3}, c_{ij,4}; c'_{ij,1}, c'_{ij,2}, c'_{ij,3}, c'_{ij,4}\right) \otimes \left(t_{ij,1}, t_{ij,2}, t_{ij,3}, t_{ij,4}; t'_{ij,1}, t'_{ij,2}, t'_{ij,3}, t'_{ij,4}\right)\right)$$

s.t. $\sum_{j=1}^{q}\left(t_{ij,1}, t_{ij,2}, t_{ij,3}, t_{ij,4}; t'_{ij,1}, t'_{ij,2}, t'_{ij,3}, t'_{ij,4}\right) = \left(a_{i,1}, a_{i,2}, a_{i,3}, a_{i,4}; a'_{i,1}, a'_{i,2}, a'_{i,3}, a'_{i,4}\right), \quad i = 1,2,3,\ldots p$

(5.1)

$$\sum_{i=1}^{p}\left(t_{ij,1}, t_{ij,2}, t_{ij,3}, t_{ij,4}; t'_{ij,1}, t'_{ij,2}, t'_{ij,3}, t'_{ij,4}\right) = \left(b_{j,1}, b_{j,2}, b_{j,3}, b_{j,4}; b'_{j,1}, b'_{j,2}, b'_{j,3}, b'_{j,4}\right), \quad j = 1,2,3,\ldots q$$

(5.2)

$$\left(t_{ij,1}, t_{ij,2}, t_{ij,3}, t_{ij,4}; t'_{ij,1}, t'_{ij,2}, t'_{ij,3}, t'_{ij,4}\right) \geq 0^{I} \qquad (5.3)$$

(5)

The goal function of technique (5) may be rewritten as indicated below, as we mentioned in the previous chapters.

$$\text{Min} \quad H\Bigg(\sum_{i=1}^{p}\sum_{j=1}^{q} c_{ij,1}\, t_{ij,1}, \sum_{i=1}^{p}\sum_{j=1}^{q} c_{ij,2}\, t_{ij,2}, \sum_{i=1}^{p}\sum_{j=1}^{q} c_{ij,3}\, t_{ij,3}, \sum_{i=1}^{p}\sum_{j=1}^{q} c_{ij,4}\, t_{ij,4},$$
$$\sum_{i=1}^{p}\sum_{j=1}^{q} c'_{ij,1}\, t'_{ij,1}, \sum_{i=1}^{p}\sum_{j=1}^{q} c'_{ij,2}\, t'_{ij,2}, \sum_{i=1}^{p}\sum_{j=1}^{q} c'_{ij,3}\, t'_{ij,3}, \sum_{i=1}^{p}\sum_{j=1}^{q} c'_{ij,4}\, t'_{ij,4}, \Bigg)$$

(6)

Constraints (5.1) & (5.2) in method (5) may be reformulated as follows

$$\left(\sum_{j=1}^{q} t_{ij,1}, \sum_{j=1}^{q} t_{ij,2}, \sum_{j=1}^{q} t_{ij,3}, \sum_{j=1}^{q} t_{ij,4}, \sum_{j=1}^{q} t'_{ij,1}, \sum_{j=1}^{q} t'_{ij,2}, \sum_{j=1}^{q} t'_{ij,3}, \sum_{j=1}^{q} t'_{ij,4}\right) = \left(a_{i,1}, a_{i,2}, a_{i,3}, a_{i,4}, a'_{i,1}, a'_{i,2}, a'_{i,3}, a'_{i,4}\right)$$

(7)

$$\left(\sum_{i=1}^{p} t_{ij,1}, \sum_{i=1}^{p} t_{ij,2}, \sum_{i=1}^{p} t_{ij,3}, \sum_{i=1}^{p} t_{ij,4}, \sum_{i=1}^{p} t'_{ij,1}, \sum_{i=1}^{p} t'_{ij,2}, \sum_{i=1}^{p} t'_{ij,3}, \sum_{i=1}^{p} t'_{ij,4}\right) = \left(b_{j,1}, b_{j,2}, b_{j,3}, b_{j,4}, b'_{j,1}, b'_{j,2}, b'_{j,3}, b'_{j,4}\right)$$

