Development of Fuzzy Inventory Theory for Deteriorating Items with Ramp Type Demand

By

Dr. Biswaranjan Mandal
Associate Professor of Mathematics
Acharya Jagadish Chandra Bose College, Kolkata
Email : drbrmajcbc@gmail.com
Dr. Bappa Saha
Assistant Professor of Physics,
Acharya Jagadish Chandra Bose College,
Kolkata, West Bengal, India,

Abstract :
The present research paper proposes an inventory theory for deteriorating items with ramp type demand under fuzzy environment. The model is solved with salvage value associated to the units deteriorating during the cycle. Fuzzy set theory is generally assumed or uncertainty nature of quantitative scenario. The demand rate, deterioration rate, holding cost, unit cost and salvage value are considered as trapezoidal fuzzy numbers. The total inventory costs for both crisp model and fuzzy model are derived. Both graded mean integration method and signed distance method are used to defuzzify the total cost function. Shortages are not allowed in the proposed paper. Further, numerical examples are given to develop crisp model and fuzzy model. A structural comparative study is demonstrated here by illustrating the model with sensitivity analysis.

Key Words: Inventory, ramp type, fuzzy, deterioration, salvage value, trapezoidal fuzzy numbers and defuzzify.

1. Introduction:

The inventor model is the oldest inventory model. In many inventory management system, the uncertainty occurs due to fuzziness and it is much closed approach to reality. In this field, many researchers like Bellman et al[1], Kao et al [2], Chou [3], Guiffrida [4], Garg[5] etc developed their inventory models considering fuzzy behaviour.

There are mostly two types of demands-linear and exponential. Both the demand patterns are not more realistic in real market. So many models have been developed under ramp type demand pattern which is generally seen in the case o any new brand of consumer goods coing to the market. This demand rate increases with time upto certain period and then stabilizes becoming constant. On this view, researchers like Baker et al [6], Alfares [7], Tripathi et al [8], Mandal [9], Biswaranjan [10], [11] are noteworthy.

All we know that Salvage value named scrap value or residual value can be defined as an estimated price or value of any non-current asset at the end of its useful life (usually more than one year) or in other words the price which the company will get after utilizing it throughout its useful life by selling in a competitive market where goods can be freely exchanged. In general, the salvage value is important because it will be the carrying value of the asset on a company's books after depreciation has been fully expensed. In some cases, salvage value may just be a value the company believes it can obtain by selling a depreciated, inoperable asset for parts. As a whole salvage value plays an important role on inventory management theory. We introduced this value in our present model.

As the inventory management plays an important role in several business sectors, the popular models are developed basically on the pillar of demand and supply. Since last few decades, several research papers are written by many researchers. They explained variety inventory models. The main goal of each model was to minimize the total inventory cost function. But due to advanced technology, market dynamics increased competition day by day in business sectors and inventory is becoming more complicated. This uncertain behaviour is explained only with the help of fuzzy theory. This theory is one of the most important tools to obtain the optimality of the function considering uncertainty cost components like holding cost, deterioration cost, ordering cost and salvage value. Zadeh [12] first discussed the new set theory named fuzzy set theory. Late many researchers like Vujosevic [13], Hsieh[14], Mishra [15], Sushil Kumar[16], Uthayakumar[17], Yadav [18] etc have developed several fuzzy inventory models under various uncertainty constraints.

In the present paper, we investigate an inventory model for decaying items under ramp type demand. Both crisp and fuzzy models are discussed to obtain the total inventory cost. Two methods named graded mean integration method and signed distance method are used to develop the fuzzy model. Shortages are not allowed in the proposed paper. Further, numerical examples are given to develop crisp model and fuzzy model. A structural comparative study is demonstrated here by illustrating the model with sensitivity analysis.

2. Definitions and Preliminaries :

We have stated the following definitions for development of the fuzzy inventory model.

a) A fuzzy set X on the given universal set is a set of order pairs and defined by

$\tilde{A} = \{(x, \lambda_{\tilde{A}}(x)) : x \in X\}$, where $\lambda_{\tilde{A}} : X \to [0,1]$ is called membership function.

b) A fuzzy number $\tilde{A}$ is a fuzzy set on the real number R, if its membership function $\lambda_{\tilde{A}}$ has the following properties

(i). $\lambda_{\tilde{A}}(x)$ is upper semi continuous.

