Ranking Fuzzy Numbers by Defuzzification in a Decision Level

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Abstract

A new method for ranking generalized trapezoidal fuzzy numbers by defuzzification method based on a combination of ill-defined magnitude called as value, ambiguity an amount of vagueness present in the ill-defined magnitude in the centroid range decision level is studied. The proposed method addresses the shortcomings in some of the existing fuzzy ranking methods by ranking symmetric fuzzy numbers having same core and different heights, same support and different cores, crisp numbers, crisp numbers having same support and different heights and fuzzy numbers having compensation of areas.

Keywords*: Fuzzy numbers, Ranking, Value, Ambiguity, Centroids, Decision Level.*

1. Introduction

Systems that we deal with in day-to-day life has, some kid of uncertainty associated with it in the form of vagueness, imprecision, ill-defined and doubtful data. As fuzzy numbers allow us to represent vague and uncertain values, they have been studied and elaborated by various researchers. To handle uncertainty in decision making problems with the help of fuzzy numbers, the results will take the form of fuzzy intervals where each element in it has a different grade of membership and it is difficult to judge a fuzzy value is greater or smaller than other. One way to handle this is, to defuzzify the fuzzy intervals and use the corresponding ordering.

Ranking fuzzy numbers is very useful in decision making, fuzzy optimization, forecasting, approximate reasoning, artificial intelligence, risk analysis and in many other applications. Ranking fuzzy numbers is a procedure to compare and order a sequence of fuzzy numbers. Even though this topic is addressed by different researchers, it is still a challenging area for many researchers as, fuzzy numbers are represented by possibility distributions and can overlap with others. Ranking fuzzy numbers was first proposed by Jain [8] and since then, a lot of research has been done on this concept. As the present research uses centroids, decision levels, value and ambiguity, we throw some light on the methods that use these concepts. Ranking fuzzy numbers by using centroids was first proposed by Yager [13] and then this method was improved by Choobineh and Li [4] which does not require normal or convex property of membership function. Later on, methods based on distance between centroid point of a fuzzy number and origin by Cheng [3], area between centroid of a fuzzy number and origin Chu and Tsao [5] came into picture. These methods are counter intuitive and failed to rank fuzzy numbers with negative support as they are formulated on incorrect centroid formula. The centroid formula has been corrected by Wang et al. [11] and using this corrected centroid formula, a revised method to Chu and Tsao's [5] work was proposed by Wang and Lee [9]. Researchers like Kim and Park [15], Liou and Wang [16], Garcia and Lamata]14] stressed that the participation of decision maker is important in ranking of fuzzy numbers and hence, several methods based on involvement of decision maker in ranking fuzzy numbers came into existence for ranking fuzzy numbers. Delgado et al. [7] introduced two real indices, value and ambiguity to capture the information contained in a fuzzy number with the help of a reducing function and using these parameters one can order fuzzy numbers.

Some of existing methods in literature have shortcomings in ranking crisp numbers, crisp numbers with different heights and same support, symmetric fuzzy numbers having different supports and same core, different core and same support and fuzzy numbers with compensation of areas. To overcome the above shortcomings, we propose a method to rank generalized trapezoidal fuzzy numbers based on centroids, value (ill-defined magnitude) and ambiguity (amount of vagueness present in the ill-defined magnitude) of a fuzzy number. In this study, a generalized trapezoidal fuzzy number $A = (a, b, c, d; w)$is mathematically considered as a trapezoid and is partitioned into three plane figures. The respective centroids of these three plane figures are combined to get a fuzzy quantity, where the decision levels will be in the centroids range $\left[\frac{w}{3}, \frac{w}{2}\right]$, w is the height of the fuzzy number and $0 \leq w \leq 1$. Using thefuzzy quantity in the parametric form, the value and ambiguity of the generalized trapezoidal fuzzy number is defined which serves as a criterion for ordering the fuzzy numbers. The ranking methods developed on value and ambiguity Delgado et al. [7] require a reducing function to diminish the contribution of lower alpha levels, but the proposed method is free from this requirement. The advantage of the proposed method is that, it removes the usage of a reducing function and overcomes the several shortcomings of the existing ranking procedures. The rest of the paper is organized as follows:

In Section 2, the definitions related to the study are presented. The proposed method on ranking fuzzy numbers is presented in Section 3 along with some prepositions related to the study. In Section 4, some numerical examples and a comparative study with other existing methods are presented and finally the conclusions are presented in Section 5.

2. Definitions

In this section, the basic definitions related to the study are presented from Ma, M. et al. [17] and Delgado et al. [7].

Definition 2.1. Fuzzy Number

A generalized fuzzy number is a fuzzy set $f: R \rightarrow [0,1]$such that

1. f is upper semi-continuous;
2. *f(x)* is monotonic increasing on $[a, b]$ and monotonic decreasing on $[c, d]$ for some real numbers *a, b, c, d* such that $a \leq b \leq c \leq d$;
3. *f(x)* = 0 outside [*a, d*];
4. *f(x)* = *w*, $b \leq x \leq c$.

A trapezoidal fuzzy number with height w, $0 \leq w \leq 1$is simply denoted by A = *(a, b, c, d; w)* is shown in Fig. 1. Its membership function is defined as

$$f_A(x) = \begin{cases} w\left(\dfrac{x-a}{b-a}\right), if\ a \leq x \leq b, \\ w, \quad if\ b \leq x \leq c, \\ w\left(\dfrac{x-d}{c-d}\right), if\ c \leq x \leq d, \\ 0, \quad otherwise. \end{cases}$$

If $w = 1$, then A is called normal trapezoidal fuzzy number, otherwise A is called a generalized trapezoidal fuzzy number. If $b = c$, then A = *(a, b, c; w)* is called a triangular fuzzy number.

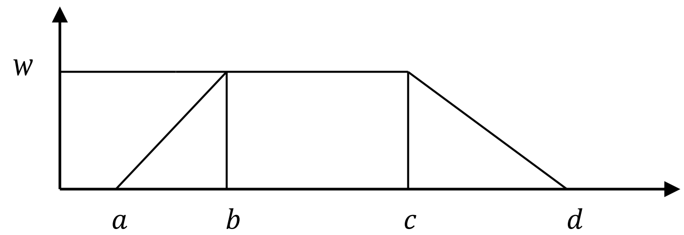

Fig.1. Generalized trapezoidal fuzzy number

Definition 2.2. Parametric Representation of Fuzzy Number

A fuzzy number A in parametric form with defuzzifiers at equal height w, $0 \leq w \leq 1$, is a pair of the functions $[\underline{a}_w(r), \overline{a}_w(r)]$; $0 \leq r \leq w$, satisfying the following conditions:

(i) $\underline{a}_w(r)$ is an increasing left continuous bounded function on $[0, w]$;

(ii) $\overline{a}_w(r)$ is an decreasing left continuous bounded function on $[0, w]$;

(iii) $\underline{a}_w(r) \leq \overline{a}_w(r)$,$0 \leq r \leq w$.

The parametric form of the trapezoidal fuzzy number$A = (a, b, c, d; w)$ for $0 \leq r \leq w$ is
$A_r = \left[a + (b-a)\frac{r}{w}, d - (d-c)\frac{r}{w}\right]$.

Definition 2.3. Value and Ambiguity of a Fuzzy Number

If A is a fuzzy number with parametric representation $[\underline{a}(r), \overline{a}(r)]$, $r \in [0,1]$ and $s: [0,1] \rightarrow [0,1]$ is a reducing function then, the value and ambiguity of the fuzzy number A with respect to the reducing function are defined by $Val(A) = \int_0^1 s(r)\{\underline{a}(r) + \overline{a}(r)\}dr$

$$Amb(A) = \int_0^1 s(r)\{\overline{a}(r) - \underline{a}(r)\}dr$$

where $s(r)$ is the reducing function and $\int_0^1 s(r)dr = 0.5$.

Definition 2.4. Fuzzy Quantity (Facchinetti et al. [10])

A fuzzy quantity is a non-convex and non-normal fuzzy set defined as the union of two or more non-normal fuzzy numbers.