(8)

Applying the coordination of two intuitionistic fuzzy integers, limitations (7) & (8) can be reformulated as shown in below constraints:

$$\begin{aligned} &\sum_{j=1}^{q} t_{ij,1} = a_{i,1}, \sum_{j=1}^{q} t_{ij,2} = a_{i,2}, \sum_{j=1}^{q} t_{ij,3} = a_{i,3}, \sum_{j=1}^{q} t_{ij,4} = a_{i,4} \qquad i = 1, 2, \ldots p, \\ &\sum_{j=1}^{q} t'_{ij,1} = a'_{i,1}, \sum_{j=1}^{q} t'_{ij,2} = a'_{i,2}, \sum_{j=1}^{q} t'_{ij,3} = a'_{i,3}, \sum_{j=1}^{q} t'_{ij,4} = a'_{i,4} \qquad i = 1, 2, \ldots p. \end{aligned} \tag{9}$$

$$\begin{aligned} &\sum_{i=1}^{p} t_{ij,1} = b_{j,1}, \sum_{i=1}^{p} t_{ij,2} = b_{j,2}, \sum_{i=1}^{p} t_{ij,3} = b_{j,3}, \sum_{i=1}^{p} t_{ij,4} = b_{j,4} \qquad j = 1, 2, \ldots q, \\ &\sum_{i=1}^{p} t'_{ij,1} = b'_{j,1}, \sum_{i=1}^{p} t'_{ij,2} = b'_{j,2}, \sum_{i=1}^{p} t'_{ij,3} = b'_{j,3}, \sum_{i=1}^{p} t'_{ij,4} = b'_{j,4} \qquad j = 1, 2, \ldots q. \end{aligned} \tag{10}$$

And the constraints (5.3)

$$\left(t_{ij,1}, t_{ij,2}, t_{ij,3}, t_{ij,4}; t'_{ij,1}, t'_{ij,2}, t'_{ij,3}, t'_{ij,4}\right) \geq 0^I$$

Correspondingly, equation (5) is transformed to the following case:

$$\text{Min} \quad H\Bigg(\sum_{i=1}^{p}\sum_{j=1}^{q} c_{ij,1}\, t_{ij,1}, \sum_{i=1}^{p}\sum_{j=1}^{q} c_{ij,2}\, t_{ij,2}, \sum_{i=1}^{p}\sum_{j=1}^{q} c_{ij,3}\, t_{ij,3}, \sum_{i=1}^{p}\sum_{j=1}^{q} c_{ij,4}\, t_{ij,4}$$
$$\sum_{i=1}^{p}\sum_{j=1}^{q} c'_{ij,1}\, t'_{ij,1}, \sum_{i=1}^{p}\sum_{j=1}^{q} c'_{ij,2}\, t'_{ij,2}, \sum_{i=1}^{p}\sum_{j=1}^{q} c'_{ij,3}\, t'_{ij,3}, \sum_{i=1}^{p}\sum_{j=1}^{q} c'_{ij,4}\, t'_{ij,4} \Bigg)$$

(11)

Equations (8), (9) and (5.3) are subject to constraints.

According to [18], the accuracy function is linear, and equation (11)'s objective function is the same as the following objective function:

$$\sum_{i=1}^{p}\sum_{j=1}^{q} H\left(c_{ij,1}t_{ij,1}, c_{ij,2}\,t_{ij,2}, c_{ij,3}\,t_{ij,3}, c_{ij,4}\,t_{ij,4}\,; c'_{ij,1}\,t'_{ij,1}, c'_{ij,2}\,t'_{ij,2}, c'_{ij,3}\,t'_{ij,3}, c'_{ij,4}\,t'_{ij,4}\right)$$

$$= \frac{1}{8} \quad \sum_{i=1}^{p}\sum_{j=1}^{q} c_{ij,1}t_{ij,1} + c_{ij,2}\,t_{ij,2}, + c_{ij,3}\,t_{ij,3} + c_{ij,4}\,t_{ij,4} + c'_{ij,1}\,t'_{ij,1} + c'_{ij,2}\,t'_{ij,2} + c'_{ij,3}\,t'_{ij,3} + c'_{ij,4}\,t'_{ij,4}$$