(ii). $\lambda_{\tilde{A}}(x)$ =0, outside some interval $[a_1, a_4]$

Then $\exists$ real numbers a_2 and a_3, $a_1 \le a_2 \le a_3 \le a_4$ such that $\lambda_{\tilde{A}}(x)$ is increasing on $[a_1, a_2]$ and decreasing on $[a_3, a_4]$ and $\lambda_{\tilde{A}}(x)$ =1 for each $x \in [a_1, a_2]$.

c) A trapezoidal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$ is represented with membership function $\lambda_{\tilde{A}}$ as

$$\lambda_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, a_1 \le x \le a_2 \\ 1, a_2 \le x \le a_3 \\ \frac{a_4-x}{a_4-a_3}, a_3 \le x \le a_4 \\ 0, otherwise \end{cases}$$

d) Suppose $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ are two trapezoidal fuzzy numbers, the arithmetical operations are defined as:

(i) $\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$

(ii) $\tilde{A} \otimes \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4)$

(iii) $\tilde{A} \Theta \tilde{B} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$

(iv) $\tilde{A} \phi \tilde{B} = (\frac{a_1}{b_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_4}{b_1})$

(v) $\alpha \otimes \tilde{A} = \{ \begin{matrix} (\alpha a_1, \alpha a_2, \alpha a_3, \alpha a_4), \alpha \ge 0 \\ (\alpha a_4, \alpha a_3, \alpha a_2, \alpha a_1), \alpha < 0 \end{matrix}$

e) Let $\tilde{A} = (a_1, a_2, a_3, a_4)$ be a trapezoidal fuzzy number, then the Graded Mean Integration Method of $\tilde{A}$ is defined as

$$P(\tilde{A}) = \frac{\frac{1}{2}\int_0^1 \alpha [A_L(\alpha) + A_G(\alpha)] d\alpha}{\int_0^1 \alpha d\alpha} = \frac{a_1 + 2a_2 + 2a_3 + a_4}{6}$$

f) Let $\tilde{A} = (a_1, a_2, a_3, a_4)$ be a fuzzy set defined on R, then the Signed Distance Method of $\tilde{A}$ is defined as

$$d(\tilde{A}, 0) = \frac{1}{2}\int_0^1 [A_L(\alpha) + A_G(\alpha)] d\alpha = \frac{a_1 + a_2 + a_3 + a_4}{4}$$

3. Assumptions and Notations:

(i) Replenishment size is constant and replenishment rate is infinite.
(ii) Lead time is zero.
(iii) T is the length of each production cycle.
(iv) h is the inventory holding cost per unit per unit time.
(v) A is the ordering cost/order.
(vi) c is the cost of each deteriorated unit.

(vii) The salvage value $\beta c (0 \le \beta \le 1)$ is associated to deteriorated units during the cycle time.

(viii) TC is the average total cost per unit time.

(ix) A constant fraction θ of the on-hand inventory deteriorates per unit time. A deteriorated item is lost.

(x). Shortages are not allowed.

(xi). The demand rate R(t) is assumed in the model

$$R(t) = D_0 \, [t - (t - \mu) H(t - \mu)], \quad D_0 > 0$$

where H(t - μ) is the well-known Heaviside's function defined as follows:

$$H(t-\mu) = 1, t \ge \mu$$
$$= 0, t < \mu$$

(xii). There is no repair or replacement of the deteriorated items.

(xiii). $\tilde{R}$ is the fuzzy demand.

(xiv). $\tilde{\theta}$ is the fuzzy deterioration rate.

(xv). $\tilde{h}$ is the fuzzy holding cost parameter.

(xvi) $\tilde{\beta}\tilde{c}$ is the fuzzy salvage parameter.

(xvii) $\tilde{c}$ is the b fuzzy purchase parameter.

(xviii) $\tilde{TC}$ is the fuzzy total cost of the system per unit time.

4. Mathematical Models:

4.1: Crisp Model

Let I(t) be the on-hand inventory at any time t. The differential equations which the on-hand inventory I(t) must satisfy during the cycle time T is the following

$$\frac{dI(t)}{dt} + \theta I(t) = -R(t), 0 \le t \le T \tag{4.1.1}$$

In this model, we assume μ < t1 and therefore the above governing equation becomes

$$\frac{dI(t)}{dt} + \theta I(t) = -D_0 t, 0 \le t \le \mu \tag{4.1.2}$$

and $$\frac{dI(t)}{dt} + \theta I(t) = -D_0 \mu, \mu \le t \le T \tag{4.1.3}$$

Boundary Conditions $i(0) = Q$ and $i(T) = 0$ (4.1.4)

Solutions of the equations (4.1.2) and (4.1.3) are the following:

$$I(t) = -D_0 (\frac{t}{\theta} - \frac{1}{\theta^2}) + e^{-\theta t} (Q - \frac{D_0}{\theta^2}), 0 \le t \le \mu \tag{4.1.5}$$

