On Vertex Strongly*-Graph of some Flower Graphs

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Abstract

A graph G(V, E) is said to be a vertex strongly*-graph if there exists a bijection $f: E \to \{1, 2, \dots . q\}$, such that for every vertex $u \epsilon V$, $\sum f(uv_i) + \prod f(uv_i)$ are distinct, where uv_i are the edges incident to the vertex u. In this paper, we will be proving that some flower graphs are vertex strongly*-graph.

1. Introduction

A graph $G(V, E)$ is a set of vertices V and edges E, each vertex $e \epsilon E$ has its end vertices in V. The graph is called a connected graph if there is a path between every two vertices. A graph with no self-loops and multiple edges is called a *simple graph*. Graph theory has a lot of applications in science and technology. Graph labeling is applied in cryptography, data science, and blockchain. Graph labeling is introduced by Rosa [1]. Gallian [5] gives different kinds of labeling. There are various kinds of edge labeling. One such work is the vertex strongly*-graph, which is introduced by Beaula and Baskar Babujee [2]. Baskar Babujee et. Al. [4] have proved that wheels, paths, crowns, fans, and umbrellas are vertex strongly*-graphs. In this paper, we will prove that some flower graphs are vertex strongly*-graphs.

Definition 1.1: Vertex strongly $*$ -graph [2]

A graph G(V, E) is said to be a *vertex strongly*-graph* if there exists a bijection $f: E \to \{1, 2, \dots . q\}$, such that for every vertex $u \epsilon V$, $\sum f(uv_i) + \prod f(uv_i)$ are distinct, where uv_i are the edges incident to the vertex u.

Definition 1.2: Flower graph [3]

The *flower graph* Fl_n is obtained from a helm graph H_n by joining each pendant vertex to the apex of the helm graph. There are three types of vertices, the apex of degree $2n$, n vertices of degree four, and n vertices of degree two. The flower graph Fl_n has $2n + 1$ vertices and $4n$ edges. For example, See Figure 1. Flower graph Fl_8

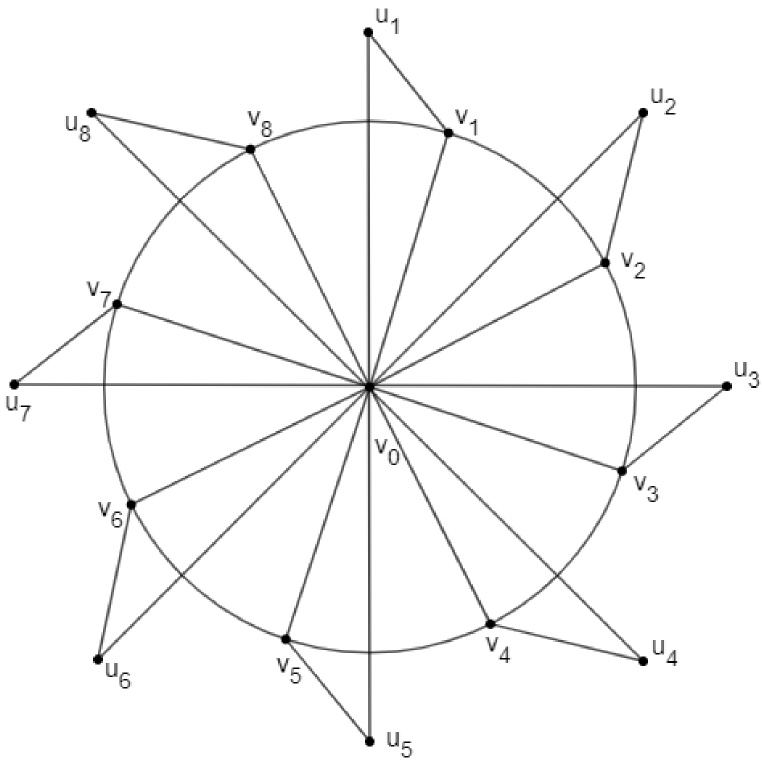

Figure 1: Flower graph Fl_8

Definition 1.3: Sunflower graph [6]

