On Vertex Strongly*-Graph of Double Comb and Closed Helm Graphs

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Abstract

A graph G(V, E) is said to be a vertex strongly*-graph if there exists a bijection $f: E \rightarrow \{1, 2, \ldots . q\}$, such that for every vertex $u \epsilon V$, $\sum f(uv_i) + \prod f(uv_i)$ are distinct, where uv_i are the edges incident to the vertex u. In this paper, we will be proving that double comb graph and closed helm are vertex strongly*-graph.

1. Introduction

A graph $G(V, E)$ is a set of vertices V and edges E, each vertex $e \epsilon E$ has its end vertices in V. The graph is called a connected graph if there is a path between every two vertices. A graph with no self-loops and multiple edges is called a *simple graph*. Graph theory has a lot of applications in science and technology. Graph labeling is applied in cryptography, data science, and blockchain. Graph labeling is introduced by Rosa [1]. Gallian [5] gives different kinds of labeling. There are various kinds of edge labeling. One such work is the vertex strongly*-graph, which is introduced by Beaula and Baskar Babujee [2]. Baskar Babujee et. Al. [4] have proved that wheels, paths, crowns, fans, and umbrellas are vertex strongly*-graphs. This paper will prove that double comb graph and closed helm are vertex strongly*-graphs.

Definition 1.1: Vertex strongly $*$ -graph [2]

A graph G(V, E) is said to be a *vertex strongly*-graph* if there exists a bijection $f: E \rightarrow \{1, 2, \ldots . q\}$, such that for every vertex $u \epsilon V$, $\sum f(uv_i) + \prod f(uv_i)$ are distinct, where uv_i are the edges incident to the vertex u.

Definition 1.2: Double Comb graph [6]

A *double comb graph* is a graph obtained from a path P_n by attaching two pendant vertices at each vertex of P_n denoted by $P_n \odot 2K_1$. For example, See Figure 1. Double comb graph $P_6 \odot 2K_1$

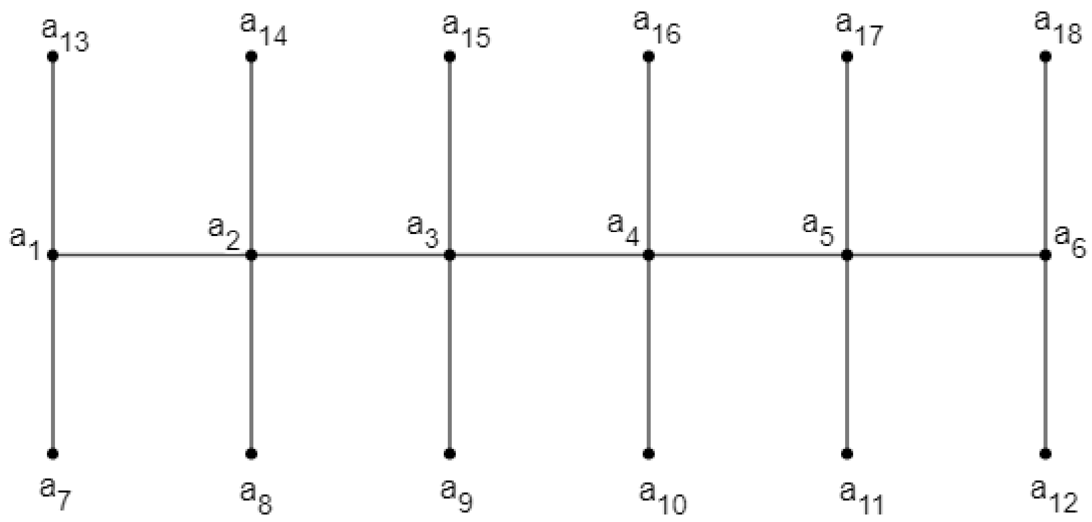

Figure 1: Double comb graph $P_6 \odot 2K_1$

Definition 1.3: Closed Helm [3]