And $I(t)=\frac{D_0\mu}{\theta}(e^{\theta(T-t)}-1), \mu\le t\le T$ (4.1.6)

From (4.1.5) and (4.1.6), we get

$$Q=\frac{D_0}{\theta^2}(1-e^{\theta\mu})+\frac{D_0\mu}{\theta}(1+e^{\theta T}-e^{\theta\mu})$$

Or, $Q=D_0\mu(\frac{\theta}{2}T^2+T-\frac{\theta}{2}\mu^2-\frac{3\mu}{2})$ (by using $e^{\theta}=1+\theta+\frac{\theta^2}{2}$ as O(θ^3)<<1) (4.1.7)

Cost Components:

1. Setup Cost $=\frac{A}{T}$ (4.1.8)
2. Holding cost = $\frac{h}{T}\int_0^T I(t)dt = \frac{h}{T}[\int_0^{\mu} I(t)dt+\int_{\mu}^{T} I(t)dt]$

$$=\frac{h}{T}[\frac{D_0\mu^2}{2\theta}-\frac{D_0\mu T}{\theta}+\frac{D_0\mu}{\theta^2}e^{\theta(T-\mu)}-(Q-\frac{D_0}{\theta^2})\frac{1}{\theta}(e^{-\theta\mu}-1)]$$

$$=\frac{hD_0\mu}{T}(-\mu^2+\frac{\mu^3\theta}{4}+\frac{\mu^4\theta^2}{4}-\frac{\mu^2\theta T}{2}+\frac{T^2}{2}+\frac{\mu\theta T^2}{2}-\frac{\mu^2\theta^2T^2}{4}] \quad (4.1.9)$$

(by using $e^{\theta}=1+\theta+\frac{\theta^2}{2}$, as O(θ^3)<<1)

3. Deteriorating cost = $\frac{c\theta}{T}\int_0^T I(t)dt$

$$=\frac{D_0\mu c\theta}{T}[-\mu^2+\frac{\mu^3\theta}{4}+\frac{\mu^4\theta^2}{4}-\frac{\mu^2\theta T}{2}+\frac{T^2}{2}+\frac{\mu\theta T^2}{2}-\frac{\mu^2\theta^2T^2}{4}] \quad (4.1.10)$$

4. The Salvage value of deteriorating items = $\frac{\beta c\theta}{T}\int_0^T I(t)dt$

$$=\frac{D_0\mu\beta c\theta}{T}[-\mu^2+\frac{\mu^3\theta}{4}+\frac{\mu^4\theta^2}{4}-\frac{\mu^2\theta T}{2}+\frac{T^2}{2}+\frac{\mu\theta T^2}{2}-\frac{\mu^2\theta^2T^2}{4}] \quad (4.1.11)$$

Therefore the average total cost per unit time is given by

TC(T) = Setup cost + holding cost + deteriorating cost-Salvage Value

$$=\frac{A}{T}+\frac{h+(1-\beta)c\theta}{T}D_0\mu[-\mu^2+\frac{\mu^3\theta}{4}+\frac{\mu^4\theta^2}{4}-\frac{\mu^2\theta T}{2}+\frac{T^2}{2}+\frac{\mu\theta T^2}{2}-\frac{\mu^2\theta^2T^2}{4}] \quad (4.1.12)$$

For minimum, the necessary condition is $\frac{dTC}{dT}=0$

Or, $\frac{D_0\mu}{2}(1+\mu\theta-\frac{\mu^2\theta^2}{2})T^2+D_0\mu^3(1-\frac{\mu\theta}{4}-\frac{\mu^2\theta^2}{4})-\frac{A}{h+(1-\beta)c\theta}=0$ (4.1.13)

Which is the equation for optimum solution.

Let T^* be the positive real root of the above equation (4.1.13), then T^* is the optimum cycle time.

The optimum average total cost of TC(T) is TC(T^*).

Note: If there be no deterioration i.e. θ = 0, the equation (4.1.13) becomes

$$\frac{D_0\mu}{2}T^2+D_0\mu^3-\frac{A}{h}=0 \text{ or } T=\sqrt{(\frac{2A}{D_0\mu h}-2\mu^2)} \quad (4.1.14)$$

4.2 Fuzzy Model :

Consider

A = 200, $\tilde{h}$ = (h_1,h_2,h_3,h_4), $\tilde{c}$ = (c_1,c_2,c_3,c_4), $\tilde{D}_0$ = $(D_{01},D_{02},D_{03},D_{04})$

$\tilde{\mu}$ = $(\mu_1,\mu_2,\mu_3,\mu_4)$, $\tilde{\theta}$ = $(\theta_1,\theta_2,\theta_3,\theta_4)$, $\tilde{\beta}$ = $(\beta_1,\beta_2,\beta_3,\beta_4)$

are as trapezoidal fuzzy numbers.