3. Proposed Method

In this section, the proposed method to rank fuzzy numbers by defuzzification based on centroids, value and ambiguity with decision levels in the range [*w/3*, *w/2*] is presented. Consider a generalized trapezoidal fuzzy number $A_i = (a_i, b_i, c_i, d_i; w_i)$ and to find a fuzzy quantity, we treat this trapezoidal fuzzy number as a trapezoid *PQRS*, shown graphically in Fig. 2. Partition this trapezoid *PQRS* into two triangular regions *PQL*, *MRS* and one rectangular region *LQRM* with $A\left(\frac{a_i+2b_i}{3},\frac{w_i}{3}\right)$, $B\left(\frac{d_i+2c_i}{3},\frac{w_i}{3}\right)$ and $C\left(\frac{b_i+c_i}{2},\frac{w_i}{2}\right)$as respective centroids. Join these centroids to get a fuzzy quantity *ABC* with decision level in the range $\left[\frac{w_i}{3},\frac{w_i}{2}\right]$. In the second step of defuzzification, the parametric form of the triangular fuzzy number is written and the value and ambiguity on this fuzzy quantity is defined which serves as a criteria for ranking generalized trapezoidal fuzzy numbers.

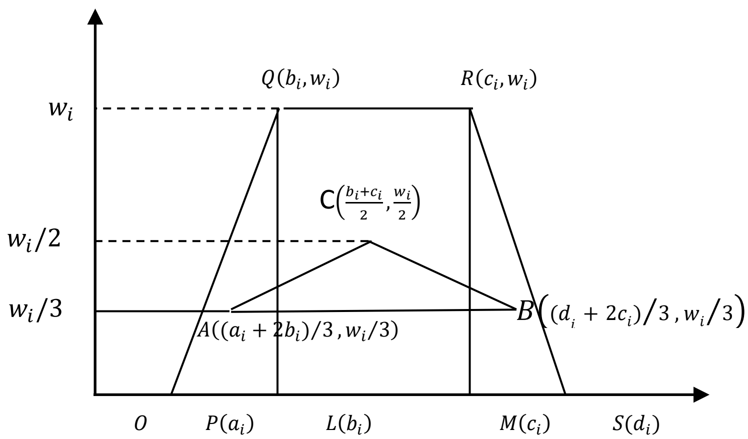

Fig. 2. Triangular fuzzy number from centroids

Definition 3.1. The fuzzy quantityA_i^* formed from generalized trapezoidal fuzzy number

$A_i=(a_i,b_i,c_i,d_i;w_i)$ with decision levels in $\left[\frac{w_i}{3},\frac{w_i}{2}\right]$ where $0\le w_i\le 1$, is defined as

$$A_i^*=\left(\frac{a_i+2b_i}{3},\frac{b_i+c_i}{2},\frac{2c_i+d_i}{3};\frac{w_i}{3},\frac{w_i}{2}\right) \quad (1)$$

Definition 3.2. The membership function for the fuzzy quantity given by Eq. (1) is defined as

$$f_{A_i^*}=\begin{cases} w_i\left(\frac{x-a_i-b_i+c_i}{3c_i-b_i-2a_i}\right), if\ \frac{a_i+2b_i}{3}\le x\le\frac{b_i+c_i}{2}, \\ \frac{w_i}{2}, \qquad if\ x=\frac{b_i+c_i}{2}, \\ w_i\left(\frac{x+b_i-c_i-d_i}{3b_i-c_i-2d_i}\right), if\ \frac{b_i+c_i}{2}\le x\le\frac{2c_i+d_i}{3}, \\ 0, \qquad otherwise. \end{cases} \quad (2)$$

Definition 3.3. The parametric form for the fuzzy quantity given by Eq. (1) with decision level α_i in $\left[\frac{w_i}{3}, \frac{w_i}{2}\right]$ where $0 \le w_i \le 1$, is defined as:

$$\left[\underline{A_i^*}(\alpha_i), \overline{A_i^*}(\alpha_i)\right] = \begin{bmatrix} (a_i + b_i - c_i) + \left(\frac{3c_i - b_i - 2a_i}{w_i}\right)\alpha_i, \\ (-b_i + c_i + d_i) + \left(\frac{3b_i - c_i - 2d_i}{w_i}\right)\alpha_i \end{bmatrix} \quad (3)$$

Definition 3.4. The value of the generalized trapezoidal fuzzy number $A_i = (a_i, b_i, c_i, d_i; w_i)$ based on the parametric form of fuzzy quantity is defined as:

$$Val\ (A_i) = \int_{w_i/3}^{w_i/2} \left[\underline{A_i^*}(\alpha_i) + \overline{A_i^*}(\alpha_i)\right] d\alpha_i \quad (4)$$

On integrating Eq. (4), we get,

$$Val(A_i) = (a_i + 5b_i + 5c_i + d_i)\left(\frac{w_i}{36}\right) \quad (5)$$

Definition 3.5. The ambiguity of the generalized trapezoidal fuzzy number

$A_i = (a_i, b_i, c_i, d_i; w_i)$ based on the parametric form of fuzzy quantity is defined as:

$$Amb\ (A_i) = \int_{w_i/3}^{w_i/2} \left[\overline{A_i^*}(\alpha_i) - \underline{A_i^*}(\alpha_i)\right] d\alpha_i \quad (6)$$

On integrating Eq. (6), we get,

$$Amb(A_i) = (-a_i - 2b_i + 2c_i + d_i)\left(\frac{w_i}{36}\right) \quad (7)$$

Definition 3.6. If $A_1 = (a_1, b_1, c_1, d_1; w_1)$ and $A_2 = (a_2, b_2, c_2, d_2; w_2)$ are two generalized trapezoidal fuzzy numbers then, for decision level $\alpha_i \in \left[\frac{w_i}{3}, \frac{w_i}{2}\right]$; $i = 1, 2$, let $A_1^*\ and\ A_2^*$ be the corresponding fuzzy quantities as defined by Eq. (2) respectively then, the following decisions are made:

(1) If $Val(A_1) > Val(A_2)$, then $A_1 \succ A_2$.
(2) If $Val(A_1) < Val(A_2)$, then $A_1 \prec A_2$.
(3) If $Val(A_1) = Val(A_2)$, then ;
    (a) if $Amb(A_1) > Amb(A_2)$, then $A_1 \prec A_2$,
    (b) if $Amb(A_1) < Amb(A_2)$, then $A_1 \succ A_2$,
    (c) if $Amb(A_1) = Amb(A_2)$, then $A_1 \sim A_2$.

Proposition 3.1. If $A_1 = (a_1, b_1, c_1, d_1; w_1)$ and $-A_1 = (-d_1, -c_1, -b_1, -a_1; w_1)$, then

$-Val(A_1) = Val(-A_1)$.

Proof. By using Eq. (5), we get, $Val(A_1) = (a_1 + 5b_1 + 5c_1 + d_1)\left(\frac{w_1}{36}\right)$

$$Val(-A_1) = (-a_1 - 5b_1 - 5c_1 - d_1)\left(\frac{w_1}{36}\right)$$

$$\text{Hence, } -Val(A_1) = Val(-A_1) .$$

Proposition 3.2. If $A_1 = (0,0,0,0,w_1)$, then $Val(A_1) = Amb(A_1) = 0$.

Proof. The proof is obvious.

Proposition 3.3. If $A\ and\ B$ are two generalized trapezoidal fuzzy numbers such that $A \succ B$, then $-A \prec -B$, provided the ranking is evaluated through their values.

Proof. Given that $A \succ B$ and as the ordering is done through values, we have $Val(A) > Val(B)$.

This implies that $-Val(A) < -Val(B)$ and $Val(-A) < Val(-B)$

$$\text{Hence, } -A \prec -B.$$

Proposition 3.4. If $A_1, A_2\ and\ A_3$ are three generalized trapezoidal fuzzy numbers such that $A_1 \prec A_2$ and $A_2 \prec A_3$ then, $A_1 \prec A_3$ through the ordering with respect to the values.

Proof. Given that $A_1 \prec A_2$ and $A_2 \prec A_3$ then, by Definition 3.6, we have $Val(A_1) < Val(A_2)$

and $Val(A_2) < Val(A_3)$.

This implies that $Val(A_1) < Val(A_3)$, hence $A_1 \prec A_3$.

4. Numerical Examples and Comparative Study

In this section, we demonstrate the proposed method first by considering some numerical examples taken from different studies and second by doing a comparative study with different existing methods in literature.

Example 4.1. Consider the symmetric fuzzy numbers with different heights

$A_1 = (0.3,0.5,0.5,0.7;1)$ and $A_2 = (0.3,0.5,0.5,0.7;0.8)$ taken from Liou and Wang [16] shown in Fig. 3.

