# Vertex strongly*-graph of some constructed planar graphs

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## Abstract

Edge labeling is one of the research topics in graph theory that has many applications in science and technology. A graph G(V, E) is said to be a vertex strongly*-graph if there exists a bijection $f: E \rightarrow \{1, 2, \ldots. q\}$, such that for every vertex $u\epsilon V$, $\sum f(uv_i) + \prod f(uv_i)$ are distinct, where u$v_i$ are the edges incident to the vertex u. In this paper, we will be proving that the following constructed planar graphs - the special cycle $C_n^+$(a graph obtained by adding a pendant edge for each vertex of the cycle $C_n$), multi-star graph $K_{1,n,n,\ldots,n(m-times)}$ and The constructed graph $P_2 + mK_1$ are vertex strongly*-graph.

## Introduction

A graph $G(V, E)$ is a set of vertices $V$ and edges $E$, each vertex $e\epsilon E$ has its end vertices in $V$. Any problem can be visualized using graphs, and this helps in finding solutions. Graph labeling is one of the research areas in graph theory that has a lot of applications in data science, blockchain, cryptography, and many more engineering field. Graph labeling is introduced by Rosa [1]. A survey on graph labeling can be found in Gallian [5], which gives updated research work. There is a lot of research work available in edge labeling. One such work is vertex strongly*-graph, which is introduced by Beaula and Baskar Babujee [2]. Baskar babujee et. Al. [4] have proved that wheels, fans, paths, crowns, and umbrellas are vertex strongly*-graphs. In this paper, we will prove that the double vertex graph of a path $U_2(P_n)$ and multi-star graph $K_{1,n,n,\ldots,n(m\ times)}$ are vertex strongly*-graphs.

**Definition 1.1: Vertex strongly $*$ -graph [2]**
A graph G(V, E) is said to be a vertex strongly*-graph if there exists a bijection $f: E \rightarrow \{1, 2, \ldots. q\}$, such that for every vertex $u\epsilon V$, $\sum f(uv_i) + \prod f(uv_i)$ are distinct, where u$v_i$ are the edges incident to the vertex u.

**Definition 1.2: Special Cycle $C_p^+$ [6]**
The special cycle $C_p^+$ is a newly constructed graph, it is constructed by adding one pendant vertex to each vertex of the cycle $C_p$. Ex: See Fig. 1

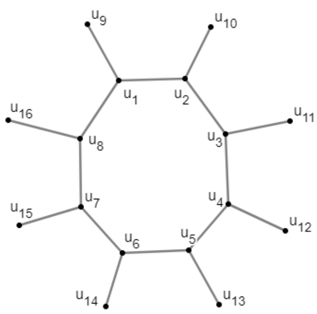


Figure 1: Special graph $C_8^+$

**Definition 1.3: Multi-star [3]**

Starting from the star graph $K_{1,n}$ with vertices $\{v_0, v_1, v_2, \dots, v_n\}$ introduce an edge to each of the pendant vertices $v_1, v_2, \dots, v_n$ to get the resulting graph $K_{1,n,n}$ with vertices $\{v_0, v_1, \dots, v_n, v_{n+1}, \dots, v_{2n}\}$, again introduce an edge to each of the pendant vertices $v_{n+1}, \dots, v_{2n}$, to get the graph $K_{1,n,n,n}$. Repeating this procedure $(m-1)$ times the resulting graph $K_{1,n,n,\dots n(m-times)}$. Ex: See Fig. 2

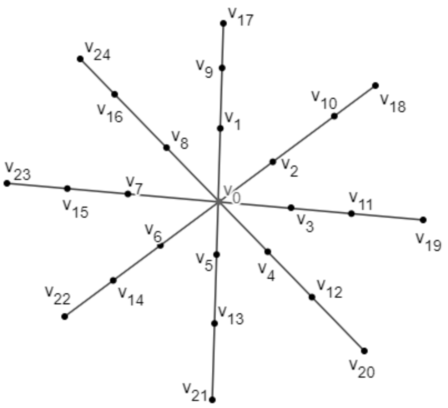


Figure 2: Multi-star $K_{1,8,8,8(3\ times)}$

**Definition 1.4: Graph $P_2 + rK_1$ [6]**

The constructed graph $P_2 + rK_1$ has vertex set $V = \{a_1, a_2, \dots, a_{r+1}, a_{r+2}\}$, and the edge set $E = \{(a_i, a_{r+1}), (a_i, a_{r+2}), (a_{r+1}, a_{r+2}), 1 \le i \le r\}$. Ex: See Fig.3

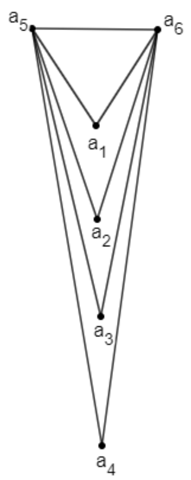


Figure 3: Graph $P_2 + 4K_1$

## 2. Main Results

**Theorem 2.1**

The special cycle $C_p^+$ is a vertex strongly*-graph.