(12)

Equation (11) is consequently changed to the deterministic equivalent case is as follows:

Min $\sum_{i=1}^{p}\sum_{j=1}^{q} H\left(c_{ij,1}t_{ij,1}, c_{ij,2}\,t_{ij,2}, c_{ij,3}\,t_{ij,3}, c_{ij,4}\,t_{ij,4}\,; c'_{ij,1}\,t'_{ij,1}, c'_{ij,2}\,t'_{ij,2}, c'_{ij,3}\,t'_{ij,3}, c'_{ij,4}\,t'_{ij,4}\right)$

s. t constraints equations (8), (9) and (5.3). (13)

The above equation (13) can be obtained by using standard linear programming models. By replacing the optimum solution of equation (13) i.e., $\bar{t}_{ij,1}, \bar{t}_{ij,2}, \bar{t}_{ij,3}, \bar{t}_{ij,4}; \bar{t}'_{ij,1}, \bar{t}'_{ij,2}, \bar{t}'_{ij,3}$ and $\bar{t}'_{ij,4}$ in $\tilde{t}^{I}_{ij} = \left(t_{ij,1}, t_{ij,2}, t_{ij,3}, t_{ij,4}\,; t'_{ij,1}, t'_{ij,2}, t'_{ij,3}, t'_{ij,4}\right)$, the excellent solution of LRIFTP (3) is obtained. Also, the entire least intuitionistic fuzzy shipping expenditure is solved by replacing the values of $\tilde{\bar{t}}^{I}_{ij} = \left(\bar{t}_{ij,1}, \bar{t}_{ij,2}, \bar{t}_{ij,3}, \bar{t}_{ij,4}\,; \bar{t}'_{ij,1}, \bar{t}'_{ij,2}, \bar{t}'_{ij,3}, \bar{t}'_{ij,4}\right)$ in $\tilde{Z}^{I}_{LR} = \sum_{i=1}^{p}\sum_{j=1}^{q} \tilde{c}^{I}_{ij} \otimes \tilde{t}^{I}_{ij}$.

The steps that make up the suggested technique are given below:

Step 1: Create specified LRIFTP.

Step 2: By using the procedure described in this segment, the data acquired in step 1 of LRIFTP are converted into its equivalent crisp linear programming problem (13).

Step 3: Determine the best solution $\bar{t}_{ij,1}, \bar{t}_{ij,2}, \bar{t}_{ij,3}, \bar{t}_{ij,4}; \bar{t}'_{ij,1}, \bar{t}'_{ij,2}, \bar{t}'_{ij,3}$ and $\bar{t}'_{ij,4}$ Using the deterministic linear programming problem (13),

Step 4: Detect an LR intuitionistic fuzzy optimal solution $\{\tilde{\bar{t}}_{ij}\}$ by substituting the values of $\bar{t}_{ij,1}, \bar{t}_{ij,2}, \bar{t}_{ij,3}, \bar{t}_{ij,4}; \bar{t}'_{ij,1}, \bar{t}'_{ij,2}, \bar{t}'_{ij,3}$ and $\bar{t}'_{ij,4}$ in $\tilde{t}_{ij} = \left(\mathrm{t}_{ij,1}, t_{ij,2}, t_{ij,3}, t_{ij,4}; t'_{ij,1}, t'_{ij,2}, t'_{ij,3}, t'_{ij,4}\right)$.

Step 5: Obtain LR intuitionistic fuzzy transportation cost by substituting the elements of $\tilde{\bar{t}}_{ij}$ in $Z^{I}_{LR} = \sum_{i=1}^{p}\sum_{j=1}^{q} \tilde{c}^{I}_{ij} \otimes \tilde{t}^{I}_{ij}$.

Step 6: Utilizing the accuracy function, determine the lowest LR intuitionistic fuzzy transportation cost in the crisp form.

5. Numerical Example

In light of this section, one numerical example is provided as indicated in the example below to illustrate the suggested methodology.

Example:

This illustrates the use of the suggested method by obtaining the most economical shipping

expenditure from two sources to three destinations, which reduces the overall expenditure of transportation and is considered by LR intuitionistic trapezoidal fuzzy elements.