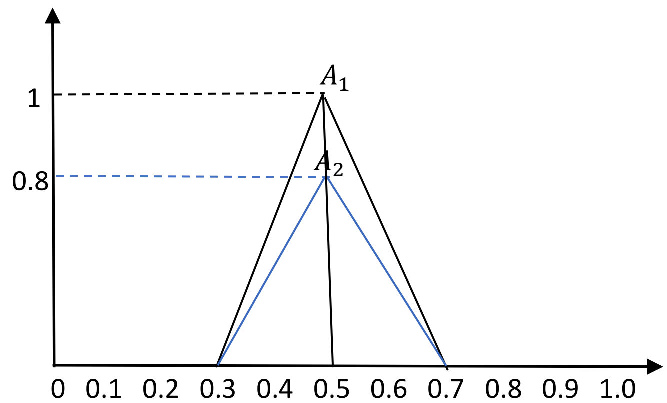

Fig. 3. *A_1 = (0.3,0.5,0.5,0.7;1), A_2 = (0.3,0.5,0.5,0.7;0.8)*

Liou and Wang ([16]) failed to discriminate the above fuzzy numbers and they ranked them as $A_1 \sim A_2$ for all values of the decision maker $\alpha \in [0,1]$.By using Eq. (5) of the proposed method, we get $Val(A_1) = 0.1667, Val(A_2) = 0.1333$ and as $Val(A_1) > Val(A_2)$ we can conclude that $A_1 \succ A_2$. This example shows that the proposed method can rank symmetric fuzzy numbers with different heights.

Example 4.2. Consider fuzzy numbers $A_1 = (5, 7, 9, 10; 1), A_2 = (6, 7, 9, 10; 0.6)$ having same core and unequal heights and a symmetric fuzzy number $A_3 = (7, 8, 9, 10; 0.4)$ taken from Liou and Wang [16] shown in Fig. 4.

By using Eq. (5) of the proposed method, we get $Val(A_1) = 2.6388, Val(A_2) = 1.6$ and $Val(A_3) = 1.1333$. As $Val(A_1) > Val(A_2) > Val(A_3)$, we conclude that $A_1 \succ A_2 \succ A_3$.

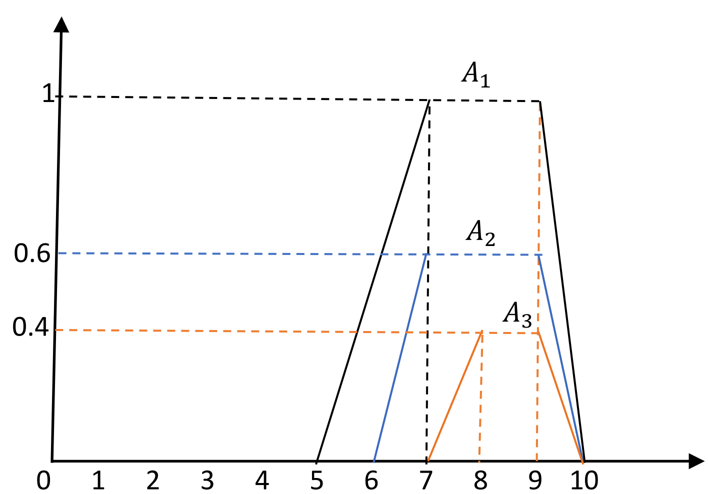

Fig. 4. *A_1 = (5,7,9,10;1), A_2 = (6,7,9,10;0.6), A_3 = (7,8,9,10;0.4)*

Liou and Wang [16] failed to discriminate the fuzzy numbers for the case of an optimistic decision maker $\alpha = 1$ and for $0 \leq \alpha < 1$, they ordered the above fuzzy numbers $A_1 \prec A_2 \prec A_3$. This is unreasonable as fuzzy number A_1 is a normal fuzzy number with highest membership value and one has to naturally prefer this than the other two fuzzy numbers. The proposed method ranks fuzzy numbers with same core, symmetric and fuzzy numbers with different heights.

Example 4.3. Consider the fuzzy numbers $A_1 = (1, 2, 2, 5; 1)$ and $A_2 = (1, 2, 2, 4; 1)$ having

same core taken from Liou and Wang [16] shown in Fig. 5

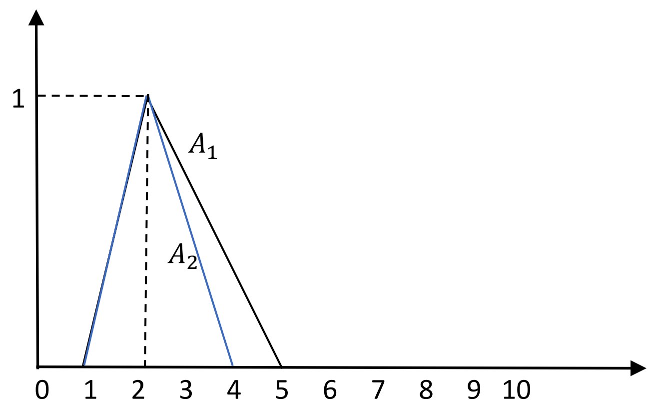

Fig. 5. *A_1 = (1, 2, 2, 5;1), A_2 = (1, 2, 2, 4;1)*

By using Eq. (5) of the proposed method we get $Val(A_1) = 0.7222$ and $Val(A_2) = 0.6944$. As $Val(A_1) > Val(A_2)$ we can conclude that $A_1 \succ A_2$. This example shows that the proposed method can rank fuzzy numbers having same core. This result is in coincidence with Liou and Wang [16] method for moderate decision maker $\alpha = 0.5$ as the maximum value of decision level in the proposed method is 0.5.

Example 4.4. Consider the symmetric fuzzy numbers $A_1 = (1, 3, 3, 5; 1)$ and

$A_2 = (2, 3, 3, 4; 1)$ having same core taken from Chu and Tsao [5] shown in Fig. 6.

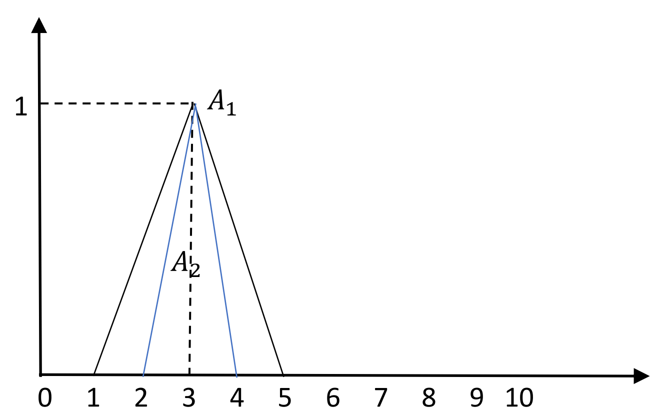

Fig. 6. *A_1 = (1, 3, 3, 5;1), A_2 = (2, 3, 3, 4; 1)*

By using Eq. (5) of the proposed method we get $Val(A_1) = Val(A_2) = 1$, hence the ranking is done through ambiguity. By using Eq. (7), we get $Amb(A_1) = 0.1111$ and $Amb(A_2) = 0.0555$. As $Amb(A_1) > Amb(A_2)$ we can conclude that $A_1 \prec A_2$. This example shows that the proposed method can rank symmetric fuzzy numbers having same core and because of this nature, many existing methods failed to rank the above fuzzy numbers. For instance, Chu and Tsao [5], Abbasbandy and Hajjari [1], Wang and Lee [9], Yao and Wu [12] ranked the above fuzzy numbers as $A_1 \sim A_2$.

Example 4.5 (Comparitive Study). In this example, a comparative study of the proposed method with different existing methods in literature like Chu and Tsao [5], Wang et al. [11], Chen and Sanguansat [20], Chen and Chen [24], Chen et al. [18], Nasseri et al. [23], Rezvani [22], Yager [13], Shureshjani and Darehmiraki [21] and Rituparna and Bijit [19] is made by considering the following sets of fuzzy numbers taken from Yao and Wu [12], shown in Fig. 7. The results are presented in Table 1.