A *closed helm* CH_n is the graph obtained by taking a helm H_n and adding edges between the pendant vertices. For example, See Figure 2. Closed helm CH_8

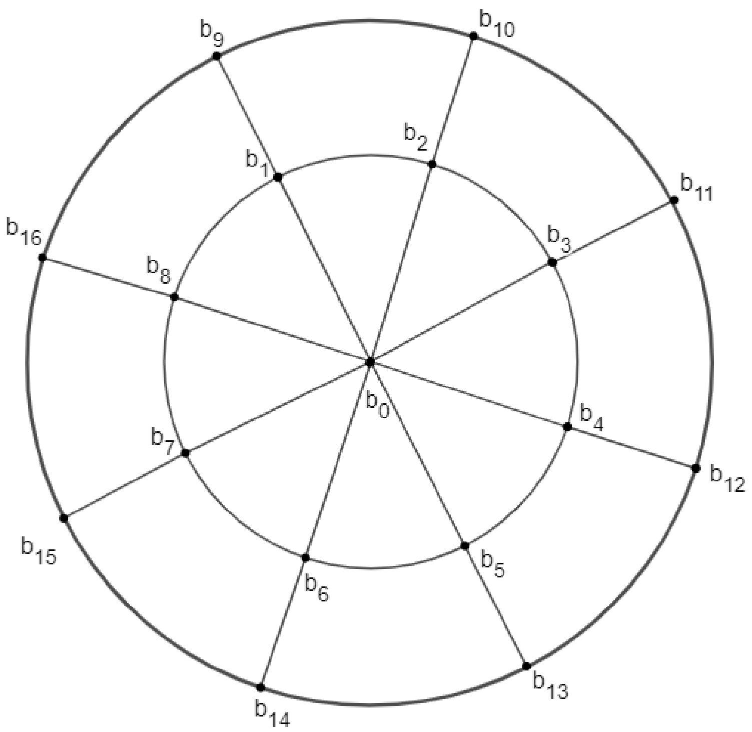

Figure 2. Closed helm CH_8

2. Main Results

Theorem 2.1

The double comb graph $P_n \odot 2K_1$ is a vertex strongly*-graph.

Proof:

Let $V = \{a_1, a_2, \ldots, a_n, a_{n+1}, \ldots, a_{2n}, a_{2n+1}, \ldots, a_{3n}\}$ be the vertex set and,
$E = \{(a_i, a_{i+1}), 1 \le i \le n-1\} \cup \{(a_i, a_{n+i}), (a_i, a_{2n+i})\ 1 \le i \le n\}$ be the edge set of the double comb graph $P_n \odot 2K_1$.

Labeling the edges of the double comb graph $P_n \odot 2K_1$ using a bijective function g defined as follows,

$g: E \to \{1, 2, 3, \ldots, 3n-1\}$, such that

$g(a_i, a_{i+1}) = i, 1 \le i \le n-1$

$g(a_i, a_{n+i}) = n + i - 1, 1 \le i \le n$

$g(a_i, a_{2n+i}) = 2n + i - 1, 1 \le i \le n$

The following Figure 3 is an example of an edge labeled double comb graph $P_6 \odot 2K_1$

