# On Vertex Strongly*-Graph of Some Graph Constructed from Wheel

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## Abstract

A graph $G(V, E)$ is said to be a vertex strongly*-graph if there exists a bijection $f: E \rightarrow \{1, 2, \ldots. q\}$, such that for every vertex $u\epsilon V$, $\sum f(uv_i) + \prod f(uv_i)$ are distinct, where u$v_i$ are the edges incident to the vertex u. This paper proves that some graphs constructed from wheel graphs are vertex strongly*-graph.

## Introduction

A graph $G(V, E)$ is a set of vertices $V$ and edges $E$, each vertex $e\epsilon E$ has its end vertices in $V$. An edge labeling of a graph is a function of assigning numbers edges so that no two edges get the same labeling. The vertex labeling of a graph is also a function of assigning numbers to vertices so that no two vertices get the same label [5]. In 1967 Graph labeling was introduced by Rosa [7]. Graph theory has a lot of applications in the field of Science and Technology. Any kind of concept can be visualized using graphs and studied further. There is a lot of research work available in graph labeling. One such work on graph labeling is vertex strongly*-graph, which was introduced by Beaula and Baskar Babujee [2]. Baskar Babujee et. Al. [3] have proved that wheels, fans, paths, crowns, and umbrellas are vertex strongly*-graphs. In this paper, we will prove that the Gear graph, Strong face wheel graph, and Helm graph are vertex strongly*-graphs. The following gives a set of definitions needed for this paper. All the basic definitions and notations are taken from [4].

**Definition 1.1: Vertex strongly $*$ -graph [2]**
A graph G(V, E) is said to be a vertex strongly*-graph if there exists a bijection $f: E \rightarrow \{1, 2, \ldots. q\}$, such that for every vertex $u\epsilon V$, $\sum f(uv_i) + \prod f(uv_i)$ are distinct, where u$v_i$ are the edges incident to the vertex u.

**Definition 1.2: Gear graph [6]**
The Gear Graph $G_n$ is obtained from the wheel $W_n$ by adding a vertex between every pair of vertices of the $n$-cycle. Ex: See Fig.1: Gear graph $G_7$.

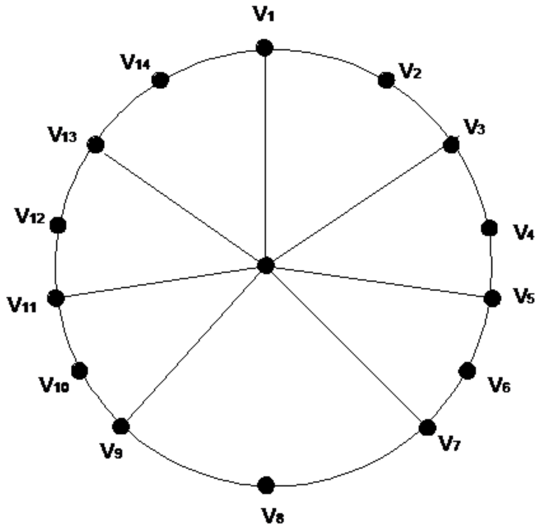


Figure 1: Gear graph $G_7$

**Definition 1.3: Strong Face Wheel Graph [1]**

Let $W_n$ be a wheel graph with $n+1$ vertices. A strong face wheel graph $W_n^*$ obtained from $W_n$ by adding a new vertex to every face of $W_n$ except the external face and joining this vertex with all vertices surrounding that face so that all faces of a new graph $W_n^*$ are isomorphic to the cycle $C_3$. Ex: See Fig. 2: Strong face Wheel graph $W_6^*$.