The total cost of the system per unit time in fuzzy sense is given by the following

$$\tilde{TC}(T)=\frac{A}{T}+\frac{\tilde{h}+(1-\tilde{\beta})\tilde{c}\tilde{\theta}}{T}\tilde{D}_0\tilde{\mu}[-\tilde{\mu}^2+\frac{\tilde{\mu}^3\tilde{\theta}}{4}+\frac{\tilde{\mu}^4\tilde{\theta}^2}{4}-\frac{\tilde{\mu}^2\tilde{\theta}T}{2}+\frac{T^2}{2}+\frac{\tilde{\mu}\tilde{\theta}T^2}{2}-\frac{\tilde{\mu}^2\tilde{\theta}^2T^2}{4}] \quad (4.2.1)$$

We defuzzify the fuzzy total cost $\tilde{TC}(T)$ using the following two methods:

1) Graded Mean Integration Method (GM)
2) Signed Distance Method (SD).

4.2.1 *Graded Mean Integration Method (GM)* :

By Graded Mean Integration Method, the total cost is given by

$$\tilde{TC}_{GM}(T)=\frac{1}{6}[\tilde{TC}_{GM1}(T)+2\tilde{TC}_{GM2}(T)+2\tilde{TC}_{GM3}(T)+\tilde{TC}_{GM4}(T)] \quad (4.2.1.1)$$

where

$$\tilde{TC}_{GM1}(T)=\frac{A}{T}+\frac{h_1+(1-\beta_1)c_1\theta_1}{T}D_{01}\mu_1[-\mu_1^2+\frac{\mu_1^3\theta_1}{4}+\frac{\mu_1^4\theta_1^2}{4}-\frac{\mu_1^2\theta T}{2}+\frac{T^2}{2}+\frac{\mu_1\theta_1T^2}{2}-\frac{\mu_1^2\theta_1^2T^2}{4}]$$

$$\tilde{TC}_{GM2}(T)=\frac{A}{T}+\frac{h_2+(1-\beta_2)c_2\theta_2}{T}D_{02}\mu_2[-\mu_2^2+\frac{\mu_2^3\theta_2}{4}+\frac{\mu_2^4\theta_2^2}{4}-\frac{\mu_2^2\theta_2T}{2}+\frac{T^2}{2}+\frac{\mu_2\theta_2T^2}{2}-\frac{\mu_2^2\theta_2^2T^2}{4}]$$

$$\tilde{TC}_{GM3}(T)=\frac{A}{T}+\frac{h_3+(1-\beta_3)c_3\theta_3}{T}D_{03}\mu_3[-\mu_3^2+\frac{\mu_3^3\theta_3}{4}+\frac{\mu_3^4\theta_3^2}{4}-\frac{\mu_3^2\theta_3T}{2}+\frac{T^2}{2}+\frac{\mu_3\theta_3T^2}{2}-\frac{\mu_3^2\theta_3^2T^2}{4}]$$

And $\tilde{TC}_{GM4}(T)=\frac{A}{T}+\frac{h_4+(1-\beta_4)c_4\theta_4}{T}D_{04}\mu_4[-\mu_4^2+\frac{\mu_4^3\theta_4}{4}+\frac{\mu_4^4\theta_4^2}{4}-\frac{\mu_4^2\theta_4T}{2}+\frac{T^2}{2}+\frac{\mu_4\theta_4T^2}{2}-\frac{\mu_4^2\theta_4^2T^2}{4}]$