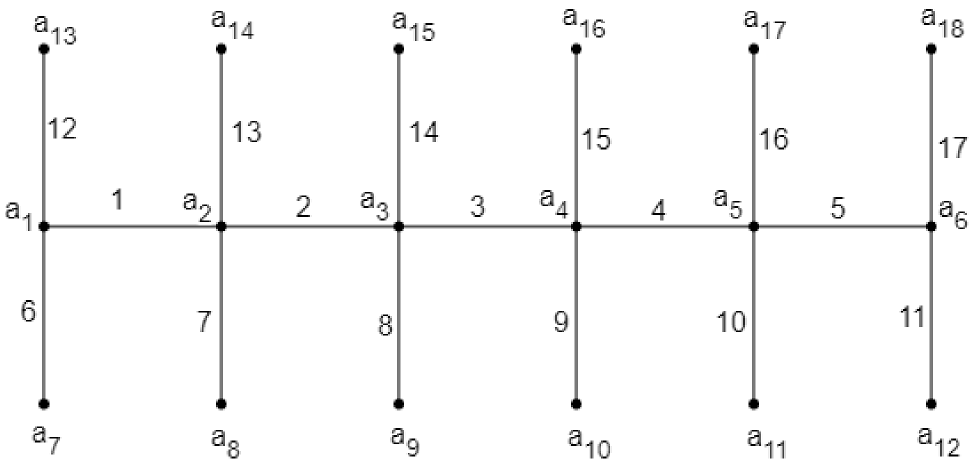

Figure 3: Edge-labeled double comb graph $P_6 \odot 2K_1$

To prove that the double comb graph $P_n \odot 2K_1$ is a vertex strongly*-graph,
The vertex calculation for each vertex is given below,

$f(a_1) = 2n^2 + 3n + 1$

$f(a_n) = (2n-1)(3 + (n-1)(3n-1))$

$f(a_i) = 3n + 4i - 3 + i(i-1)(n+i-1)(2n+i-1), 2 \le i \le n-1$

$f(a_{n+i}) = 2(n+i-1), 1 \le i \le n$

$f(a_{2n+i}) = 2(2n + i - 1), 1 \leq i \leq n$

To prove that the above calculations are distinct for each vertex.

Case (a): Consider $f(a_{n+i})$ and $f(a_{n+i+1})$

$f(a_{n+i}) = 2(n + i - 1), 1 \leq i \leq n$

$f(a_{n+i+1}) = 2(n + i + 1 - 1)$

$f(a_{n+i+1}) = 2(n + i - 1)+2$

$f(a_{n+i+1}) = f(a_{n+i}) + 2$

$f(a_{n+i+1}) \neq f(a_{n+i})$ for $1 \leq i \leq n$

Case (b): Consider $f(a_{n+i})$ and $f(a_{n+i+1})$

$f(a_{2n+i}) = 2(2n + i - 1), 1 \leq i \leq n$

$f(a_{2n+i+1}) = 2(2n + i + 1 - 1)$

$f(a_{2n+i+1}) = 2(2n + i - 1)+2$

$f(a_{2n+i+1}) = f(a_{2n+i}) + 2$

$f(a_{2n+i+1}) \neq f(a_{2n+i})$ for $1 \leq i \leq n$

Therefore, the vertex calculation is distinct for each vertex.

Hence the double comb graph $P_n \odot 2K_1$ is a vertex strongly*-graph.

Theorem 2.2

The closed helm CH_n is a vertex strongly*-graph.

Proof:

Let $V = \{b_0, b_1, b_2, \dots, b_n, b_{n+1}, b_{n+2}, \dots, b_{2n}\}$ be the vertex set and,
$E = \{(b_0, b_i), (b_i, b_{i+1}), (b_i, b_{n+i}), (b_{n+i}, b_{n+i+1})\ 1 \leq i \leq n - 1\} \cup$
$\{(b_0, b_n), (b_n, b_1), (b_n, b_{2n}), (b_{2n}, b_{n+1})\}$ be the edge set of the closed helm CH_n.

Labeling the edges of the closed helm CH_n, using a bijective function g as defined below.

$g: E \to \{1, 2,3, \dots ,4n\}$, such that

$g(b_0, b_i) = i, 1 \leq i \leq n$

$g(b_i, b_{i+1}) = n + i, 1 \leq i \leq n - 1$

$g(b_n, b_1) = 2n$

$g(b_i, b_{n+i}) = 2n + i, 1 \leq i \leq n$

$g(b_{n+i}, b_{n+i+1}) = 3n + i, 1 \leq i \leq n - 1$

$g(b_{2n}, b_{n+1}) = 4n$

The following Figure 4 is an example of the edge labeled closed helm CH_8.