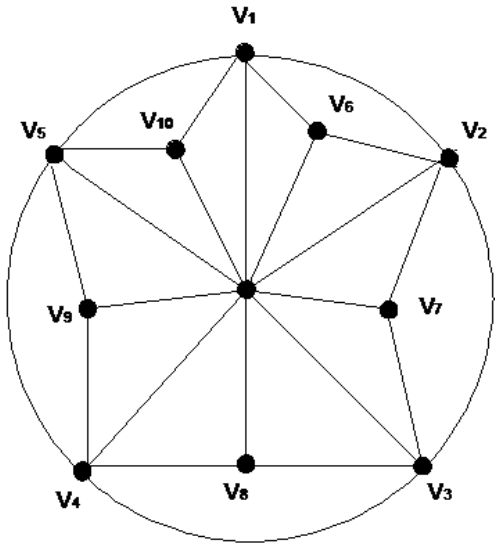


Figure 2: Strong face Wheel graph $W_6^*$

**Definition 1.4: Helm graph [6]**

The *helm* $H_n, n \geq 3$ is the graph obtained from a wheel $W_n$ by attaching a pendant edge at each rim vertex. Ex: See Fig. 3: Helm $H_8$.

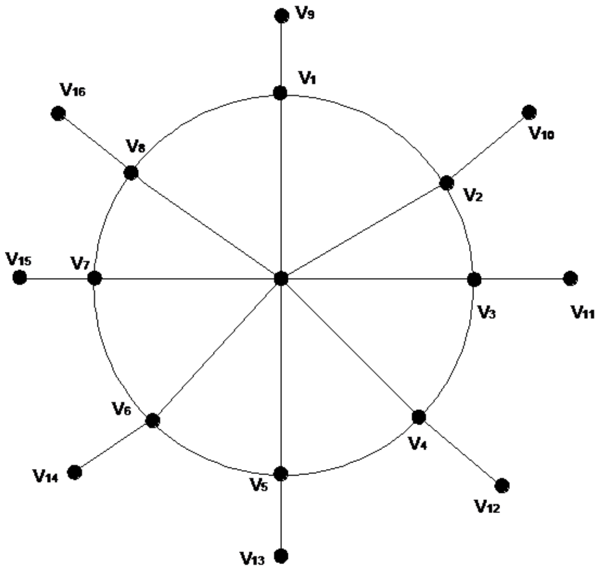

Figure 3: Helm $H_8$

## 2. Main Results

### Theorem 2.1

The Gear Graph $G_p(p \geq 3)$ is obtained from the wheel $W_p(p \geq 3)$ by adding a vertex between every pair of vertices of the $p$-cycle is a vertex strongly*-graph.

### Proof:

The Gear Graph $G_p(p \geq 3)$ is obtained from the wheel $W_p(p \geq 3)$ by adding a vertex between every pair of vertices of the $p$-cycle.
Let $V = \{u_1, u_2, \dots, u_p, u_{p+1}, \dots, u_{2p}, u_{2p+1}\}$ be the vertex set and,
$E = \{(u_i, u_{i+1}), (u_{2p}, u_1), 1 \leq i \leq 2p - 1\} \cup \{(u_{2p+1}, u_{2i-1}), 1 \leq i \leq p\}$ be the edge set of the Gear Graph $G_p$.

Labeling the edges of the Gear Graph $G_p$, using a bijective function $g$ defined as follows,

$g: E \rightarrow \{1, 2,3, \dots ,3p\}$, such that
$g(u_i, u_{i+1}) = p + i, 1 \leq i \leq 2p - 1$
$g(u_{2p}, u_1) = 3p,$
$g(u_{2p+1}, u_{2i-1}) = i, 1 \leq i \leq p$