From the equation (4.2.1.1), we get

$$\tilde{TC}_{GM}(T)=\frac{A}{T}+\frac{1}{6}\frac{h_1+(1-\beta_1)c_1\theta_1}{T}D_{01}\mu_1[-\mu_1^2+\frac{\mu_1^3\theta_1}{4}+\frac{\mu_1^4\theta_1^2}{4}-\frac{\mu_1^2\theta T}{2}+\frac{T^2}{2}+\frac{\mu_1\theta_1T^2}{2}-\frac{\mu_1^2\theta_1^2T^2}{4}]$$
$$+\frac{1}{3}\frac{h_2+(1-\beta_2)c_2\theta_2}{T}D_{02}\mu_2[-\mu_2^2+\frac{\mu_2^3\theta_2}{4}+\frac{\mu_2^4\theta_2^2}{4}-\frac{\mu_2^2\theta_2T}{2}+\frac{T^2}{2}+\frac{\mu_2\theta_2T^2}{2}-\frac{\mu_2^2\theta_2^2T^2}{4}]$$
$$+\frac{1}{3}\frac{h_3+(1-\beta_3)c_3\theta_3}{T}D_{03}\mu_3[-\mu_3^2+\frac{\mu_3^3\theta_3}{4}+\frac{\mu_3^4\theta_3^2}{4}-\frac{\mu_3^2\theta_3T}{2}+\frac{T^2}{2}+\frac{\mu_3\theta_3T^2}{2}-\frac{\mu_3^2\theta_3^2T^2}{4}]$$
$$+\frac{1}{6}\frac{h_4+(1-\beta_4)c_4\theta_4}{T}D_{04}\mu_4[-\mu_4^2+\frac{\mu_4^3\theta_4}{4}+\frac{\mu_4^4\theta_4^2}{4}-\frac{\mu_4^2\theta_4T}{2}+\frac{T^2}{2}+\frac{\mu_4\theta_4T^2}{2}-\frac{\mu_4^2\theta_4^2T^2}{4}] \quad (4.2.1.2)$$

The necessary condition for the minimization of the average cost $\tilde{TC}_{GM}(T)$ is

$$\frac{d\tilde{TC}_{GM}(T)}{dT}=0$$

Or, $-6A+\{h_1+(1-\beta_1)c_1\theta_1\}D_{01}\mu_1\{\frac{T^2}{2}(1+\mu_1\theta_1-\frac{\mu_1^2\theta_1^2}{2})+\mu_1^2(1-\frac{\mu_1\theta_1}{4}-\frac{\mu_1^2\theta_1^2}{4})\}$

$$+2\{h_2+(1-\beta_2)c_2\theta_2\}D_{02}\mu_2\{\frac{T^2}{2}(1+\mu_2\theta_2-\frac{\mu_2^2\theta_2^2}{2})+\mu_2^2(1-\frac{\mu_2\theta_2}{4}-\frac{\mu_2^2\theta_2^2}{4})\}$$
$$+2\{h_3+(1-\beta_3)c_3\theta_3\}D_{03}\mu_3\{\frac{T^2}{2}(1+\mu_3\theta_3-\frac{\mu_3^2\theta_3^2}{2})+\mu_3^2(1-\frac{\mu_3\theta_3}{4}-\frac{\mu_3^2\theta_3^2}{4})\}$$
$$+\{h_4+(1-\beta_4)c_4\theta_4\}D_{04}\mu_4\{\frac{T^2}{2}(1+\mu_4\theta_4-\frac{\mu_4^2\theta_4^2}{2})+\mu_4^2(1-\frac{\mu_4\theta_4}{4}-\frac{\mu_4^2\theta_4^2}{4})\}=0 \quad (4.2.1.3)$$

which gives the optimum values of T.

$\tilde{TC}_{GM}(T)$ is minimum only if $\frac{d^2\tilde{TC}_{GM}(T)}{dT^2}>0$ would be satisfied for T>0.

The optimal total cost $\tilde{TC}^*_{GM}(T)$ is obtained by putting the optimal value T in the equation (4.2.1.2).

4.2.2 *Signed Distance Method (SD):*

By Signed Distance Method, the total cost is given by

$$\tilde{TC}_{SD}(T)=\frac{1}{4}[\tilde{TC}_{SD1}(T)+\tilde{TC}_{SD2}(T)+\tilde{TC}_{SD3}(T)+\tilde{TC}_{SD4}(T)] \quad (4.2.2.1)$$

$$\tilde{TC}_{GM1}(T)=\frac{A}{T}+\frac{h_1+(1-\beta_1)c_1\theta_1}{T}D_{01}\mu_1[-\mu_1^2+\frac{\mu_1^3\theta_1}{4}+\frac{\mu_1^4\theta_1^2}{4}-\frac{\mu_1^2\theta T}{2}+\frac{T^2}{2}+\frac{\mu_1\theta_1T^2}{2}-\frac{\mu_1^2\theta_1^2T^2}{4}]$$
$$\tilde{TC}_{GM2}(T)=\frac{A}{T}+\frac{h_2+(1-\beta_2)c_2\theta_2}{T}D_{02}\mu_2[-\mu_2^2+\frac{\mu_2^3\theta_2}{4}+\frac{\mu_2^4\theta_2^2}{4}-\frac{\mu_2^2\theta_2T}{2}+\frac{T^2}{2}+\frac{\mu_2\theta_2T^2}{2}-\frac{\mu_2^2\theta_2^2T^2}{4}]$$
$$\tilde{TC}_{GM3}(T)=\frac{A}{T}+\frac{h_3+(1-\beta_3)c_3\theta_3}{T}D_{03}\mu_3[-\mu_3^2+\frac{\mu_3^3\theta_3}{4}+\frac{\mu_3^4\theta_3^2}{4}-\frac{\mu_3^2\theta_3T}{2}+\frac{T^2}{2}+\frac{\mu_3\theta_3T^2}{2}-\frac{\mu_3^2\theta_3^2T^2}{4}]$$