Take a look at an LRIFTP that has two sources and three destinations. The following table 1 lists the LR intuitionistic fuzzy costs, supply, and requirements for an amount of an item from the i^th^ source to the j^th^ destination.

Table 1:

	D_1	D_2	D_3	Supply
S_1	(20,30,10,10; 15,35,10,10)	(60,70,10,20; 55,75,10,20)	(90,110,10,10; 85,115,10,10)	80,100,20,20; 70,110,20,20)
S_2	(70,80,10,10; 65,85,10,10)	(80,100,10,20; 75,115,10,10)	(30,50,10,10; 25,35,10,30)	(60,80,20,20; 50,90,20,20)
Demand	(50,70,20,20; 40,80,20,20)	(30,40,10,10; 25,45,10,10)	(60,70,10,10; 55,75,10,10)	

The transportation chart above is evenly distributed. Therefore, LRIFTP is expressed mathematically as follows:

$$
\begin{aligned}
Min \quad Z_{LR}^{I} = &\left((20,30,10,10;15,35,10,10)\otimes t_{11}^{I}\right)+\left((60,70,10,20;55,75,10,20)\otimes t_{12}^{I}\right)\\
&+\left((90,110,10,10;85,115,10,10)\otimes t_{13}^{I}\right)+\left((70,80,10,10;65,85,10,10)\otimes t_{21}^{I}\right)\\
&+\left((80,100,10,20;75,115,10,10)\otimes t_{22}^{I}\right)+\left(30,50,10,10;25,35,10,30)\otimes t_{23}^{I}\right)\\
s.t. \quad & t_{11}^{I}+t_{12}^{I}+t_{13}^{I}=(80,100,20,20;70,110,20,20)\\
& t_{21}^{I}+t_{22}^{I}+t_{23}^{I}=(60,80,20,20;50,90,20,20)\\
& t_{11}^{I}+t_{21}^{I}=(50,70,20,20;40,80,20,20)\\
& t_{12}^{I}+t_{22}^{I}=(30,40,10,10;25,45,10,10)\\
& t_{13}^{I}+t_{23}^{I}=(60,70,10,10;55,75,10,10)
\end{aligned}
$$

$t_{11}^{I}, t_{12}^{I}, t_{13}^{I}, t_{21}^{I}, t_{22}^{I}, t_{23}^{I}$, are non negative LR intuitionistic trapezoidal fuzzy elements. (14)

In accord with the methodology described in segment 4 and in light of methodology (13), the formulated LRIFTP (14) is converted into a clear linear programming problem as follows:

$$
\begin{aligned}
Min = {} & \frac{1}{8}\left(40t_{11,1}+60t_{11,2}-10t_{11,3}+10t_{11,4}+30t'_{11,1}+70t'_{11,2}-10t'_{11,3}+10t'_{11,4}\right)\\
& +\frac{1}{8}\left(120t_{12,1}+140t_{12,2}-10t_{12,3}+20t_{12,4}+110t'_{12,1}+150t'_{12,2}-10t'_{12,3}+20t'_{12,4}\right)\\
& +\frac{1}{8}\left(180t_{13,1}+220t_{13,2}-10t_{13,3}+10t_{13,4}+170t'_{13,1}+230t'_{13,2}-10t'_{13,3}+10t'_{13,4}\right)\\
& +\frac{1}{8}\left(140t_{21,1}+160t_{21,2}-10t_{21,3}+10t_{21,4}+130t'_{21,1}+170t'_{21,2}-10t'_{21,3}+10t'_{21,4}\right)\\
& +\frac{1}{8}\left(160t_{22,1}+200t_{22,2}-10t_{22,3}+20t_{22,4}+150t'_{22,1}+230t'_{22,2}-10t'_{22,3}+10t'_{22,4}\right)\\
& +\frac{1}{8}\left(60t_{23,1}+100t_{23,2}-10t_{23,3}+10t_{23,4}+50t'_{23,1}+70t'_{23,2}-10t'_{23,3}+30t'_{23,4}\right)
\end{aligned}
$$