The following Figure 4 gives the edge labeled Gear Graph $G_7$

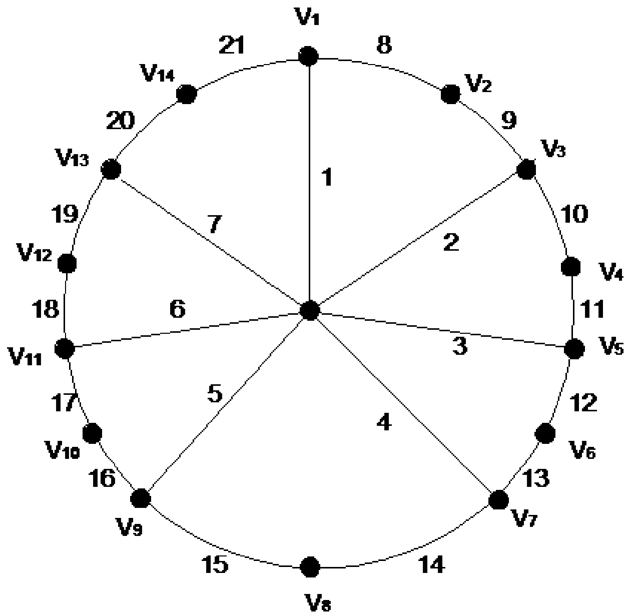


Figure 4: Gear Graph $G_7$

To prove that the Gear Graph $G_p$ is a vertex strongly*-graph,

The vertex calculation for each vertex is given below,

$f(u_1) = 4p + 2 + (3p)(p + 1)$
$f(u_{2i-1}) = 2p + 5i - 3 + i(p + 2i - 2)(p + 2i - 1), 2 \leq i \leq p$
$f(u_{2i}) = 2p + 4i - 1 + (p + 2i - 1)(p + 2i), 1 \leq i \leq p$

To prove that the above calculation is distinct for each vertex.
Case (a): Consider $f(u_{2i-1})$ and $f(u_{2i+1})$
$f(u_{2i-1}) = 2p + 5i - 3 + i(p + 2i - 2)(p + 2i - 1), 2 \leq i \leq p$

$f(u_{2i+1}) = 2p + 5(i + 1) - 3 + (i + 1)(p + 2(i + 1) - 2)(p + 2(i + 1) - 1), 2 \leq i \leq p$

$f(u_{2i+1}) = f(u_{2i-1}) + 5 + (i + 1)(p + 2i)(p + 2i + 1) + 2i(2p + 4i - 1)$

$f(u_{2i+1}) \neq f(u_{2i-1})$ for $2 \leq i \leq p$

Case (a): Consider $f(u_{2i})$ and $f(u_{2(i+1)})$
$f(u_{2i}) = 2p + 4i - 1 + (p + 2i - 1)(p + 2i), 1 \leq i \leq p$

$f(u_{2(i+1)}) = 2p + 4(i + 1) - 1 + (p + 2(i + 1) - 1)(p + 2(i + 1)), 1 \leq i \leq p$

$f(u_{2(i+1)}) = f(u_{2i}) + 2(2p + 4i + 1) + 4$

$f\left(u_{2(i+1)}\right) \neq f(u_{2i})$ for $2 \leq i \leq p$

Every pair of vertices have distinct labeling.

Hence the Gear Graph $G_p$is a vertex strongly*-graph.

**Theorem 2.2**
The strong face wheel graph $W_n^*$ is a vertex strongly*-graph.

**Proof:**

Let $V = \{v_1, \dots, v_n, v_{n+1}, \dots, v_{2n}, v_{2n+1}\}$ be the vertex set and,
$E = \{(v_{2n+1}, v_i), 1 \le i \le 2n\} \cup \{(v_{n+i}, v_i), (v_{n+i}, v_{i+1}), (v_i, v_{i+1})\ 1 \le i \le n-1\} \cup \{(v_{2n}, v_1), (v_{2n}, v_n), (v_n, v_1)\}$ be the edge set of the strong face wheel graph $W_n^*$.