And $\tilde{TC}_{GM4}(T)=\frac{A}{T}+\frac{h_4+(1-\beta_4)c_4\theta_4}{T}D_{04}\mu_4[-\mu_4^2+\frac{\mu_4^3\theta_4}{4}+\frac{\mu_4^4\theta_4^2}{4}-\frac{\mu_4^2\theta_4T}{2}+\frac{T^2}{2}+\frac{\mu_4\theta_4T^2}{2}-\frac{\mu_4^2\theta_4^2T^2}{4}]$

From the equation (4.2.2.1), we get

$$\tilde{TC}_{SD}(T) = \frac{A}{T} + \frac{1}{4}\frac{h_1+(1-\beta_1)c_1\theta_1}{T}D_{01}\mu_1[-\mu_1^2+\frac{\mu_1^3\theta_1}{4}+\frac{\mu_1^4\theta_1^2}{4}-\frac{\mu_1^2\theta T}{2}+\frac{T^2}{2}+\frac{\mu_1\theta_1T^2}{2}-\frac{\mu_1^2\theta_1^2T^2}{4}]$$
$$+\frac{1}{4}\frac{h_2+(1-\beta_2)c_2\theta_2}{T}D_{02}\mu_2[-\mu_2^2+\frac{\mu_2^3\theta_2}{4}+\frac{\mu_2^4\theta_2^2}{4}-\frac{\mu_2^2\theta_2T}{2}+\frac{T^2}{2}+\frac{\mu_2\theta_2T^2}{2}-\frac{\mu_2^2\theta_2^2T^2}{4}]$$
$$+\frac{1}{4}\frac{h_3+(1-\beta_3)c_3\theta_3}{T}D_{03}\mu_3[-\mu_3^2+\frac{\mu_3^3\theta_3}{4}+\frac{\mu_3^4\theta_3^2}{4}-\frac{\mu_3^2\theta_3T}{2}+\frac{T^2}{2}+\frac{\mu_3\theta_3T^2}{2}-\frac{\mu_3^2\theta_3^2T^2}{4}]$$
$$+\frac{1}{4}\frac{h_4+(1-\beta_4)c_4\theta_4}{T}D_{04}\mu_4[-\mu_4^2+\frac{\mu_4^3\theta_4}{4}+\frac{\mu_4^4\theta_4^2}{4}-\frac{\mu_4^2\theta_4T}{2}+\frac{T^2}{2}+\frac{\mu_4\theta_4T^2}{2}-\frac{\mu_4^2\theta_4^2T^2}{4}] \quad (4.2.2.2)$$

The necessary condition for the minimization of the average cost $\tilde{TC}_{SD}(T)$ is

$$\frac{d\tilde{TC}_{SD}(T)}{dT}=0$$

Or, $$-4A + \{h_1+(1-\beta_1)c_1\theta_1\}D_{01}\mu_1\{\frac{T^2}{2}(1+\mu_1\theta_1-\frac{\mu_1^2\theta_1^2}{2})+\mu_1^2(1-\frac{\mu_1\theta_1}{4}-\frac{\mu_1^2\theta_1^2}{4})\}$$
$$+\{h_2+(1-\beta_2)c_2\theta_2\}D_{02}\mu_2\{\frac{T^2}{2}(1+\mu_2\theta_2-\frac{\mu_2^2\theta_2^2}{2})+\mu_2^2(1-\frac{\mu_2\theta_2}{4}-\frac{\mu_2^2\theta_2^2}{4})\}$$
$$+\{h_3+(1-\beta_3)c_3\theta_3\}D_{03}\mu_3\{\frac{T^2}{2}(1+\mu_3\theta_3-\frac{\mu_3^2\theta_3^2}{2})+\mu_3^2(1-\frac{\mu_3\theta_3}{4}-\frac{\mu_3^2\theta_3^2}{4})\}$$
$$+\{h_4+(1-\beta_4)c_4\theta_4\}D_{04}\mu_4\{\frac{T^2}{2}(1+\mu_4\theta_4-\frac{\mu_4^2\theta_4^2}{2})+\mu_4^2(1-\frac{\mu_4\theta_4}{4}-\frac{\mu_4^2\theta_4^2}{4})\} = 0 \quad (4.2.2.3)$$

which gives the optimum values of T.