A *sunflower planar graph* Sf_n is obtained from a wheel graph with vertices $a_0, a_1, a_2, \dots, a_n$ (a_0 is central vertex and $a_1, a_2, \dots, a_n$ are rim vertices) and additional vertices $b_1, b_2, \dots, b_n$ such that b_j is joined to a_j and a_{j+1} is taken modulo n. For example, See Figure 2. Sunflower graph Sf_8

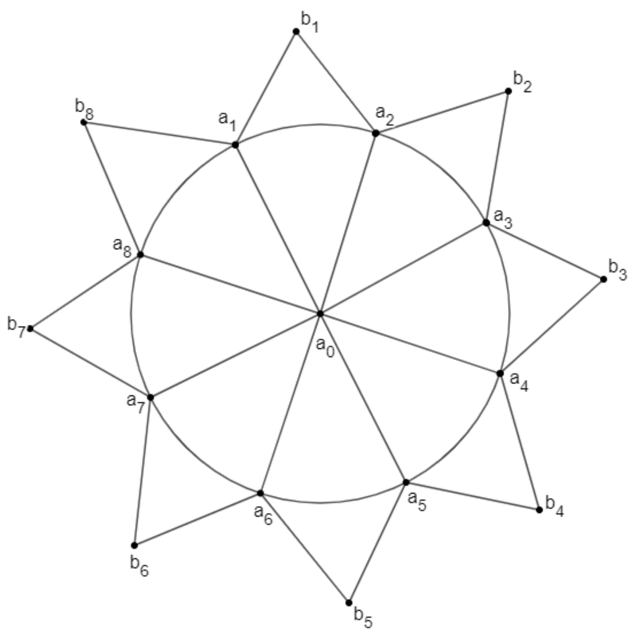

Figure 2. Sunflower graph Sf_8

2. Main Results

Theorem 2.1

The flower graph $fl_n$is a vertex strongly*-graph.

Proof:

Let $V = \{v_0, v_1, v_2, \dots, v_n, u_1, u_2, \dots, u_n\}$ be the vertex set and,
$E = \{(v_0, v_i), (v_0, u_i), (v_i, u_i) 1 \le i \le n\} \cup \{(v_i, v_{i+1}), (v_n, v_1) \ 1 \le i \le n-1\}$ be the edge set of the flower graph fl_n.

Labeling the edges of the flower graph fl_n using a bijective function g defined as follows,

$g: E \to \{1, 2,3, \dots ,4n\}$, such that
$g(v_0, v_i) = i, 1 \le i \le n$
$g(v_0, u_i) = n + i, 1 \le i \le n$
$g(v_i, u_i) = 2n + i, 1 \le i \le n$
$g(v_i, v_{i+1}) = 3n + i, 1 \le i \le n - 1$
The following Figure 3 is an example of the edge-labeled flower graph fl_8

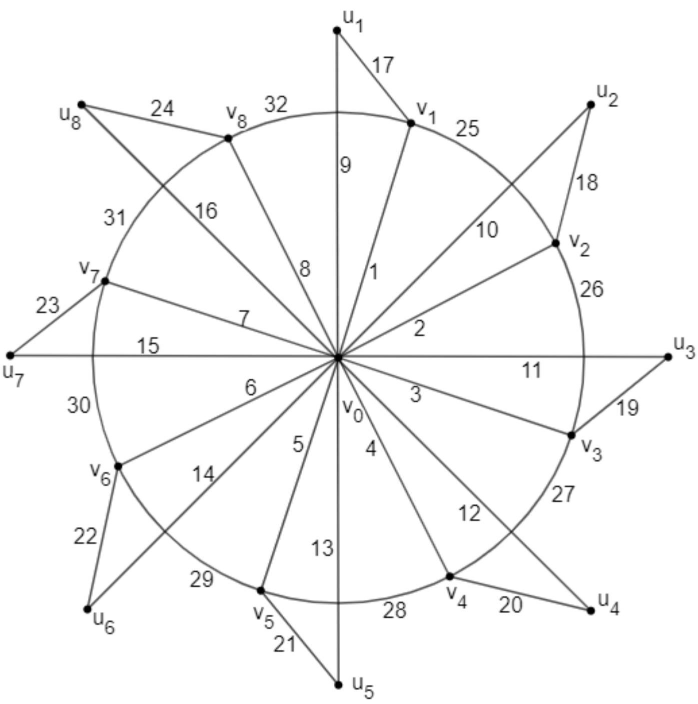

Figure 3: Edge-labeled flower graph fl_8

To prove that the flower graph fl_n is a vertex strongly*-graph,
The vertex calculation for each vertex is given below,