**Proof:**

Let $V = \{u_1, u_2, \dots, u_p, u_{p+1}, \dots, u_{2p}\}$ be the vertex set and,
$E = \{(u_i, u_{i+1}), (u_i, u_{p+i}), (u_p, u_1), (u_p, u_{2p})\ 1 \leq i \leq n-1\}$ be the edge set of the special cycle $C_p^+$.

Labeling the edges of the special cycle $C_p^+$ using a bijective function $g$ defined as follows,

$g: E \rightarrow \{1, 2,3, \dots ,2p\}$, such that
$g(u_i, u_{i+1}) = i$, $1 \leq i \leq p-1$
$g(u_p, u_1) = p$,
$g(u_i, u_{p+i}) = p + i$, $1 \leq i \leq p$

To prove that the special cycle $C_p^+$ is a vertex strongly*-graph,

The vertex calculation for each vertex is given below,

$f(u_1) = 2(p+1) + p(p+1)$
$f(u_i) = p + 3i - 1 + i(i-1)(p+i)$, $2 \leq i \leq p$

To prove that the above calculation is distinct for each vertex.
$f(u_i) = p + 3i - 1 + i(i-1)(p+i)$, $2 \leq i \leq p$
$f(u_{i+1}) = p + 3(i+1) - 1 + (i+1)(i+1-1)(p+i+1)$
$f(u_{i+1}) = p + 3i - 1 + i(i-1)(p+i) + i(i+1) + 3$
$f(u_{i+1}) = f(u_i) + i(i+1) + 3$
$f(u_{i+1}) \neq f(u_i)$ for $2 \leq i \leq p-1$

Hence the special cycle $C_p^+$ is a vertex strongly*-graph.

**Theorem 2.2**
The multi-star graph $K_{1,n,n,\dots n(m-times)}$ is a vertex strongly*-graph.

**Proof:**

Let $V = \{v_0, v_1, \dots, v_n, v_{n+1}, \dots, v_{2n}, v_{2n+1}, \dots, v_{mn}\}$ be the vertex set and,
$E = \{(v_0, v_i), (u_i, u_{i+nj})\ 1 \leq i \leq n, 1 \leq j \leq m\}$ be the edge set of the multi-star graph $K_{1,n,n,\dots n(m-times)}$.

Labeling the edges of the multi-star graph $K_{1,n,n,\dots n(m-times)}$, using a bijective function $g$ defined as follows,

$g: E \rightarrow \{1, 2,3, \dots , mn\}$, such that
$g(v_0, v_i) = i$, $1 \leq i \leq n$
$g(v_i, v_{i+j}) = i + nj$, $1 \leq i \leq n$, $1 \leq j \leq m$

To prove that the multi-star graph $K_{1,n,n,\dots n(m-times)}$ is a vertex strongly*-graph,

The vertex calculation for each vertex is given below,

$f(v_0) = \frac{n(n+1)}{2} + n!$

$f(v_i) = n + 2i + i(n + i), 1 \le i \le n$

$f(v_{jn+i}) = (jn + i)((j + 1)n + i) + (2j + 1)n + 2i, 1 \le i \le n, 1 \le j \le m - 2$

$f(v_{(m-1)n+i}) = 2((m - 1)n + i), 1 \le i \le n$

As the numbers are in ascending order, the calculations are not equal for different vertices.

Therefore, the calculated values are distinct for each vertex.

Hence multi-star graph $K_{1,n,n,\dots n(m-times)}$ is a vertex strongly*-graph.

**Theorem 2.3**

The constructed graph $P_2 + rK_1$ is a vertex strongly*-graph.
**Proof:**

Let $V = \{a_1, a_2, \dots, a_{r+1}, a_{r+2}\}$ be the vertex set and
$E = \{(a_i, a_{r+1}), (a_i, a_{r+2}), (a_{r+1}, a_{r+2}), 1 \le i \le r\}$ be the edge set of the constructed graph $P_2 + rK_1$.

Labeling the edges of the graph $P_2 + rK_1$, using a bijective function $g$ defined as follows,

$g: E \to \{1, 2,3, \dots, r, r + 1, r + 2\}$, such that

$g(a_{r+1}, a_{r+2}) = 2r + 1.$
$g(a_{r+1}, a_i) = i, 1 \le i \le r.$
$g(a_{r+2}, a_i) = r + i, 1 \le i \le r.$

To prove that the graph $P_2 + rK_1$ is a vertex strongly*-graph.

The vertex calculation for each vertex is given below,

$f(a_{r+1}) = \frac{r(r+1)}{2} + 2r + 1 + r!\,(2r + 1)$

$f(a_{r+2}) = r^2 + \frac{r(r+1)}{2} + 2r + 1 + \frac{(2r)!}{r!}(2r + 1)$

$f(a_i) = r + 2i + i(r + i), 1 \le i \le r$

As the numbers are in ascending order, the calculations are not equal for different vertices.

Therefore, the calculated values are distinct for each vertex.

Hence the graph $P_2 + rK_1$ is a vertex strongly*-graph.

## Conclusion:

This paper gives the results on vertex strongly*-graph on some constructed planar graphs. This work can be extended for different graph structures. This labeling has good applications in radio signal assignments, network analysis, cryptography and many more engineering related fields.

## References

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