On Vertex strongly*-graph of some constructed graphs

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Abstract

A graph G(V, E) is said to be a vertex strongly*-graph if there exists a bijection $f: E \rightarrow \{1, 2, \dots . q\}$, such that for every vertex $u\epsilon V$, $\sum f(uv_i) + \prod f(uv_i)$ are distinct, where uv_i are the edges incident to the vertex u. In this paper, we will be proving that the prism $C_n \times P_2$ and the special fan graph f_n^+ are vertex strongly*-graph.

Introduction

A graph $G(V, E)$ is a set of vertices V and edges E, each vertex $e\epsilon E$ has its end vertices in V. Any problem can be visualized using graphs, which helps find solutions. Graph labeling is one of the research areas in graph theory that has a lot of applications in data science, blockchain, cryptography, and many more engineering field. Graph labeling is introduced by Rosa [1]. A survey on graph labeling can be found in Gallian [5], which gives updated research work. There is a lot of research work available in edge labeling. One such work is the vertex strongly*-graph, which is introduced by Beaula and Baskar Babujee [2]. Baskar Babujee et. Al. [4] have proved that wheels, paths, crowns, fans, and umbrellas are vertex strongly*-graphs. In this paper, we will prove that the prism $C_n \times P_2$ and special fan graph f_n^+ are vertex strongly*-graphs.

Definition 1.1: Vertex strongly $*$ -graph [2]

A graph G(V, E) is said to be a *vertex strongly*-graph* if there exists a bijection $f: E \rightarrow \{1, 2, \dots . q\}$, such that for every vertex $u\epsilon V$, $\sum f(uv_i) + \prod f(uv_i)$ are distinct, where uv_i are the edges incident to the vertex u.

Definition 1.2: Cartesian Product of two graphs [3]

The *Cartesian product* $G_1 \times G_2$ of two graphs G_1 and G_2 is the simple graph with $V_1 \times V_2$ as its vertex set and two vertices (u_1, v_1) and (u_2, v_2) are adjacent in $G_1 \times G_2$ if and only if $u_1 = u_2$ and, v_1 is adjacent to v_2 in G_2, or u_1 is adjacent to u_2 in G_1 and $v_1 = v_2$. Ex: Prism $C_8 \times P_2$, the following Figure 1. Prism $C_8 \times P_2$ is an example of a Cartesian product of C_8 and P_2.

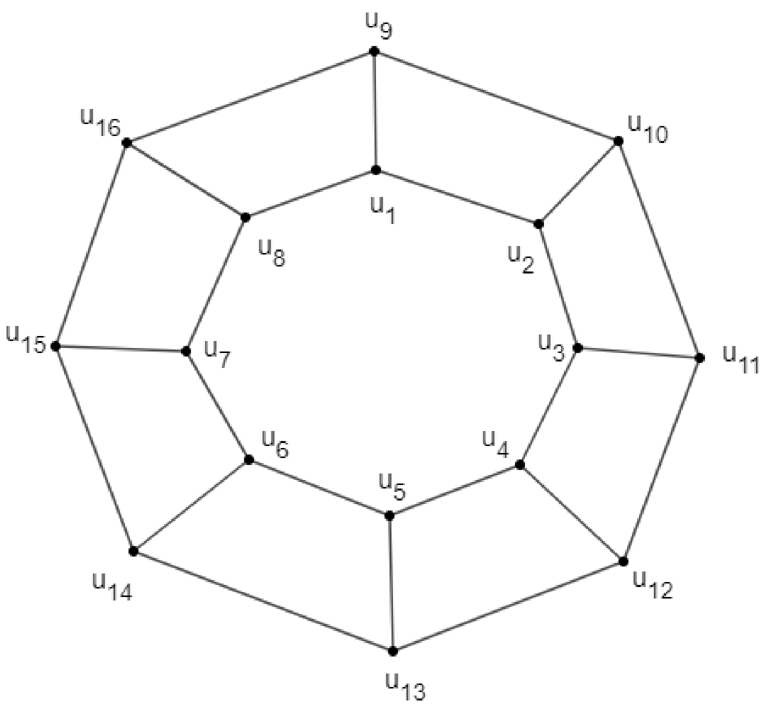

Figure 1: Prism $C_8 \times P_2$

Definition 1.3: Special Fan graph f_n^+ [6]

The *Fan graph* f_n is obtained by taking $n-3$ concurrent chords in a cycle C_n. The vertex at which all the chords are concurrent is called the *apex vertex*. In other words, f_n is the join $P_n + K_1$. The special fan graph f_n^+ is a graph obtained by adding a pendant vertex to each vertex of the fan graph f_n. Figure 2. Special Fan graph f_5^+ is an example of a special fan graph.