$f(v_0) = (2n)! + n(2n + 1)$
$f(v_1) = 9n + 3$
$f(v_i) = 8n + 4i - 1 + (i)(3n + i - 1)(3n + i)(2n + i), 2 \le i \le n$
$f(u_i) = 3n + 2i + (n + i)(2n + i), 1 \le i \le n$

To prove that the above calculations are distinct for each vertex.
Case (a): Consider $f(v_i)$ and $f(v_{i+1})$, $2 \leq i \leq n-1$
$f(v_i) = 8n + 4i - 1 + (i)(3n + i - 1)(3n + i)(2n + i)$,
$f(v_{i+1}) = 8n + 4(i + 1) - 1 + (i + 1)(3n + i + 1 - 1)(3n + i + 1)(2n + i + 1)$
$f(v_{i+1}) = f(v_i) + 4 + (3n + 2i)(3n + i + 1)(2n + i + 1) + i(5n + 2i + 1)(3n + i - 1)$
$f(v_{i+1}) \neq f(v_i)$ for $2 \leq i \leq n-1$

Case (b): Consider $f(u_i)$ and $f(u_{i+1})$, $1 \leq i \leq n$
$f(u_i) = 3n + 2i + (n + i)(2n + i)$
$f(u_{i+1}) = 3n + 2(i + 1) + (n + i + 1)(2n + i + 1)$
$f(u_{i+1}) = 3n + 2i + (n + i)(2n + i) + 3n + 2i + 3$
$f(u_{i+1}) = f(u_i) + 3n + 2i + 3$
$f(u_{i+1}) \neq f(u_i)$ for $1 \leq i \leq n$

Therefore, the vertex calculation is distinct for each vertex.
Hence the flower graph fl_n is a vertex strongly*-graph.

Theorem 2.2
The Sunflower graph Sf_n is a vertex strongly*-graph.

Proof:
Let $V = \{a_0, a_1, a_2, \dots, a_n, b_1, b_2, \dots, b_n\}$ be the vertex set and,
$E = \{(a_0, a_i), (a_i, a_{i+1}), (a_n, a_1)\ (b_i, a_i), (b_i, a_{i+1}), (a_0, a_n), (b_n, a_n), (b_n, a_1)\ 1 \leq i \leq n - 1\}$ be the edge set of the Sunflower graph Sf_n.

Labeling the edges of the Sunflower graph Sf_n, using a bijective function g as defined below.

$g: E \to \{1, 2, 3, \dots, 4n\}$, such that
$g(a_0, a_i) = i$, $1 \leq i \leq n$
$g(a_i, a_{i+1}) = n + i$, $1 \leq i \leq n-1$
$g(a_1, a_n) = 2n$
$g(b_i, a_i) = 2n + 2i - 1$, $1 \leq i \leq n-1$
$g(b_i, a_{i+1}) = 2n + 2i$, $1 \leq i \leq n-1$
$g(b_n, a_n) = 4n - 1$
$g(b_n, a_1) = 4n$

The following Figure 4 is an example of the edge labeled Sunflower graph Sf_8.