Labeling the edges of the strong face wheel graph $W_n^*$ using a bijective function $g$ defined as follows,

$g: E \to \{1, 2,3, \dots ,2n + 1\}$, such that
$g(v_{2n+1}, v_i) = i, 1 \le i \le 2n$
$g(v_i, v_{i+1}) = 4n + i, 1 \le i \le n - 1,$
$g(v_{n+i}, v_i) = 2n + i, 1 \le i \le n - 1$
$g(v_{n+i}, v_{i+1}) = 2n + i + 1, 1 \le i \le n - 1$
$g(v_n, v_1) = 5n$
$g(v_{2n}, v_1) = 4n$
$g(v_{2n}, v_n) = 4n - 1$

The following Figure 5 is an example of the edge labeling of strong face wheel graph $W_5^*$

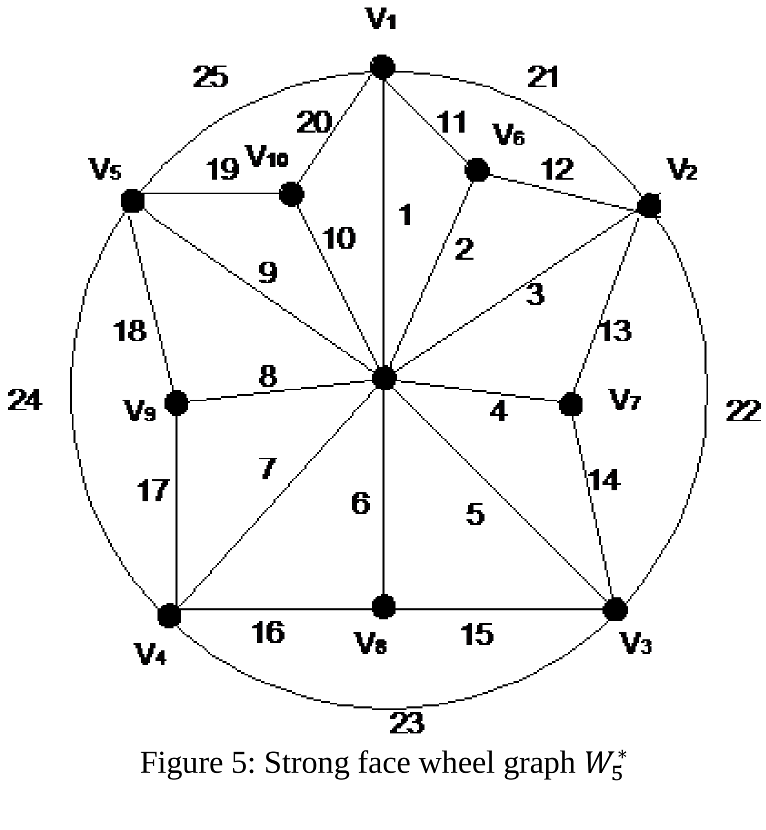

Figure 5: Strong face wheel graph $W_5^*$

To prove that the strong face wheel graph $W_n^*$is a vertex strongly*-graph,

The vertex calculation for each vertex is given below,

$f(v_{2n+1}) = \frac{2n(2n+1)}{2} + (2n)!$

$f(v_1) = 5n + 4n + 1 + 4n + 2n + 1 + 20n^2(4n + 1)(2n + 1),$

$f(v_i) = (14n + 8i - 5) + 2(4n + i - 1)(4n + i)(n + i - 1)(2n + 2i - 1)(2i - 1),$

$2 \leq i \leq n$

$f(v_i) = (2i - 1) + 2i + 2(i - n) + 4i(2i - 1)(i - n), \quad n + 1 \leq i \leq 2n$

As the numbers are increasing, the calculations are not equal for different vertices.

Therefore, the calculated values are distinct for each vertex.

Hence the strong face wheel graph $W_n^*$ is a vertex strongly*-graph.

**Theorem 2.3**

The helm $H_n, n \geq 3$ is a vertex strongly*-graph.