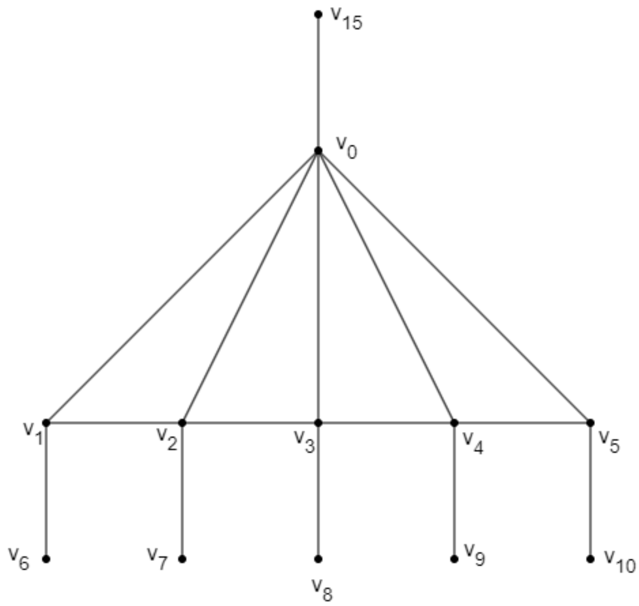

Figure 2: Special Fan graph f_5^+

2. Main Results

Theorem 2.1

The Prism $C_n \times P_2$ is a vertex strongly*-graph.

Proof:

Let $V = \{u_1, u_2, \dots, u_n, u_{n+1}, \dots, u_{2n}\}$ be the vertex set and,
$E = \{(u_i, u_{i+1}), (u_{n+i}, u_{n+i+1}), (u_i, u_{n+i}), (u_n, u_1), (u_{2n}, u_{n+1})\ 1 \leq i \leq n-1\}$ be the edge set of the Prism $C_n \times P_2$.

Labeling the edges of the Prism $C_n \times P_2$ using a bijective function g defined as follows,

$g: E \rightarrow \{1, 2, 3, \dots, 2n\}$, such that
$g(u_i, u_{i+1}) = i,\ 1 \leq i \leq n-1$
$g(u_{n+i}, u_{n+i+1}) = 2n + i,\ 1 \leq i \leq n-1$
$g(u_i, u_{n+i}) = n + i,\ 1 \leq i \leq n$
$g(u_n, u_1) = n,$
$g(u_{2n}, u_{n+1}) = 3n$
The following Figure 3 gives an edge labeled prism $C_8 \times P_2$.

