Optimization by Morphological Filters for multi-criteria shortest path problems in multi-modal transport network

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Abstract

The Multi-Criteria Shortest Path (MCSP) problem gets more challenging to solve due to numerous contradictory criteria and multimodality in transport networks which complicates the problem inducing difficulties in decision-making. Several studies in the literature show that metaheuristics are often used to solve this sort of problem. In this paper, we propose a new approach based on the Optimization by Morphological Filters (OMF) algorithm applied for the first time to solve the multi-criteria shortest path problem in a multimodal transport network. To ameliorate and adapt the algorithm to the MCSP problem, two neighborhood calculation systems are proposed: Neighborhood in depth and Neighborhood in width. To test the approach, we propose a model of the multimodal transport road network of a real geographical area, by using a multivalued graph with three means of transportation. The comparison demonstrates that our technique outperforms several other proposed techniques including Non-dominated Sorting Genetic Algorithm (NSGA-II), with a six-fold improvement in processor execution time.

Keywords: *Multimodal transport network, Multi-criteria optimization, Shortest path, Optimization by Morphological Filters, Graphs modelization.*

1. Introduction

As traffic grows and road designs become more complex, managing and optimizing road networks becomes increasingly difficult. One of the most challenging problems is the Multi-Criteria Shortest Path (MCSP) problem, especially when optimizing contradictory criteria like time and cost or distance and safety. The problem becomes even more challenging in multimodal transport networks, where several means of transportation are involved. The MCSP is a known NP-hard [1] problem, and researchers often use metaheuristics to provide approximate solutions. However, most research has been done on small multimodal networks, and the multimodal aspect of real networks has often been ignored, as the case of Hedi et al [2] and Boussedjra et al [3]. While Sauvanet and Néron [4] tested methods on real networks, they too ignored the multimodal aspect.

We propose a new model for solving the MCSP problem in multimodal transport road networks by considering distance, duration, cost, walking time and quality of the path as optimization criteria. The model is tested on the city of Oran in Algeria, which includes tram, buse, and taxis as modes of transportation. The proposed approach adapts a new metaheuristic called Optimization by Morphological Filters (OMF), we include two neighborhood calculation systems. The objective is to achieve a more thorough exploration of the search space and a faster search for the global optimum.

2. LITERATURE REVIEW

The review of the literature reveals a number of factors that are taken into consideration for modeling and solving the MCSP problem in multimodal transport networks.

2.1. Single mean versus multi means of transportation

Several studies have addressed the MCSP problem in transportation networks using various modeling and solving methods. Sauvanet and Néron [4] and Ravalet and Bussière [5] have solved the single mean MCSP issue using bicycles, while Hadj Khalifa et al [6] have used a person guidance system for foot travel. Ayed et al [7] address multimodal transport issues including rail, bus, and air. Boussedjra et al [3] try to solve Intermodal Shortest Path using three modes of transportation. Guiwu and Dong [8] addressed multi-objective optimization for multimodal transportation with roads, railways, and waterways, and Dib et al [9] focused on Railway, bus, tram, and pedestrian transportation. Zhou et al [10] proposed a multimodal transport network model including cars, buses, trains, bicycles, and pedestrians, and Zhang et al [11] studied a multimodal discrete network for optimizing car, bus, rail transit, and slow traffic modes such as walking and bike-sharing.

2.2. Graph modeling

Two different models for the multimodal MCSP problem have been proposed. The first involves modeling the graph in multi-layers, which complicates the modeling but allows for standard algorithm resolution. The second model deals with a single multivalued graph, making implementation more challenging. Boussedjra et al [3] and Dib et al [9] modeled each mode of transportation as a separate layer in a multimodal graph, while Zhang et al [11] have developed a model based on two types of transport networks. Dotoli et al [12] proposed a decomposition of the graph using multi-agent systems. A transfer graph structure has been used by Ayed et al in [7], while Kamel and Slim [13] used a simple multivalued dynamic graph model that was later extended with additional parameters.

2.3. Multi-criteria modeling

Multi-criteria problems are NP-hard [1] and require well-adapted modeling and powerful resolution approaches. In contrast to single-criteria problems that can be easily solved by conventional algorithms like Dijkstra [14] or Bellman [15]. Some researchers use criteria aggregation to transform the problem into a single criterion problem. This strategy has been followed by Boussedjra et al [3], Zhou et al [10], Kamel and Slim [13], Beirigo and Dos Santos [16], Sur et al [17] and Mouilah and Belkadi [18]. While others attempt to resolve it through appropriate techniques. Savanet and Néron [4] proposed the BCA* algorithm, which uses the concept of Pareto front and the Tchebysheff metric [19] to compute non-dominated best compromise solutions. Sedeño-Noda and Raith [20] proposed a new Dijkstra-like method to solve the bi-objective case. Dib et al [9] developed a multi-criteria routing algorithm to solve an itinerary planning problem. Camargo-Perez et al [21] implemented the Analytic Hierarchy Technique to solve the passenger transfer node location problem for a public transportation system.

2.4. Exact methods versus metaheuristics

Exact approaches are not commonly used to solve the MCSP problems due to implementation difficulties for large graphs. However, some researchers have proposed extensions to exact methods, such as Best Compromise A* (BCA*) used by Sauvanet and Néron [4] and Multi-Objective A* (MOA*) used by Stewart and White [22]. Most researchers prefer using metaheuristics to solve the MCSP problem, such as genetic algorithms used by Kamel and Slim [13] and Wang et al [23], ant colony algorithms used by Mouilah and Belkadi [18], Non-dominated Sorting Genetic Algorithm (NSGA-II) [24] implemented by Beirigo and Dos Santos [16] and Chitra and Potti [25], simulated annealing used by Mu et al [26], and an adaptive and discrete real Bat algorithms was proposed by Sur and Shukla [27]. These techniques have been applied to solve various types of MCSP problems, including multi-criteria path optimization in a multimodal transport network, bi-objective travel planning problem, and shortest path problem with traffic restriction in a road network.