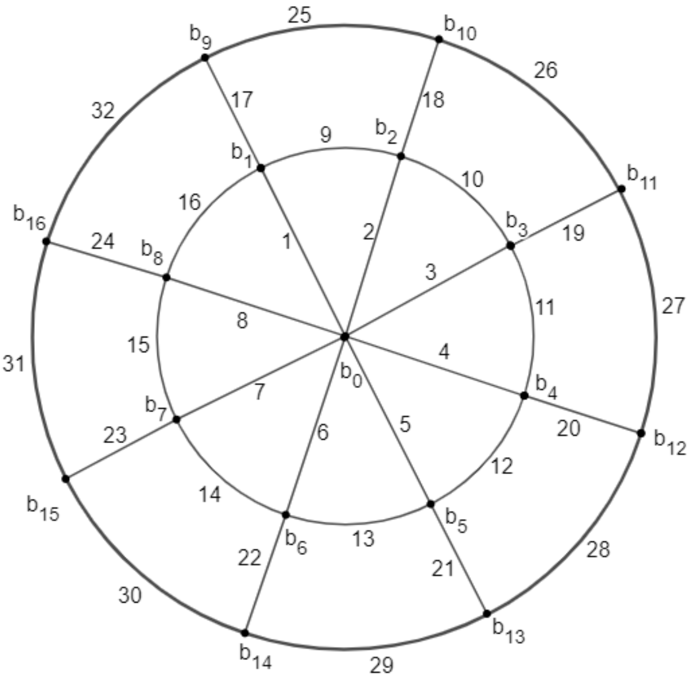

Figure 3: Edge labeled closed helm CH_8

To prove that the closed helm CH_n is a vertex strongly*-graph,
The vertex calculation for each vertex is given below,

$f(b_0) = n! + \frac{n(n+1)}{2}$

$f(b_1) = 5n + 3 + (2n)(2n + 1)(n + 1)$

$f(b_i) = 4n + 4i - 1 + i(n + i - 1)(n + i)(2n + i)$, $2 \leq i \leq n$

$f(b_{n+1}) = 9n + 2 + (4n)(3n + 1)(2n + 1)$

$f(b_{n+i}) = 8n + 3i - 1 + (2n + i)(3n + i - 1)(3n + i)$, $2 \leq i \leq n$

To prove that the above calculations are distinct for each vertex.

Case (a): Consider $f(b_i)$ and $f(b_{i+1})$, $2 \leq i \leq n$

$f(b_i) = 4n + 4i - 1 + i(n + i - 1)(n + i)(2n + i)$

$f(b_{i+1}) = 4n + 4(i + 1) - 1 + (i + 1)(n + i + 1 - 1)(n + i + 1)(2n + i + 1)$

$f(b_{i+1}) = f(b_i) + 4 + (n + 2i)(n + i + 1)(2n + i + 1) + i(n + i - 1)(3n + 2i + 1)$

$f(b_{i+1}) \neq f(b_i)$ for $2 \leq i \leq n$

Case (b): Consider $f(b_{n+i})$ and $f(b_{n+i+1})$, $2 \leq i \leq n$

$f(b_{n+i}) = 8n + 3i - 1 + (2n + i)(3n + i - 1)(3n + i)$

$f(b_{n+i+1}) = 8n + 3(i + 1) - 1 + (2n + i + 1)(3n + i + 1 - 1)(3n + i + 1)$

$f(b_{n+i+1}) = f(b_{n+i}) + 3 + (3n + i)(7n + 3i + 1)$

$f(b_{n+i+1}) \neq f(b_{n+i})$ for $2 \leq i \leq n$

Therefore, the calculated values are distinct for each vertex.
Hence closed helm CH_n is a vertex strongly*-graph.

Conclusion:

This paper gives the results on vertex strongly*-graph on double comb and closed helm graphs. This work can be extended to different graph structures. This labeling has good applications in radio signal assignments, network analysis, cryptography, and other engineering-related fields.

References

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