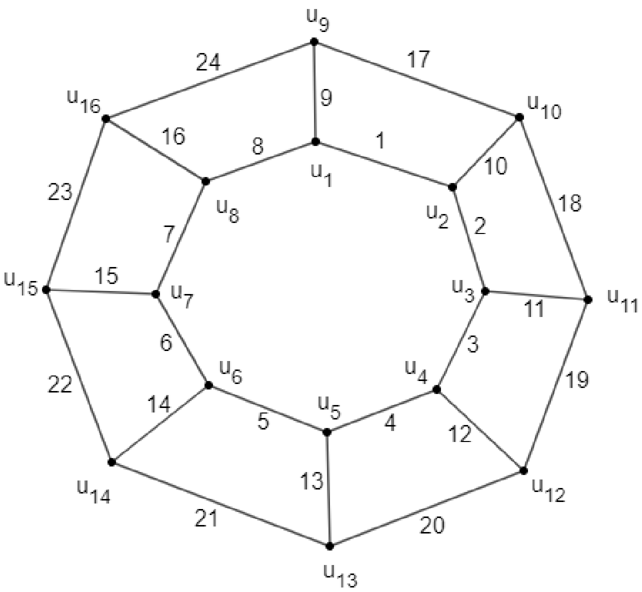

Figure 3: Edge labeled prism $C_8 \times P_2$

To prove that the Prism $C_n \times P_2$ is a vertex strongly*-graph,

The vertex calculation for each vertex is given below,

$f(u_1) = (n+1)(n+2)$

$f(u_{11}) = 6n + 2 + 3n(n+1)(2n+1)$

$f(u_i) = n + 3i - 1 + i(i-1)(n+i), 2 \leq i \leq n$

$f(u_{n+i}) = 5n + 3i - 1 + (n+i)(2n+i-1)(2n+i), 2 \leq i \leq n$

To prove that the above calculations are distinct for each vertex.

Case (a): Consider $f(u_i)$ and $f(u_{i+1})$

$f(u_i) = n + 3i - 1 + i(i-1)(n+i), 2 \leq i \leq n$

$f(u_{i+1}) = n + 3(i+1) - 1 + (i+1)(i+1-1)(n+i+1)$

$f(u_{i+1}) = n + 3i - 1 + i(i-1)(n+i) + i(2n+3i+1) + 3$

$f(u_{i+1}) = f(u_i) + i(2n+3i+1) + 3$

$f(u_{i+1}) \neq f(u_i)$ for $2 \leq i \leq n-1$

Case (b): Consider $f(u_{n+i})$ and $f(u_{n+i+1})$

$f(u_{n+i}) = 5n + 3i - 1 + (n+i)(2n+i-1)(2n+i), 2 \leq i \leq n$

$f(u_{n+i+1}) = 5n + 3(i+1) - 1 + (n+i+1)(2n+(i+1)-1)(2n+i+1),$

$f(u_{n+i+1}) = 5n + 3i - 1 + (n+i)(2n+i-1)(2n+i) +$
$(n+i)(2n+i-1)(2n+i+1) + (2n+i+1)(3n+2i)$

$f(u_{n+i+1}) = f(u_{n+i}) + (n+i)(2n+i-1)(2n+i+1) + (2n+i+1)(3n+2i)$

$f(u_{n+i+1}) \neq f(u_{n+i})$ for $2 \leq i \leq n-1$

Hence the Prism $C_n \times P_2$ is a vertex strongly*-graph.

Theorem 2.2

The special fan graph f_5^+ is a vertex strongly*-graph.

Proof:

Let $V = \{v_0, v_1, \dots, v_n, v_{n+1}, \dots, v_{2n}\}$ be the vertex set and,

$E = \{(v_0, v_i), (v_i, v_{n+i})\ 1 \leq i \leq n\} \cup \{(v_i, v_{i+1})\ 1 \leq i \leq n-1\}$ be the edge set of the special fan graph f_n^+.

Labeling the edges of the special fan graph f_n^+, using a bijective function g defined as follows,

$g: E \to \{1, 2, 3, \dots, 3n\}$, such that

$g(v_0, v_i) = i, 1 \leq i \leq n$

$g(v_i, v_{n+i}) = 2n - 1 + i, 1 \leq i \leq n$

$g(v_i, v_{i+1}) = n + i, 1 \leq i \leq n - 1$

To prove that the special fan graph f_n^+ is a vertex strongly*-graph,

The vertex calculation for each vertex is given below,

$f(v_0) = 3n + \frac{n(n+1)}{2} + 3n(n!)$

$f(v_i) = 4n + 4i - 2 + i(n + i - 1)(n + i)(2n + i - 1), 1 \leq i \leq n - 1$

$f(v_{n+i}) = 2(2n + i - 1), 1 \leq i \leq n$

$f(v_1) = 3n + 2 + (2n)(n + 1)$

$f(v_n) = 6n - 2 + n(2n - 1)(3n - 1)$

As the numbers are in ascending order, the calculations are not equal for different vertices.

Therefore, the calculated values are distinct for each vertex.

Hence special fan graph f_n^+ is a vertex strongly*-graph.

Conclusion:

This paper gives the results on vertex strongly*-graph on some constructed planar graphs. This work can be extended to different graph structures. This labeling has good applications in radio signal assignments, network analysis, cryptography, and other engineering-related fields.

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