$\tilde{TC}_{GM}(T)$ is minimum only if $\frac{d^2\tilde{TC}_{SD}(T)}{dT^2}$ >0 would be satisfied for T>0.

The optimal total cost $\tilde{TC}^*_{SD}(T)$ is obtained by putting the optimal value T in the equation (4.2.2.2).

5. Numerical example:

The following examples are considered to illustrate the preceding two inventory models namely crisp model and fuzzy model.

Example 1: (Crisp Model):

The values of the parameters be as follows
A=200 per order; h = \$ 15 per unit; c = \$ 20 per unit; D_0 = 150 units; μ = 0.25 year; θ = 0.01; β = 0.2.

Solving the equation (4.1.13) with the help of computer using the above parameter values, we find the following optimum outputs

T^* = 0.759 year; Q^* = 14.53 units and TC^* = \$ 432.83

It is also checked that this solution satisfies the sufficient condition for optimality.

Example 2: (Fuzzy Model):

Consider the fuzzy parameters are

A=200 per order; $\tilde{h} = (6, 10, 14, 18)$; $\tilde{c} = (10, 15, 20, 25)$; $\tilde{D}_0 = (80, 100, 120, 150)$;

$\tilde{\mu} = (0.23, 0.25, 0.27, 0.30)$; $\tilde{\theta} = (0.01, 0.03, 0.05, 0.07)$; $\tilde{\beta} = (0.1, 0.2, 0.3, 0.5)$.

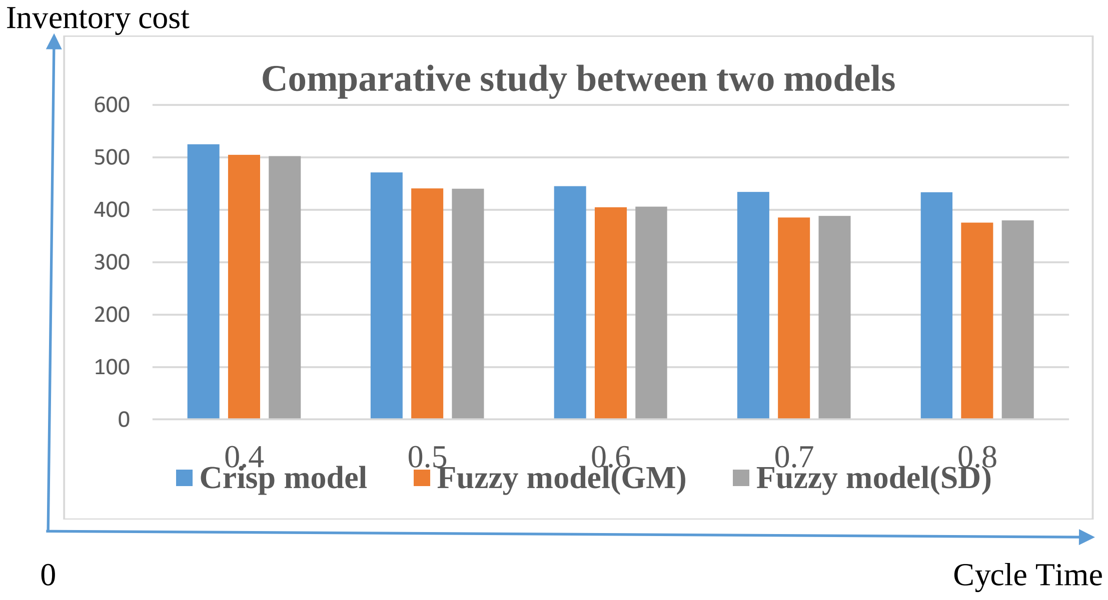
Solving the equations (4.2.1.3) and (4.2.2.3) with the help of computer using the above values of fuzzy parameters, we find the following optimum outputs

Fuzzy Model	T^*	$\tilde{TC}^*$
Graded Mean Integration Method	0.519 year	\$ 432.29
Signed Distance Method	0.609 year	\$ 404.15

Sensitivity analysis and Pictorial Presentation.

The table indicates the comparative study between two types of inventory models under the optimal average costs and cycle time. Also a pictorial presentation is furnished on the basis of following data. The results of this analysis are shown in the following table.