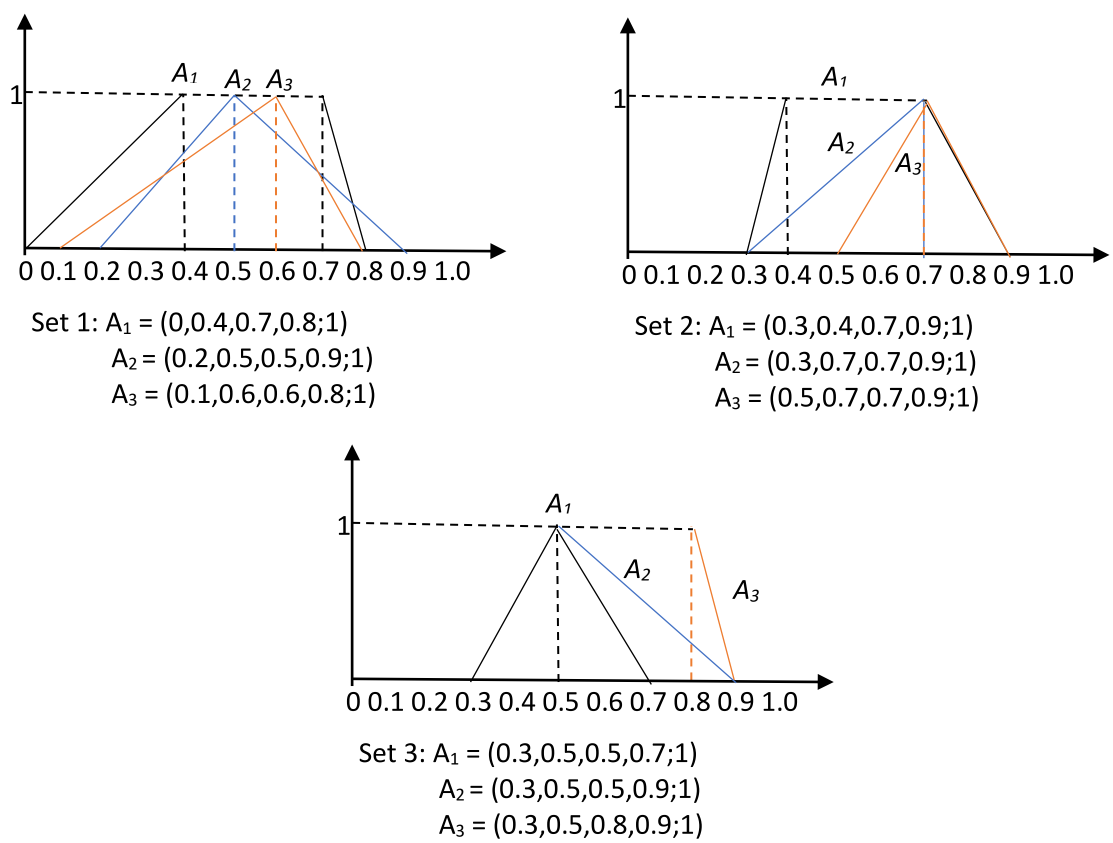

Fig. 7. Three sets of fuzzy numbers (Yao and Wu [12])

Set (1) : $A_1 = (0, 0.4, 0.7, 0.8; 1.0), A_2 = (0.2, 0.5, 0.5, 0.9; 1.0), A_3 = (0.1, 0.6, 0.6, 0.8; 1.0)$

Set (2): $A_1 = (0.3, 0.4, 0.7, 0.9; 1.0), A_2 = (0.3, 0.7, 0.7, 0.9; 1.0), A_3 = (0.5, 0.7, 0.7, 0.9; 1.0)$

Set (3): $A_1 = (0.3, 0.5, 0.5, 0.7; 1.0), A_2 = (0.3, 0.5, 0.5, 0.9; 1.0), A_3 = (0.3, 0.5, 0.8, 0.9; 1.0)$

Table 1									
Methods	Set 1			Set 2			Set 3		
	A_1	A_2	A_3	A_1	A_2	A_3	A_1	A_2	A_3
Chu and Tsao [5]	0.2440	0.2624	0.2619	0.2847	0.3248	0.3500	0.2500	0.2747	0.3152
Ranking order	$A_1 \prec A_3 \prec A_2$			$A_1 \prec A_2 \prec A_3$			$A_1 \prec A_2 \prec A_3$		
Wang et al. [11]	0.6284	0.6289	0.6009	0.7289	0.7157	0.7753	0.6009	0.6574	0.7646
Ranking order	$A_3 \prec A_1 \prec A_2$			$A_2 \prec A_1 \prec A_3$			$A_1 \prec A_2 \prec A_3$		
Chen and Sanguansat [20]	0.4750	0.5250	0.5250	0.5750	0.6500	0.7000	0.5000	0.5500	0.6250
Ranking order	$A_1 \prec A_2 \sim A_3$			$A_1 \prec A_2 \prec A_3$			$A_1 \prec A_2 \prec A_3$		
Chen and Chen [24]	0.3494	0.4079	0.4043	0.4508	0.5193	0.6017	0.4298	0.4394	0.4901
Ranking order	$A_1 \prec A_3 \prec A_2$			$A_1 \prec A_2 \prec A_3$			$A_1 \prec A_2 \prec A_3$		
Chen et al. [18]	0.4269	0.4667	0.4773	0.5168	0.6046	0.6511	0.4444	0.4889	0.5747
Ranking order	$A_1 \prec A_2 \prec A_3$			$A_1 \prec A_2 \prec A_3$			$A_1 \prec A_2 \prec A_3$		
Nasseri et al. [23]	1.3934	1.4413	1.4446	1.6281	1.7188	1.8615	1.4615	1.5188	1.7281
Ranking order	$A_1 \prec A_2 \prec A_3$			$A_1 \prec A_2 \prec A_3$			$A_1 \prec A_2 \prec A_3$		
Rezvani [22]	0.0060	0.0973	0.0730	0.0524	0.1050	0.1269	0.0685	0.1050	0.0892
Ranking order	$A_1 \prec A_3 \prec A_2$			$A_1 \prec A_2 \prec A_3$			$A_1 \prec A_3 \prec A_2$		
Yager [13]	0.4636	0.5333	0.5000	0.5777	0.6333	0.7000	0.5000	0.5667	0.6222
Ranking order	$A_1 \prec A_3 \prec A_2$			$A_1 \prec A_2 \prec A_3$			$A_1 \prec A_2 \prec A_3$		
Shureshjani and Darehmiraki [21]									
$\boldsymbol{\alpha = 0.1}$	0.8685	0.9405	0.9585	1.0305	1.1790	1.2600	0.9000	0.9810	1.1295
Ranking order	$A_1 \prec A_2 \prec A_3$			$A_1 \prec A_2 \prec A_3$			$A_1 \prec A_2 \prec A_3$		
$\boldsymbol{\alpha = 0.5}$	0.5125	0.5125	0.5625	0.5625	0.6750	0.7000	0.5000	0.5250	0.6375
Ranking order	$A_1 \sim A_2 \prec A_3$			$A_1 \prec A_2 \prec A_3$			$A_1 \prec A_2 \prec A_3$		
$\boldsymbol{\alpha = 0.8}$	0.2140	0.2020	0.2340	0.2220	0.2760	0.2800	0.[12]	0.2040	0.2580
Ranking order	$A_2 \prec A_1 \prec A_3$			$A_1 \prec A_2 \prec A_3$			$A_1 \prec A_2 \prec A_3$		
Rituparna and Bijit [19]									
$\boldsymbol{\alpha = 0.1}$	0.4959	0.5112	0.5454	0.5607	0.6606	0.6930	0.4950	0.5274	0.6273
Ranking order	$A_1 \prec A_2 \prec A_3$			$A_1 \prec A_2 \prec A_3$			$A_1 \prec A_2 \prec A_3$		
$\boldsymbol{\alpha = 0.5}$	0.3875	0.3833	0.4250	0.4208	0.5083	0.5250	0.3750	0.3917	0.4792
Ranking order	$A_2 \prec A_1 \prec A_3$			$A_1 \prec A_2 \prec A_3$			$A_1 \prec A_2 \prec A_3$		
$\boldsymbol{\alpha = 0.8}$	0.1928	0.1817	0.2108	0.1997	0.2485	0.2520	0.1800	0.1835	0.2323
Ranking order	$A_2 \prec A_1 \prec A_3$			$A_1 \prec A_2 \prec A_3$			$A_1 \prec A_2 \prec A_3$		
Proposed method									
$\boldsymbol{\alpha \in [1/3, 1/2]}$	0.175	0.1694	0.1917	0.1861	0.2278	0.2333	0.1667	0.1722	0.2139
Ranking order	$A_2 \prec A_1 \prec A_3$			$A_1 \prec A_2 \prec A_3$			$A_1 \prec A_2 \prec A_3$		

From Table 1, it can be seen that for Set (1), the ranking results of one method conflict the other and the results of the proposed method are consistent with Rituparna and Bijit [19] method for decision levels $\alpha = 0.5\ and\ \alpha = 0.8$ and Shureshjani and Darehmiraki [21] method for decision level $\alpha = 0.8$ and for $\alpha = 0.5$ this method failed to discriminate fuzzy numbers $A_1\ and\ A_2$ and the method proposed by Chen and Sanguansat [20] failed to discriminate fuzzy numbers $A_2\ and\ A_3$.

For Set (2), the ranking results of the proposed method coincides with all other methods except the method proposed by Wang et al. [11]. This method failed to rank fuzzy numbers A_1 and A_2 having same right spreads which is unreasonable.