$$
\begin{aligned}
s.t.\quad & t_{11,1}+t_{12,1}+t_{13,1}=80,\quad t'_{11,1}+t'_{12,1}+t'_{13,1}=70,\\
& t_{11,2}+t_{12,2}+t_{13,2}=100,\quad t'_{11,2}+t'_{12,2}+t'_{13,2}=110\\
& t_{11,3}+t_{12,3}+t_{13,3}=20,\quad t'_{11,3}+t'_{12,3}+t'_{13,3}=20,\\
& t_{11,4}+t_{12,4}+t_{13,4}=20,\quad t'_{11,4}+t'_{12,4}+t'_{13,4}=20,\\
& t_{21,1}+t_{22,1}+t_{23,1}=60,\quad t'_{21,1}+t'_{22,1}+t'_{23,1}=50,\\
& t_{21,2}+t_{22,2}+t_{23,2}=80,\quad t'_{21,2}+t'_{22,2}+t'_{23,2}=90,\\
& t_{21,3}+t_{22,3}+t_{23,3}=20,\quad t'_{21,3}+t'_{22,3}+t'_{23,3}=20,\\
& t_{21,4}+t_{22,4}+t_{23,4}=20,\quad t'_{21,4}+t'_{22,4}+t'_{23,4}=20,\\
& t_{11,1}+t_{21,1}=50,\quad t'_{11,1}+t'_{21,1}=40,\\
& t_{11,2}+t_{21,2}=70,\quad t'_{11,2}+t'_{21,2}=80,\\
& t_{11,3}+t_{21,3}=20,\quad t'_{11,3}+t'_{21,3}=20,\\
& t_{11,4}+t_{21,4}=20,\quad t'_{11,4}+t'_{21,4}=20,\\
& t_{12,1}+t_{22,1}=30,\quad t'_{12,1}+t'_{22,1}=25,\\
& t_{12,2}+t_{22,2}=40,\quad t'_{12,2}+t'_{22,2}=45,\\
& t_{12,3}+t_{22,3}=10,\quad t'_{12,3}+t'_{22,3}=10,\\
& t_{12,4}+t_{22,4}=10,\quad t'_{12,4}+t'_{22,4}=10,\\
& t_{13,1}+t_{23,1}=60,\quad t'_{13,1}+t'_{23,1}=55,\\
& t_{13,2}+t_{23,2}=70,\quad t'_{13,2}+t'_{23,2}=75,\\
& t_{13,3}+t_{23,3}=10,\quad t'_{13,3}+t'_{23,3}=10,\\
& t_{13,4}+t_{23,4}=10,\quad t'_{13,4}+t'_{23,4}=10,
\end{aligned}
$$

$$t_{ij,1},t_{ij,2},t_{ij,3},t_{ij,4},t'_{ij,1},t'_{ij,2},t'_{ij,3},t'_{ij,4}\ge 0\quad i=1,2,\ j=1,2,3 \tag{15}$$

The optimal solution of the crisp linear programming problem (15) is determined as follows:

$$
\begin{aligned}
&\bar{t}_{11,1}=50,\bar{t}_{11,2}=70,\ \bar{t}_{11,3}=0,\ \bar{t}_{11,4}=0,\ \bar{t}'_{11,1}=40,\bar{t}'_{11,2}=80,\bar{t}'_{11,3}=0,\bar{t}'_{11,4}=10,\\
&\bar{t}_{12,1}=30,\bar{t}_{12,2}=30,\bar{t}_{12,3}=10,\bar{t}_{12,4}=10,\bar{t}'_{12,1}=25,\bar{t}'_{12,2}=30,\bar{t}'_{12,3}=10,\bar{t}'_{12,4}=0,\\
&\bar{t}_{13,1}=0,\ \bar{t}_{13,2}=0,\ \bar{t}_{13,3}=10,\ \bar{t}_{13,4}=10,\ \bar{t}'_{13,1}=5,\bar{t}'_{13,2}=0,\bar{t}'_{13,3}=10,\bar{t}'_{13,4}=10,\\
&\bar{t}_{21,1}=0,\bar{t}_{21,2}=0,\bar{t}_{21,3}=0,\bar{t}_{21,4}=0,\ \bar{t}'_{21,1}=0,\ \bar{t}'_{21,2}=0,\ \bar{t}'_{21,3}=20,\ \bar{t}'_{21,4}=10,\\
&\bar{t}_{22,1}=0,\bar{t}_{22,2}=10,\bar{t}_{22,3}=0,\bar{t}_{22,4}=0,\ \bar{t}'_{22,1}=0,\ \bar{t}'_{22,2}=15,\ \bar{t}'_{22,3}=0,\ \bar{t}'_{22,4}=10,\\
&\bar{t}_{23,1}=60,\bar{t}_{23,2}=70,\bar{t}_{23,3}=0,\bar{t}_{23,4}=0,\ \bar{t}'_{23,1}=50,\bar{t}'_{23,2}=75,\bar{t}'_{23,3}=0,\bar{t}'_{23,4}=0\ .
\end{aligned}
\tag{16}
$$

