# On Vertex strongly*-graph of some constructed graphs

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**Abstract**

A graph G(V, E) is said to be a vertex strongly*-graph if there exists a bijection $f: E\ \{1, 2, \ldots.q\}$, such that for every vertex $u \epsilon V$, $\sum \quad f(uv_i) + \prod \quad f(uv_i)$ are distinct, where $uv_i$ are the edges incident to the vertex $u$. In this paper, we will be proving that the comb graph $P_5 \odot K_1$ and the bi-star $B_{m,n}$ are vertex strongly*-graph.

## Introduction

A graph $G(V, E)$ is a set of vertices $V$ and edges $E$, each vertex $e \epsilon E$ has its end vertices in $V$. Any problem can be visualized using graphs, which helps find solutions. Graph labeling is one of the research areas in graph theory that has a lot of applications in data science, blockchain, cryptography, and many more engineering field. Graph labeling is introduced by Rosa [1]. A survey on graph labeling can be found in Gallian [5], which gives updated research work. There is a lot of research work available in edge labeling. One such work is the vertex strongly*-graph, which is introduced by Beaula and Baskar Babujee [2]. Baskar Babujee et. Al. [4] have proved that wheels, paths, crowns, fans, and umbrellas are vertex strongly*-graphs. In this paper, we will prove that the comb graph $P_5 \odot K_1$ and the bi-star $B_{m,n}$ are vertex strongly*-graphs.

**Definition 1.1: Vertex strongly $*$ -graph [2]**

A graph G(V, E) is said to be a *vertex strongly*-graph* if there exists a bijection $f: E\ \{1, 2, \ldots.q\}$, such that for every vertex $u \epsilon V$, $\sum \quad f(uv_i) + \prod \quad f(uv_i)$ are distinct, where $uv_i$ are the edges incident to the vertex $u$.

**Definition 1.2: Comb graph $P_n \odot K_1$ [3]**

The *comb graph* $P_n \odot K_1$ is a graph obtained from the path $P_n$ by adding a pendant vertex to each vertex of $P_n$. Figure 1. Comb graph $P_5 \odot K_1$ is an example of a comb graph.

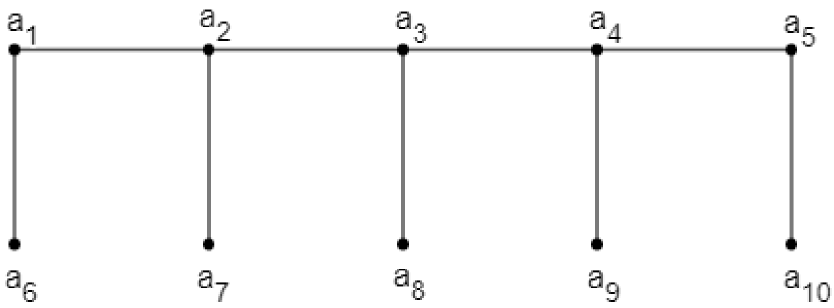


Figure 1: Comb graph $P_5 \odot K_1$

**Definition 1.3: Bi-star $B_{m,n}$ [6]**

The *bi-star* $B_{m,n}$ is a graph obtained from $K_2$ by joining $m$ pendant edges to one end of $K_2$ and $n$ pendant edges to the other end of $K_2$. Figure 2. Bi-star $B_{5,6}$ is an example of a bi-star.

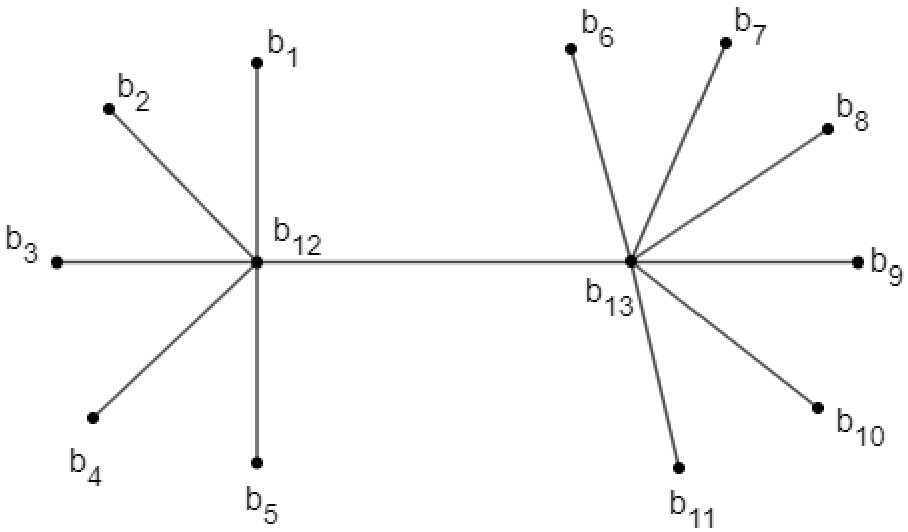


Figure 2: Bi-star $B_{5,6}$

**2. Main Results**

**Theorem 2.1**

The comb graph $P_n \odot K_1$ is a vertex strongly*-graph.

**Proof:**

Let $V = \{a_1, a_2, \dots, a_n, a_{n+1}, \dots, a_{2n}\}$ be the vertex set and,
$E = \{(a_i, a_{i+1}), 1 \leq i \leq n-1\} \cup \{(a_i, a_{n+i}),\ 1 \leq i \leq n\}$ be the edge set of the comb graph $P_n \odot K_1$.