**Proof:**

Let $V = \{a_1, a_2, \dots, a_r, a_{r+1}, \dots, a_{2r}, a_{2r+1}\}$ be the vertex set and $E = \{(a_i, a_{i+1}), (a_r, a_1), 1 \leq i \leq r - 1\} \cup \{(a_{2n+1}, a_i), (a_i, a_{n+i}), 1 \leq i \leq r\}$ be the edge set of the helm $H_n, n \geq 3$.

Labeling the edges of the helm graph $H_n$using a bijective function $g$ defined as follows,

$g: E \rightarrow \{1, 2, 3, \dots, r, r + 1, \dots, 2r + 1\}$, such that

$g(a_r, a_1) = 2r.$
$g(a_{2r+1}, a_i) = i, 1 \leq i \leq r.$
$g(a_i, a_{i+1}) = r + i, 1 \leq i \leq r - 1.$
$g(a_i, a_{r+i}) = 2r + i, 1 \leq i \leq r.$

Figure 6 is an example of the edge labeled Helm graph $H_8$

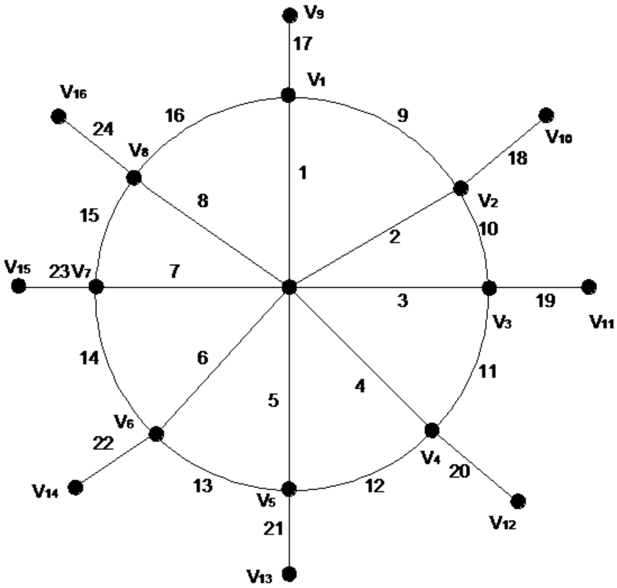

Figure 6: Helm graph $H_8$

To prove that the helm graph $H_n$ is a vertex strongly*-graph.

The vertex calculation for each vertex is given below,

$f(a_{2r+1}) = \frac{r(r+1)}{2} + r!$

$f(a_i) = 4(r+i) - 1 + i(r+i-1)(r+i)(2r+i), 2 \leq i \leq r$

$f(a_i) = 2(r+i), r+1 \leq i \leq 2r$

$f(a_1) = 5r + 3$

Case (a): Consider $f(a_i)$ and $f(a_{i+1})$, $2 \leq i \leq r$

$f(a_i) = 4(r+i) - 1 + i(r+i-1)(r+i)(2r+i),$

$f(a_{i+1}) = 4(r+i+1) - 1 + (i+1)(r+i+1-1)(r+i+1)(2r+i+1),$

$f(a_{i+1}) = f(a_i) + i(r+i-1)(3r+2i+1) + (r+2i)(r+i+1)(2r+i+1) + 4,$

$f(a_{i+1}) \neq f(a_i)$ for $2 \leq i \leq r$

Case (a): Consider $f(a_i)$ and $f(a_{i+1})$, $r+1 \leq i \leq 2r$

$f(a_i) = 2(r+i),$

$f(a_{i+1}) = 2(r+i+1),$

$f(a_{i+1}) = f(a_i) + 2,$

$f(a_{i+1}) \neq f(a_i)$ for $r+1 \leq i \leq 2r$

Therefore, the calculated values are distinct for each vertex.

Hence the helm graph $H_n$ is a vertex strongly*-graph.

## Conclusion:

This paper gives the results on vertex strongly*-graph on some constructed planar graphs. This work can be extended to different graph structures. This labeling has good applications in radio signal assignments, network analysis, cryptography, and many more engineering-related fields.

## References


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