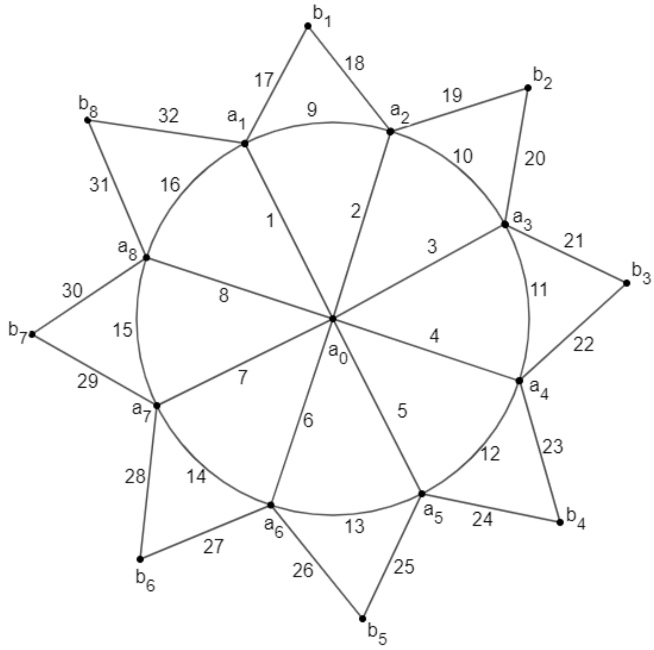

Figure 3: Edge labeled Sunflower graph Sf_8

To prove that the Sunflower graph Sf_n is a vertex strongly*-graph,

The vertex calculation for each vertex is given below,

$f(a_0) = n! + \frac{n(n+1)}{2}$
$f(a_i) = 6n + 7i - 4 + i(n + i - 1)(n + i)(2n + 2i - 2)(2n + 2i - 1), 2 \leq i \leq n$
$f(a_1) = 9n + 3 + 8n^2(2n + 1)(n + 1)$
$f(b_i) = 4n + 4i - 1 + (2n + 2i - 1)(2n + 2i), 1 \leq i \leq n$

To prove that the above calculations are distinct for each vertex.
Case (a): Consider $f(a_i)$ and $f(a_{i+1})$, $2 \leq i \leq n$
$f(a_i) = 6n + 7i - 4 + i(n + i - 1)(n + i)(2n + 2i - 2)(2n + 2i - 1)$
$f(a_{i+1}) = 6n + 7(i + 1) - 4 + i(n + i + 1 - 1)(n + i + 1)(2n + 2(i + 1) - 2)(2n + 2(i + 1) - 1),$
$f(a_{i+1}) = f(a_i) + 7 + 2(n + 2i)(n + i + 1)(n + i)(2n + 2i + 1)$
$+12i(n + i - 1)(n + i)^2$
$f(a_{i+1}) \neq f(a_i)$ for $2 \leq i \leq n$

Case (b): Consider $f(b_i)$ and $f(b_{i+1})$, $1 \leq i \leq n$
$f(b_i) = 4n + 4i - 1 + (2n + 2i - 1)(2n + 2i),$
$f(b_{i+1}) = 4n + 4(i + 1) - 1 + (2n + 2(i + 1) - 1)(2n + 2(i + 1)),$
$f(b_{i+1}) = f(b_i) + 8n + 8i + 6$

$f(b_{i+1}) \neq f(b_i)$ for $1 \leq i \leq n$

Therefore, the calculated values are distinct for each vertex.
Hence Sunflower graph Sf_n is a vertex strongly*-graph.

Conclusion:

This paper gives the results on vertex strongly*-graph on some flower-related graphs. This work can be extended to different graph structures. This labeling has good applications in radio signal assignments, network analysis, cryptography, and other engineering-related fields.

References

1. A Rosa, On certain valuation of the vertices of a graph, Theory of graphs, Proceedings of the Symposium, Rome, Gordon and Breach, New York, 349-355, 1967.
2. J Baskar Babujee and C Beaula, On vertex strongly*-graph, Proceed. Internat. Conf. Math. and Comput. Sci., 25-26, July 2008.
3. A Fathima Banu, S Chelliah and M P Syed Ali Nisaya, Even vertex tetrahedral mean graphs, World Scientific News, Volume 156, 26-39, 2021
4. J Baskar Babujee, K Kannan and V Vishnupriya, Vertex Strongly *-graphs, Internat. J. Analyzing Components and Combin. Biology in Math., Volume 2, 19-25.
5. J A Gallian, A Dynamic Survey of Graph Labeling, Electronic Journal of Combinatorics, 2020.
6. M Majeed, N Idrees, S Nasir, F B Farooq, On certain prime cordial families of graphs, Journal of Taibah University for Science, 14:1, 579-584.
DOI 10.1080/16583655.2020.1756561.