By putting the values of $\bar{t}_{ij,1},\bar{t}_{ij,2},\bar{t}_{ij,3},\bar{t}_{ij,4},\bar{t}'_{ij,1},\bar{t}'_{ij,2},\bar{t}'_{ij,3}$ and $\bar{t}'_{ij,4}$ in $\tilde{\bar{t}}_{ij}^{I}=\left(\bar{t}_{ij,1},\bar{t}_{ij,2},\bar{t}_{ij,3},\bar{t}_{ij,4};\bar{t}'_{ij,1},\bar{t}'_{ij,2},\bar{t}'_{ij,3},\bar{t}'_{ij,4}\right)$, the LR intuitionistic fuzzy optimal solution of LRIFTP is determined as follows:

$$
\begin{aligned}
&\tilde{\bar{t}}_{11}^{I}=(50,70,0,0;40,80,0,10),\ \tilde{\bar{t}}_{12}^{I}=(30,30,10,10;25,30,10,0),\\
&\tilde{\bar{t}}_{13}^{I}=(0,0,10,10;5,0,10,10),\ \ \tilde{\bar{t}}_{21}^{I}=(0,0,0,0;0,0,20,10),\\
&\tilde{\bar{t}}_{22}^{I}=(0,10,0,0;0,15,0,10),\ \ \tilde{\bar{t}}_{23}^{I}=(60,70,0,0;50,75,0,0)
\end{aligned}
\tag{17}
$$

By giving the values of $\bar{t}_{ij,1},\bar{t}_{ij,2},\bar{t}_{ij,3},\bar{t}_{ij,4},\bar{t}'_{ij,1},\bar{t}'_{ij,2},\bar{t}'_{ij,3}$ and $\bar{t}'_{ij,4}$ in the objective function of the LRIFTP (14), the total LR intuitionistic fuzzy transportation cost is obtained as follows:

$$\tilde{\bar{Z}}_{LR}^{I}=\sum_{i-1}^{2}\sum_{j=1}^{3}\tilde{c}_{ij}^{I}\otimes\tilde{\bar{t}}_{ij}^{I}=(4600,8700,200,300;3650,9400,400,400).$$

$\therefore$ The following is the overall minimal LR intuitionistic fuzzy transportation cost employing accuracy function (1) in crisp form:

$$H\left(\tilde{Z}_{LR}^{I}\right)=H\left(\sum_{i=1}^{p}\sum_{j=1}^{q}\tilde{c}_{ij}^{I}\otimes\tilde{t}_{ij}^{I}\right)=6600.$$

6. Conclusion:

In this paper, a unique technique for balancing LR intuitionistic fuzzy transportation problems is proposed. In this problem, the supply and demand of the commodity, as well as the transportation costs, are all represented as LR intuitionistic trapezoidal fuzzy numbers. The suggested approach for solving LRIFTP based on traditional linear programming problems was used to present a numerical example and provide the LR intuitionistic fuzzy optimal solution. Additionally, using the accuracy function, the minimum LR intuitionistuc fuzzy transportation cost was calculated in crisp form. The suggested method has the benefit of determining an optimal LR intuitionistic fuzzy solution, and the best values are non- negative LR intuitionistic fuzzy numbers. This approach is straightforward to comprehend and use in practical situations.

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