Cycle Time(T^*) (year)	Optimum value of TC(T^*) for (\$)		
	Crisp Model	Fuzzy Model (***Graded Mean Integration Method)***	Fuzzy Model (***Signed Distance Method)***
0.4	525.03	504.95	502.24
0.5	471.28	440.79	440.04
0.6	444.95	404.87	406.30
0.7	434.28	385.09	388.12
0.8	433.40	375.39	379.87



Analyzing the results given in the above table and pictorial presentation, the following observations are made:

(i) The total inventory optimum costs(TC(T^*)) decrease with the increase in the values of the optimum cycle time(T^*) for both crisp model and fuzzy model.

(ii) It is also seen that the minimum inventory cost attains mainly for the fuzzy model under Graded Mean Integration Method in compare to others.

Concluding remarks:

In this study, we have carried out two types of inventory models for deteriorating items with ramp type demand and salvage in nature. The demand rate, deterioration rate, holding cost, unit cost and salvage value are considered as trapezoidal fuzzy numbers. The total inventory costs for both crisp model and fuzzy model are derived. Both graded mean integration method and signed distance method are used to defuzzify the total cost function. Shortages are not allowed in the proposed paper. Further, numerical examples are given to develop crisp model and fuzzy model. A structural comparative study is demonstrated here by illustrating the model with sensitivity analysis. Further this model can be developed assuming Weibull distributed deterioration and cubic demand rate with fully or partially backlogged shortages.

References:

(1) Bellman R. and Zadeh L.A. (1970) : "Decision making in a fuzzy environment", Mgmt Sci., 17, pp. 141-164.

(2) Kao C. and Hsu W. K. (2002) : "Lot size-reorder point inventory model with fuzzy demands ", Computers and Mathematics with Applications., 43, pp. 1291-1302.

(3) Chou C. C.(2009) : "Fuzzy economic order quantity inventory model", Int. J. of Innovative Computing , Information and Control, 5(9), pp. 2585-2592.

(4) Guiffrida A. L. (2010) : "Fuzzy inventory models ", Non classical views, Ch-8, M. Y. Jaber(Ed.), CRC Press, FL, Boca Raton, pp. 173-190.

(5) Garg H. (2017) : "Some picture fuzzy aggregation operators and their applications to multicriteria decision-making ", Arab. J. Sci. Eng., pp. 1-16.

(6) Baker R. C. and Urban T. L. (1988) : "A deterministic inventor system with an inventory level dependent demand rate", J. Oper. Res. Soc., 17, pp. 823-831.

(7) Alfares H. K. (2007) : "Inventory model with stock dependent demand rate and variable holding cost ", Int. J. Prod. Eco., 108, pp. 259-265.

(8) Tipathi R. P. and Singh D. (2015) : "Inventory model with stock dependent demand and different holding cost functions ", Int. J. Indus. & Syst. Eng., 21(1), pp. 14-19.

(9) Mandal B. and Pal A.K. (1988) : "Order Level inventory system with ramp type demand rate", J. International Mathematics, 1(1), pp. 49-66.

(10) Biswaranjan Mandal (2010) : "An EOQ inventory model for Weibull distributed deteriorated items under ramp type demand and shortages", OPSEARCH, Indian J. of Operations Research, 47 (2), pp.158-165.

(11) Biswaranjan Mandal (2020) : "A Weibull Distributed Ameliorating Inventory Model with Ramp Type Demand and Shortages", IISRR- International Journal of Research, 6(III), ISSN 2394-885X, 31, pp. 69-75.

(12) .Zadeh (1965) : " Fuzzy sets", Information and Control, 8(3), pp. 338-353.

(13). Vujosevic M. and Petrovic D. (1996) : " EOQ formula when inventory cost is fuzzy", Int. J. Prod. Eco., 45, pp. 499-504.

(14). Hsieh C. H. (2002) : " Optimization of fuzzy production inventory models", Information Sciences., 146, pp. 29-40

(15). Mishra N. and Soni J. K.(2012) : " An inventory model with fuzzy deterioration rate and fine production rate", IOSR J. of Math (IOSR-JM), 4(4), pp. 1-9.

(16). Kumar S. and Rajpu U. S.(2015) : " Fuzzy inventory model for deteriorating items with time dependent demand and partial backlogging ", Appl Math., 6, pp. 496-509.

(17). Uttayakumar R. and Karuppasamy S. K.(2016) : " A fuzzy inventory model with lot size dependent ordering cost in healthcare industries", Oper. Res. & Appl. : An Int. J. (ORAJ), 3(1), pp. 17-23.

(18). Yadav V, Chaturvedi B. K. and Malik A. K. (2022) : " Development of fuzzy inventory mode under decreasing demand and increasing deterioration rate", Int. J. on Future Revolution in Comp. Sc & Communication Eng., 8(4), pp. 1-7.