For Set (3), the ranking results of the proposed method coincides with all other methods except the method proposed by Rezvani [22] which failed to rank fuzzy numbers A_1 and A_2 having same core.

Example 4.6 (Comparative Study). Four sets of fuzzy numbers are taken from Chen and Chen [25], and a comparative study is carried out with some existing methods in literature like Chu and Tsao [5], Wang et al. [11], Chen and Sanguansat [20], Chen and Chen [24], Chen et al. [18], Nasseri et al. [23], Rezvani [22], Yager [13], Shureshjani and Darehmiraki [21] and Rituparna and Bijit [19] shown in Fig. 8. The results are presented in Table 2.

Set (1) : $A_1 = (0.1, 0.3, 0.3, 0.5; 1.0), A_2 = (0.3, 0.5, 0.5, 0.7; 1.0)$

Set (2) : $A_1 = (0.1, 0.4, 0.4, 0.5; 1.0), A_2 = (0.2, 0.3, 0.3, 0.6; 1.0)$

Set (3) : $A_1 = (0.1, 0.3, 0.3, 0.5; 1.0), A_2 = (0.2, 0.3, 0.3, 0.4; 1.0)$

Set (4) : $A_1 = (0.1, 0.3, 0.3, 0.5; 0.8), A_2 = (0.1, 0.3, 0.3, 0.5; 1.0)$

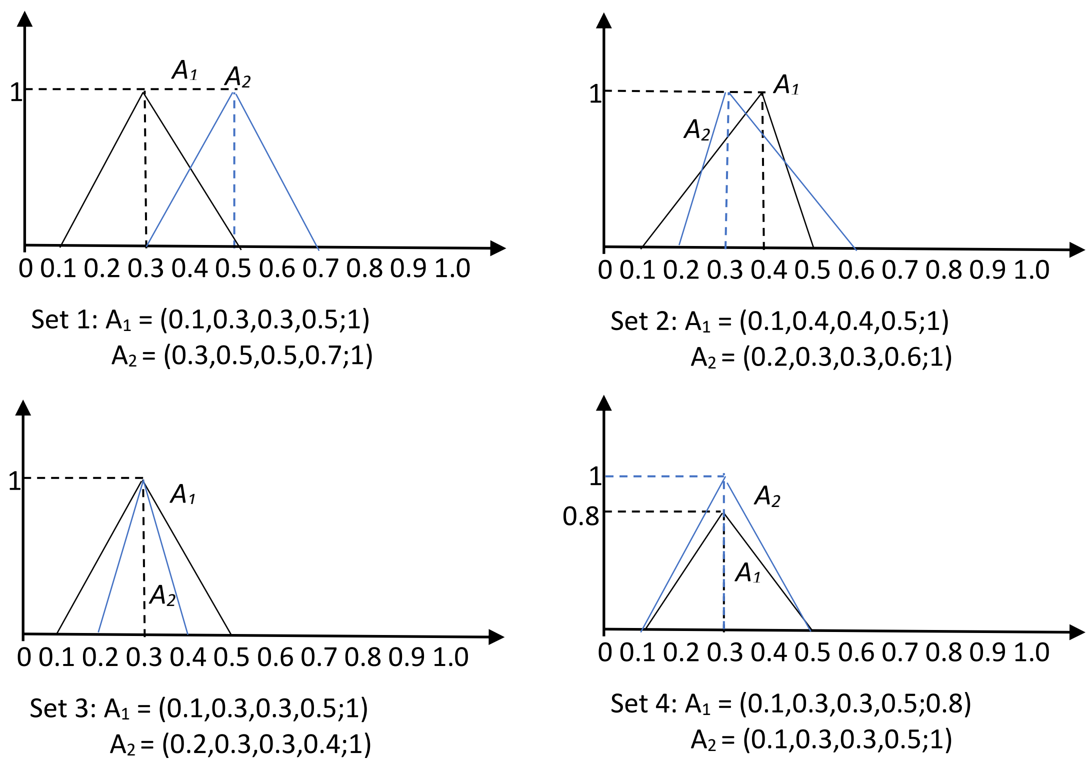

Fig. 8. Four sets of fuzzy numbers (Chen and Chen [25])

For Set (1), the ranking order of the proposed method using the values of the fuzzy numbers A_1 and A_2 is $A_1 \prec A_2$. This result is in agreement with all other methods as shown in Table 2.

For Set (2), the ranking order of the proposed method using the values of the fuzzy numbers A_1 and A_2 is $A_1 \succ A_2$. This result coincides with human intuition, as one prefers A_1 to A_2 because of their core values and compensation of areas and with methods proposed by Chen et al. [18], Shureshjani and Darehmiraki [21] and Rituparna and Bijit [19]. The methods proposed by Chu and Tsao [5], Chen and Sanguansat [20], Chen and Chen [24] and Nasseri et al. [23] failed to discriminate the fuzzy numbers whereas, ranking orders proposed by Wang et al. [11], Rezvani [22] and Yager [13] are unreasonable.

Table 2

Methods	Set 1		Set 2		Set 3		Set 4	
	A_1	A_2	A_1	A_2	A_1	A_2	A_1	A_2
Chu and Tsao [5]	0.1500	0.2500	0.1746	0.1746	0.1500	0.1500	0.1200	0.1500
Ranking order	$A_1 \prec A_2$		$A_1 \sim A_2$		$A_1 \sim A_2$		$A_1 \prec A_2$	
Wang et al. [11]	0.4484	0.6009	0.4714	0.4955	0.4485	0.4485	0.4014	0.4485
Ranking order	$A_1 \prec A_2$		$A_1 \prec A_2$		$A_1 \sim A_2$		$A_1 \prec A_2$	
Chen and Sanguansat [20]	0.3000	0.4000	0.3500	0.3500	0.3000	0.3000	0.2824	0.3000
Ranking order	$A_1 \prec A_2$		$A_1 \sim A_2$		$A_1 \sim A_2$		$A_1 \prec A_2$	
Chen and Chen [24]	0.2578	0.4298	0.2983	0.2983	0.2578	0.2774	0.2063	0.3000
Ranking order	$A_1 \prec A_2$		$A_1 \sim A_2$		$A_1 \prec A_2$		$A_1 \prec A_2$	
Chen et al. [18]	0.2553	0.4444	0.3043	0.2978	0.2553	0.2553	0.2462	0.2553
Ranking order	$A_1 \prec A_2$		$A_1 \succ A_2$		$A_1 \sim A_2$		$A_1 \prec A_2$	
Nasseri et al. [23]	1.0615	1.4615	1.1622	1.1622	1.0615	1.0901	0.8885	1.0615
Ranking order	$A_1 \prec A_2$		$A_1 \sim A_2$		$A_1 \prec A_2$		$A_1 \prec A_2$	
Rezvani [22]	0.0296	0.0685	0.0296	0.0466	0.0297	0.0238	0.0286	0.0297
Ranking order	$A_1 \prec A_2$		$A_1 \prec A_2$		$A_1 \succ A_2$		$A_1 \prec A_2$	
Yager [13]	0.3000	0.5000	0.3333	0.3666	0.3000	0.3000	0.3000	0.3000
Ranking order	$A_1 \prec A_2$		$A_1 \prec A_2$		$A_1 \sim A_2$		$A_1 \sim A_2$	
Shureshjani and Darehmiraki [21]								
$\boldsymbol{\alpha = 0.1}$	0.5400	0.9000	0.6390	0.6210	0.5400	0.5400	0.4200	0.5400
Ranking order	$A_1 \prec A_2$		$A_1 \succ A_2$		$A_1 \sim A_2$		$A_1 \prec A_2$	
$\boldsymbol{\alpha = 0.5}$	0.3000	0.5000	0.3750	0.3250	0.3000	0.3000	0.1800	0.3000
Ranking order	$A_1 \prec A_2$		$A_1 \succ A_2$		$A_1 \sim A_2$		$A_1 \prec A_2$	
$\boldsymbol{\alpha = 0.8}$	0.1200	0.[12]	0.1560	0.1240	0.1200	0.1200	0.0000	0.1200
Ranking order	$A_1 \prec A_2$		$A_1 \succ A_2$		$A_1 \sim A_2$		$A_1 \prec A_2$	
Rituparna and Bijit [19]								
$\boldsymbol{\alpha = 0.1}$	0.2970	0.4950	0.3636	0.3294	0.0648	0.0324	0.1890	0.2970