Labeling the edges of the comb graph $P_n \odot K_1$ using a bijective function $g$ defined as follows,

$g: E \rightarrow \{1, 2,3, \dots ,2n - 1\}$, such that
$g(a_i, a_{i+1}) = i, 1 \leq i \leq n - 1$
$g(a_i, a_{n+i}) = n + i - 1, 1 \leq i \leq n$

The following Figure 3 is an example of the edge labeled comb graph $P_5 \odot K_1$

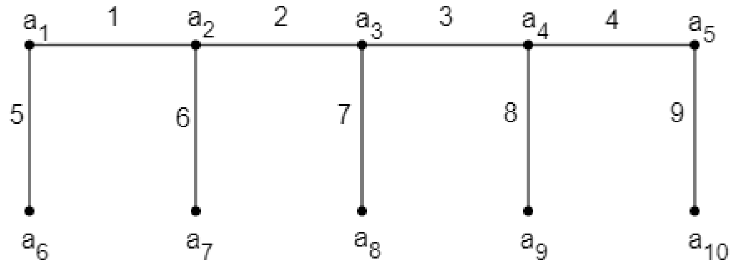


Figure 3: Edge labeled Comb graph $P_5 \odot K_1$

To prove that the comb graph $P_n \odot K_1$ is a vertex strongly*-graph,
The vertex calculation for each vertex is given below,

$f(a_1) = 2n + 1$
$f(a_n) = 3n - 2 + (n - 1)(2n - 1)$
$f(a_i) = n + 3i - 2 + i(i - 1)(n + i - 1), 2 \leq i \leq n$
$f(a_{n+i}) = 2(n + i - 1), 1 \leq i \leq n$

To prove that the above calculations are distinct for each vertex.
Case (a): Consider $f(a_i)$ and $f(a_{n+i})$
$f(a_{n+i}) = 2(n + i - 1), 1 \leq i \leq n$

$$f(a_{n+i+1}) = 2(n + i + 1 - 1)$$

$f(a_{n+i+1}) = 2(n + i - 1)+2$

$$f(a_{n+i+1}) = f(a_{n+i}) + 2$$

$f(a_{n+i+1}) \neq f(a_{n+i})$ for $1 \leq i \leq n$
Therefore, the vertex calculation is distinct for each vertex.
The following Figure 5 is an example of the vertex calculated comb graph $P_5 \odot K_1$

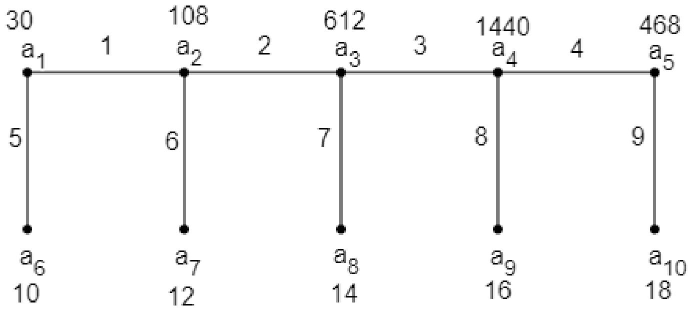


Figure 5: vertex calculated comb graph $P_5 \odot K_1$

Hence the comb graph $P_n \odot K_1$ is a vertex strongly*-graph.

**Theorem 2.2**

The bi-star $B_{m,n}$ is a vertex strongly*-graph.

**Proof:**

Let $V = \{b_1, b_2, \dots, b_m, b_{m+1}, \dots, b_{m+n}, b_{m+n+1}, b_{m+n+2}\}$ be the vertex set and,
$E = \{(b_{m+n+1}, b_i), (b_{m+n+2}, b_j), (b_{m+n+1}, b_{m+n+2})\ 1 \le i \le m, 1 \le j \le n\}$ be the edge set of the bi-star $B_{m,n}$.

Labeling the edges of the bi-star $B_{m,n}$, using a bijective function $g$ defined as follows,

$g: E \to \{1, 2,3, \dots, m + n + 1\}$, such that

$g(b_{m+n+1}, b_i) = i + 1,\ 1 \le i \le m$

$g(b_{m+n+2}, b_j) = m + j + 1,\ 1 \le i \le n$

$$g(b_{m+n+1}, b_{m+n+2}) = 1$$

The following Figure 6 is an example of the edge labeled bi-star $B_{5,6}$.

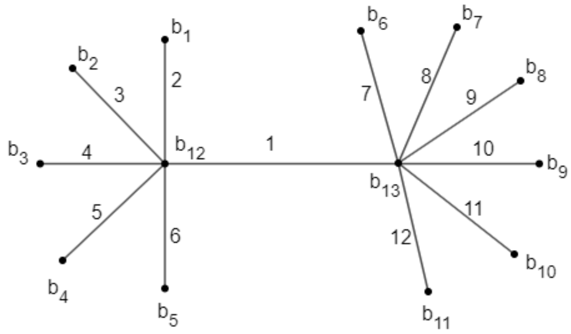


Figure 3: Edge labeled bi-star $B_{5,6}$

To prove that the bi-star $B_{m,n}$ is a vertex strongly*-graph,

The vertex calculation for each vertex is given below,

$f(b_{m+n+1}) = m! + \frac{m(m+1)}{2}$

$$f(b_{m+n+2}) = \frac{(m+n+1)!}{(m+1)!} + \frac{n(2m+n+3)}{2} + 1$$

$f(b_i) = 2(i+1), 1 \leq i \leq m$

$f(b_{m+j}) = 2(m+j+1), 1 \leq j \leq n$

As the numbers are in ascending order, the calculations are not equal for different vertices.

Therefore, the calculated values are distinct for each vertex.

Hence bi-star $B_{m,n}$ is a vertex strongly*-graph.

**Conclusion:**

This paper gives the results on vertex strongly*-graph on some constructed planar graphs. This work can be extended to different graph structures. This labeling has good applications in radio signal assignments, network analysis, cryptography, and other engineering-related fields.

**References**


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