Ranking order	$A_1 \prec A_2$		$A_1 \succ A_2$		$A_1 \prec A_2$		$A_1 \prec A_2$	
$\boldsymbol{\alpha = 0.5}$	0.2250	0.3750	0.2833	0.2416	0.0333	0.0166	0.1170	0.2250
Ranking order	$A_1 \prec A_2$		$A_1 \succ A_2$		$A_1 \prec A_2$		$A_1 \prec A_2$	
$\boldsymbol{\alpha = 0.8}$	0.1080	0.1800	0.1405	0.1114	0.0693	0.0350	0.0000	0.1080
Ranking order	$A_1 \prec A_2$		$A_1 \succ A_2$		$A_1 \prec A_2$		$A_1 \prec A_2$	
Proposed method								
$\boldsymbol{\alpha} \in [1/3, 1/2]$	0.1000	0.1667	0.1278	0.1055	0.0111	0.0055	0.0800	0.1000
Ranking order	$A_1 \prec A_2$		$A_1 \succ A_2$		$A_1 \prec A_2$		$A_1 \prec A_2$	

In Set (3), fuzzy numbers are symmetric and having same core. For the proposed method $Val(A_1) = Val(A_2) = 0.1$ and hence, the ranking order is decided by using ambiguity. As $Amb(A_1) = 0.0111, Amb(A_2) = 0.0055$ and $Amb(A_1) > Amb(A_2)$, the ranking order of the proposed method by using the ambiguity of the fuzzy numbers A_1 and A_2 is $A_1 \prec A_2$. The result is consistent with the methods proposed by Chen and Chen [18] and Nasseri et al. [23]. The methods proposed by Chu and Tsao [5], Wang et al. [11], Chen and Sanguansat [20], Chen et al. [18], Yager [13] and Shureshjani and Darehmiraki [21] failed to discriminate the fuzzy numbers, though both numbers are different in nature and by intuition, the ranking by Rezvani [22] is unreasonable.

For Set (4), the ranking order of the proposed method using the values of the fuzzy numbers A_1 and A_2 is $A_1 \prec A_2$. This result is in agreement with all other methods as shown in Table 2 except the method proposed by Yager [13] which failed to discriminate the given fuzzy numbers.

Example 4.7 (Comparative Study). Four sets of fuzzy numbers are taken from Chen et al. [18], and a comparative study is carried out with some existing methods in literature like Chu and Tsao [5], Wang et al. [11], Chen and Sanguansat [20], Chen and Chen [24], Chen et al. [18], Nasseri et al. [23], Rezvani [22], Yager [13], Shureshjani and Darehmiraki [21] and Rituparna and Bijit [19], shown in Fig. 9. The results are presented in Table 3.

Set (1) : $A_1 = (-0.5, -0.3, -0.3, -0.1; 1.0), A_2 = (0.1, 0.3, 0.3, 0.5; 1.0)$

Set (2) : $A_1 = (0, 0.4, 0.6, 0.8; 1.0), A_2 = (0.2, 0.5, 0.5, 0.9; 1.0), A_3 = (0.1, 0.6, 0.7, 0.8; 1.0)$

Set (3) : $A_1 = (0.3, 0.5, 0.5, 1.0; 1.0), A_2 = (0.1, 0.6, 0.6, 0.8; 1.0)$

Set (4) : $A_1 = (0.1, 0.2, 0.4, 0.5; 1.0), A_2 = (1.0, 1.0, 1.0, 1.0; 1.0)$

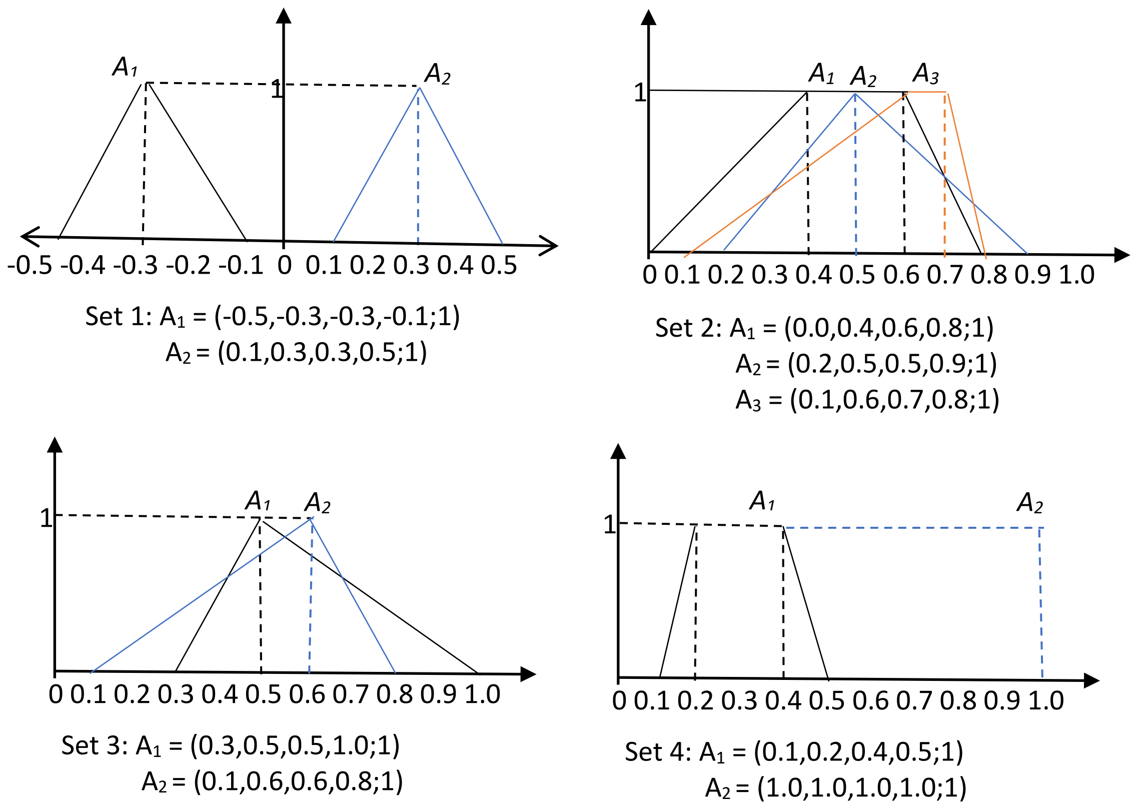

Fig. 9. Four sets of fuzzy numbers (Chen et al. [18])

From Table 3, we can observe that for Set (1), the ranking order of the proposed method using the values of the fuzzy numbers A_1 and A_2 is $A_1 \prec A_2$. By intuition, this result is in agreement with methods proposed by Chu and Tsao [5], Chen and Sanguansat [20], Chen and Chen [24], Chen et al. [18], Yager [13], Shureshjani and Darehmiraki [21] and Rituparna and Bijit [19]. The methods proposed by, Wang et al. [11] and Rezvani [22] failed to discriminate the fuzzy numbers and the ranking order of Nasseri et al. [23] is unreasonable.

Table 3

Methods	Set 1		Set 2			Set 3		Set 4	
	A_1	A_2	A_1	A_2	A_3	A_1	A_2	A_1	A_2
Chu and Tsao [5]	-0.150	0.1500	0.2281	0.2624	0.2784	0.2869	0.2619	0.1500	###
Ranking order	$A_1 \prec A_2$		$A_1 \prec A_2 \prec A_3$			$A_1 \succ A_2$		***	
Wang et al. [11]	0.4485	0.4485	0.5946	0.6289	0.6452	0.6864	0.6009	0.5362	###
Ranking order	$A_1 \sim A_2$		$A_1 \prec A_2 \prec A_3$			$A_1 \succ A_2$		***	
Chen and Sanguansat [20]	-0.300	0.300	0.450	0.525	0.550	0.575	0.525	0.300	1.000
Ranking order	$A_1 \prec A_2$		$A_1 \prec A_2 \prec A_3$			$A_1 \succ A_2$		$A_1 \prec A_2$	

Chen and Chen [24]	-0.257	0.257	0.3354	0.4079	0.4196	0.4428	0.4043	0.2537	1.0000
Ranking order	$A_1 \prec A_2$		$A_1 \prec A_2 \prec A_3$			$A_1 \succ A_2$		$A_1 \prec A_2$	
Chen et al. [18]	-0.255	0.255	0.4000	0.4666	0.5057	0.5111	0.4773	0.2553	1.0000
Ranking order	$A_1 \prec A_2$		$A_1 \prec A_2 \prec A_3$			$A_1 \succ A_2$		$A_1 \prec A_2$	
Nasseri et al. [23]	0.1385	1.0615	1.3188	1.4413	1.5227	1.5446	1.4447	1.0900	2.5000
Ranking order	$A_1 \succ A_2$		$A_1 \prec A_2 \prec A_3$			$A_1 \succ A_2$		$A_1 \prec A_2$	
Rezvani [22]	0.0297	0.0297	0.0442	0.0973	0.0505	0.1265	0.0730	1.0072	###
Ranking order	$A_1 \sim A_2$		$A_1 \prec A_3 \prec A_2$			$A_1 \succ A_2$			
Yager [13]	-0.300	0.3000	0.4400	0.5333	0.5250	0.6000	0.5000	0.3000	###
Ranking order	$A_1 \prec A_2$		$A_1 \prec A_3 \prec A_2$			$A_1 \succ A_2$		***	
Shureshjani and Darehmiraki [21]									
$\boldsymbol{\alpha = 0.1}$	-0.540	0.5400	0.8190	0.9405	1.0080	1.0215	0.9585	0.5400	1.8000
Ranking order	$A_1 \prec A_2$		$A_1 \prec A_2 \prec A_3$			$A_1 \succ A_2$		$A_1 \prec A_2$	
$\boldsymbol{\alpha = 0.5}$	-0.300	0.3000	0.4750	0.5125	0.6000	0.5375	0.5625	0.3000	1.0000
Ranking order	$A_1 \prec A_2$		$A_1 \prec A_2 \prec A_3$			$A_1 \prec A_2$		$A_1 \prec A_2$	
$\boldsymbol{\alpha = 0.8}$	-0.120	0.1200	0.1960	0.2020	0.2520	0.2060	0.2340	0.1200	0.3900
Ranking order	$A_1 \prec A_2$		$A_1 \prec A_2 \prec A_3$			$A_1 \prec A_2$		$A_1 \prec A_2$	
Rituparna and Bijit [19]									
$\boldsymbol{\alpha = 0.1}$	-0.297	0.2970	0.4626	0.5111	0.5787	0.5436	0.5454	0.2970	0.9900
Ranking order	$A_1 \prec A_2$		$A_1 \prec A_2 \prec A_3$			$A_1 \prec A_2$		$A_1 \prec A_2$	
$\boldsymbol{\alpha = 0.5}$	-0.225	0.2250	0.3583	0.3833	0.4542	0.4000	0.4250	0.2250	0.7500
Ranking order	$A_1 \prec A_2$		$A_1 \prec A_2 \prec A_3$			$A_1 \prec A_2$		$A_1 \prec A_2$	
$\boldsymbol{\alpha = 0.8}$	-0.108	0.1080	0.1765	0.1817	0.2271	0.1852	0.2108	0.1080	0.3600
Ranking order	$A_1 \prec A_2$		$A_1 \prec A_2 \prec A_3$			$A_1 \prec A_2$		$A_1 \prec A_2$	
Proposed method									
$\boldsymbol{\alpha} \in [{}^{\mathbf{1}}/_{\mathbf{3}}, {}^{\mathbf{1}}/_{\mathbf{2}}]$	-0.100	0.1000	0.0111	0.1694	0.2242	0.0250	0.1916	0.1000	0.3333
Ranking order	$A_1 \prec A_2$		$A_1 \prec A_2 \prec A_3$			$A_1 \prec A_2$		$A_1 \prec A_2$	
### means that the method cannot calculate the ranking value of the fuzzy number * means that the ranking order cannot be established by the method**									

For Set (2) fuzzy numbers, the ranking order of the proposed method using the values is $A_1 \prec A_2 \prec A_3$. This result coincides with all the methods shown in Table 3, other than the methods proposed by Rezvani [22] and Yager [13] as the results produced by these methods are unreasonable by intuition.

For Set (3), the ranking order of the proposed method coincides with the methods proposed by Shureshjani and Darehmiraki [21] for higher decision levels 0.5 and 0.8 and for all decision levels of Rituparna and Bijit [19] method and the ranking results of other methods are unreasonable.

In set (4), the fuzzy number A_2 is a crisp number and it is clear that from intuition, the ranking order should be $A_1 \prec A_2$. The proposed method and the methods proposed by Chen and Sanguansat [20], Chen and Chen [24], Chen et al. [18], Nasseri et al. [23], Shureshjani and Darehmiraki [21] and Rituparna and Bijit [19] are in complete agreement with intuition. The methods Chu and Tsao [5], Wang et al. [11], Rezvani [22] and Yager [13] failed to calculate the ranking value of the crisp fuzzy number and eventually no ordering was made by these methods.

Example 4.8 (Comparative Study). Four sets of fuzzy numbers are taken from Bortolan and Degani [2], and a comparative study is carried out with some existing methods in literature like Chu and Tsao [5], Wang et al. [11], Chen and Sanguansat [20], Chen and Chen [24], Chen et al. [18], Nasseri et al. [23], Rezvani [22], Yager [13], Shureshjani and Darehmiraki [21] and Rituparna and Bijit [19], shown in Fig. 10. The results are presented in Table 4.

Set (1) : $A_1 = (0.1, 0.1, 0.1, 0.1; 0.8), A_2 = (-0.1, -0.1, -0.1, -0.1; 1.0)$

Set (2): $A_1 = (0.3, 0.4, 0.6, 0.7; 1.0), A_2 = (0.4, 0.5, 0.5, 0.6; 1.0)$

Set (3) : $A_1 = (1.0, 1.0, 1.0, 1.0; 0.5), A_2 = (1.0, 1.0, 1.0, 1.0; 1.0)$

Set (4): $A_1 = (0.4, 0.5, 0.5, 1.0; 1.0), A_2 = (0.4, 0.7, 0.7, 1.0; 1.0), A_3 = (0.4, 0.9, 0.9, 1.0; 1.0)$

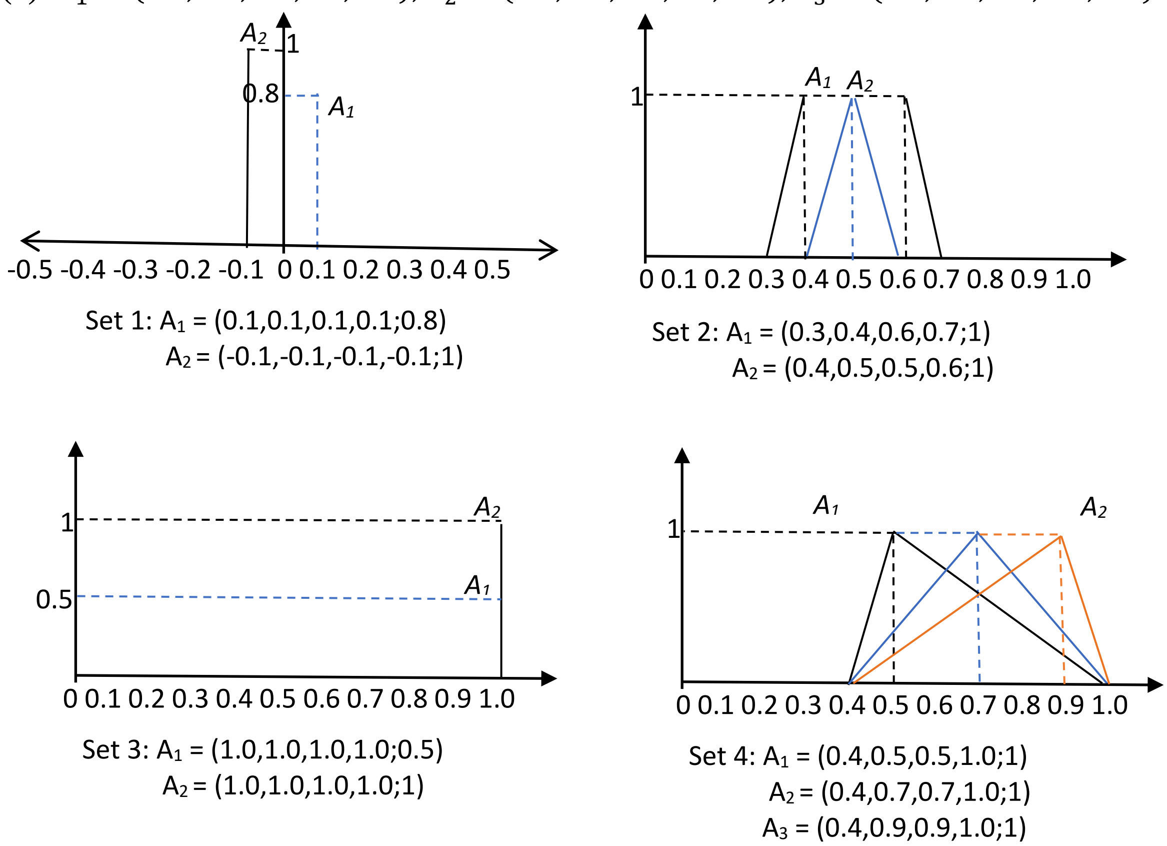

Fig. 10. Four sets of fuzzy numbers (Bortolan and Degani [2])

Table 4

Methods	Set 1		Set 2		Set 3		Set 4		
	A_1	A_2	A_1	A_2	A_1	A_2	A_1	A_2	A_3
Chu and Tsao [5]	###	###	0.2500	0.2500	###	###	0.2991	0.3500	0.3993
Ranking order	***		$A_1 \sim A_2$		***		$A_1 \prec A_2 \prec A_3$		
Wang et al. [11]	###	###	0.6689	0.6009	###	###	0.7157	0.7753	0.8359
Ranking order	***		$A_1 \succ A_2$		***		$A_1 \prec A_2 \prec A_3$		
Chen and Sanguansat [20]	0.0941	-0.100	0.5000	0.5000	0.8000	1.0000	0.6000	0.7000	0.8000
Ranking order	$A_1 \succ A_2$		$A_1 \sim A_2$		$A_1 \prec A_2$		$A_1 \prec A_2 \prec A_3$		
Chen and Chen [24]	0.0800	-0.100	0.4228	0.4623	0.5000	1.0000	0.4721	0.5623	0.6295
Ranking order	$A_1 \succ A_2$		$A_1 \prec A_2$		$A_1 \prec A_2$		$A_1 \prec A_2 \prec A_3$		
Chen et al. [18]	0.0780	-0.081	0.4444	0.4444	1.0000	1.0000	0.5333	0.6512	0.7805
Ranking order	$A_1 \succ A_2$		$A_1 \sim A_2$		$A_1 \sim A_2$		$A_1 \prec A_2 \prec A_3$		
Nasseri et al. [23]	0.5200	-0.300	1.4900	1.4900	2.1250	2.5000	1.6227	1.8174	2.0227
Ranking order	$A_1 \succ A_2$		$A_1 \sim A_2$		$A_1 \prec A_2$		$A_1 \prec A_2 \prec A_3$		
Rezvani [22]	0.0000	0.0000	0.0196	0.0627	###	###	0.1366	0.1366	0.1366
Ranking order	$A_1 \sim A_2$		$A_1 \prec A_2$		***		$A_1 \sim A_2 \sim A_3$		
Yager [13]	###	###	0.5000	0.5000	###	###	0.6333	0.7000	0.7666
Ranking order	***		$A_1 \sim A_2$		***		$A_1 \prec A_2 \prec A_3$		
Shureshjani and Darehmiraki [21]									
$\boldsymbol{\alpha = 0.1}$	0.1400	-0.180	0.9000	0.9000	0.8000	1.8000	1.0620	1.2600	1.4589
Ranking order	$A_1 \succ A_2$		$A_1 \sim A_2$		$A_1 \prec A_2$		$A_1 \prec A_2 \prec A_3$		
$\boldsymbol{\alpha = 0.5}$	0.0600	-0.100	0.5000	0.5000	0.0000	1.0000	0.5500	0.7000	0.8500
Ranking order	$A_1 \succ A_2$		$A_1 \sim A_2$		$A_1 \prec A_2$		$A_1 \prec A_2 \prec A_3$		
$\boldsymbol{\alpha = 0.8}$	0.0000	-0.040	0.1200	0.1200	0.0000	0.4000	0.2080	0.2800	0.3519
Ranking order	$A_1 \succ A_2$		$A_1 \sim A_2$		$A_1 \prec A_2$		$A_1 \prec A_2 \prec A_3$		
Rituparna and Bijit [19]									
$\boldsymbol{\alpha = 0.1}$	0.0630	-0.099	0.1314	0.0324	0.2400	0.9900	0.5598	0.6929	0.8262
Ranking order	$A_1 \succ A_2$		$A_1 \prec A_2$		$A_1 \prec A_2$		$A_1 \prec A_2 \prec A_3$		
$\boldsymbol{\alpha = 0.5}$	0.0390	-0.075	0.0917	0.0170	0.0000	0.7500	0.4083	0.5249	0.6416
Ranking order	$A_1 \succ A_2$		$A_1 \prec A_2$		$A_1 \prec A_2$		$A_1 \prec A_2 \prec A_3$		
$\boldsymbol{\alpha = 0.8}$	0.0000	-0.036	0.0395	0.0035	0.0000	0.3600	0.1869	0.2519	0.3170
Ranking order	$A_1 \succ A_2$		$A_1 \prec A_2$		$A_1 \prec A_2$		$A_1 \prec A_2 \prec A_3$		
Proposed method									
$\boldsymbol{\alpha \in [1/3, 1/2]}$	0.0266	-0.033	0.0222	0.0055	0.1666	0.3333	0.1777	0.2333	0.2888
Ranking order	$A_1 \succ A_2$		$A_1 \prec A_2$		$A_1 \prec A_2$		$A_1 \prec A_2 \prec A_3$		
### means that the method cannot calculate the ranking value of the fuzzy number									
***** means that the ranking order cannot be established by the method**									

In Table 4, it is very obvious that for Set (1), the ranking order should be $A_1 \succ A_2$ and the proposed method along with Chen and Sanguansat [20], Chen and Chen [24], Chen et al. [18], Nasseri [23], Shureshjani and Darehmiraki [21] and Rituparna and Bijit [19] produced consistent results. The methods proposed by Chu and Tsao [18], Wang et al. [11] and Yager [13] failed to calculate the ranking values of the given fuzzy numbers whereas, the ranking result of Rezvani [22] is very unreasonable.

In Set (2), vagueness in A_1 is more than A_2 hence one will prefer A_2 than A_1 and the ordering should be $A_1 \prec A_2$. For the proposed method $Val(A_1) = Val(A_2) = 0.1666$ and hence, the ranking order is decided by using ambiguity. As $Amb(A_1) = 0.0222, Amb(A_2) = 0.0055$ and $Amb(A_1) > Amb(A_2)$, the ranking order is $A_1 \prec A_2$. The ranking order of the proposed method and the methods proposed by Chen and Chen [24], Rezvani [22] and Rituparna and Bijit [19] coincide with this argument. The methods proposed by Chu and Tsao [5], Chen and Sanguansat [20], Chen et al. [18], Nasseri et al. [23], Yager [13] and Shureshjani and Darehmiraki [21] failed to discriminate the fuzzy numbers and even the method proposed by Wang et al. [11] is unreasonable.

In Set (3), A_1, A_2 are crisp numbers with different heights and hence there exists no vagueness. Intuitively, one would prefer A_2 than A_1 with respect to the normality issue and the ordering should be $A_1 \prec A_2$. The proposed method and other methods proposed by Chen and Sanguansat [20], Chen and Chen [24], Nasseri et al. [23], Shureshjani and Darehmiraki [21] and Rituparna and Bijit [19] are consistent with the ranking order $A_1 \prec A_2$. The methods proposed by Chu and Tsao [5], Wang et al. [11], Rezvani [22] and Yager [13] failed to calculate the ranking values of the crisp fuzzy numbers. The ranking order of Chen et al. [18] is quite unreasonable and failed to discriminate the given fuzzy numbers.

For Set (4), the ranking order of the fuzzy numbers through values is $A_1 \prec A_2 \prec A_3$ and this result is consistent with all other methods shown in Table 4, except the method proposed by Rezvani [22] which is unreasonable. It is also interesting to note that the ambiguity of all the given three fuzzy numbers are same.

5. Conclusions

This current research proposes a new method on ranking generalized trapezoidal fuzzy numbers by defuzzification method based on centroids, value which is the ill-defined magnitude, ambiguity an amount of vagueness present in the ill-defined magnitude and decision levels in the range $\left[\frac{w}{3}, \frac{w}{2}\right]$ where w is the height of the fuzzy number and $0 \leq w \leq 1$. The proposed method can rank different types of fuzzy numbers and address the shortcomings of ranking symmetric fuzzy numbers having same core and different heights, same support and different cores, crisp numbers, crisp numbers having same support and different heights and fuzzy numbers having compensation of areas. Another advantage of the proposed method is that it removes the usage of a reducing function to diminish the contribution of lower alpha levels. Moreover, the simplicity of the method is that the formulas for calculating the ranking values of fuzzy numbers through value and ambiguity are very simple. The effectiveness of the proposed method in overcoming the shortcomings in some of the existing fuzzy ranking methods is demonstrated through the rigorous comparative study presented in Section 4 and it is numerically very simple as well effective for implementation.

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