In this paper, we investigate the MCSP problem in a multimodal transport network. The proposed model is built as a multivalued graph in which each edge holds data relating on the network and the means of transportation. Three means are considered: taxi, tram and Bus. For the resolution of the MCSP, an extension and an adaptation of a new metaheuristic known as Optimization by Morphological Filters (OMF) is given to determine a set of non-dominated solutions that represent the Pareto-optimal.

3. FORMULATION AND MODELING

In this paper the MCSP problem is studied through the use of a single multivalued graph model: $G = (A, B, X, M)$. A is the set of nodes, B is the set of edges, X is the set of criteria and M is the set of means of transportation, where there are three means of transportation (taxi, tram, and bus) to provide the user with a variety of options for getting around. Figure 1 show an example of modelization. The path P from start node 4 to destination node 15 is presented as follow: (4-8-12-11-10-9-14-15). Criterion of P are: D_P=21, C_P=190, T_P=28, W_P=11 and Q_P=74.

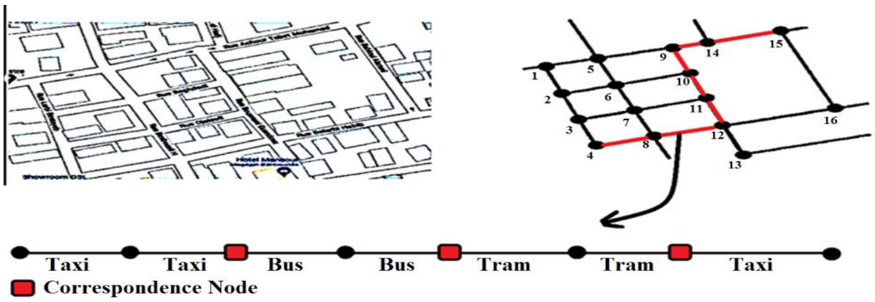

Figure 1. Modeling transport network by a graph

The considered criteria are: D, C, T, W which are to be minimized and Q to be maximized:

- **Distance (*D*):** refers to the length of the path between start and destination nodes.
- **Cost (*C*):** indicates the global monetary costs related to the journey. That includes the cost of all means of transportation used during the travel.
- **Time (*T*):** reflects the journey time. It depends on the parameters associated with edges (speed limit, path type, built-up area, presence of congestion points, road surface condition, etc.) and on the walk time.
- **Walk time (*W*):** This criterion is affected by the number of times a mode is used as well as the waiting time associated with each mean of transportation. Walking time relative to each correspondence node is another parameter affecting this criterion.
- **Quality (*Q*):** This criterion is related to the road sections (presence of obstacles, condition of the road surface, type of road, etc.) and the comfort linked to the used means of transportation.

For each edge connecting the nodes i and j, using mean k (for taxi $k = 0$, for tram $k = 1$ and for Bus $k = 2$), Table 1 presents the model's parameters estimated.

Table 1. The Model's parameters

Notation	Description	Concerned Criterion
$\boldsymbol{d_{ij}}$	The distance	*D*
$\boldsymbol{c_{ij}^k}$	Cost using mean k	*C*
$\boldsymbol{p^k}$	Pricing of taken mean k	
$\boldsymbol{f^k}$	Monetary travel cost of mean k (only for Taxi)	

$\boldsymbol{t_{ij}^k}$	Travel time using mean k	*T*
$\boldsymbol{\pi_{ij}}$	Congestion penalty, it affects the speed s_{ij}	
$\boldsymbol{s_{ij}}$	Limited speed	
$\Delta\boldsymbol{t_{ij}^k}$	Delay time using mean k	
$\boldsymbol{n_{ij}^k}$	Number of stops of mean k	
$\boldsymbol{a^k}$	Average time to stop of mean k	
$\boldsymbol{w^k}$	The duration of each mean's average wait	*W*
$\Delta\boldsymbol{w_x}$	The average walking delay relative to each correspondence node *x*	
$\boldsymbol{q_{ij}^k}$	Overall quality using mean k	*Q*
$\boldsymbol{r_{ij}}$	Road condition	
$\boldsymbol{m^k}$	Comfort linked to mean k	

For each path P connecting starting node S and destination node E, the related parameters are presented in Table 2.

Table 2. The path's parameters

Notation	Description
$\boldsymbol{e_P}$	Number of edges on *P*
$\boldsymbol{N_P^k}$	Number of times that mean k is taken on *P*
$\boldsymbol{CN_P}$	Set of correspondence nodes on path *P*
$\boldsymbol{I_P}$	The specified criterion *I* of the path *P*

The following formulas are used to calculate each criterion.

- **Distance**

$$D_P = \sum_{[i,j]\in P} d_{ij} \quad (1)$$

- **Cost**

$$C_P = \sum_{k=0}^{|M|} N_P^k \times p^k + \sum_{[i,j]\in p} c_{ij}^k \quad (2)$$

$$c_{ij}^k = d_{ij} \times f^k \quad (3)$$

The monetary cost is proportional to the overall travel cost and number of times that a mean is taken that given by (2). The overall travel cost is affected by the distance and the travel cost of each mean of transportation as indicate formula (3). If the mean of transportation is tram (*k=2*) or bus (*k=3*), only the paid is taken into consideration. For this we have: $f^2 = f^3 = 0$. The cost of travel depends on the number of times a mean is taken on *P* (N_P^k) and overall travel cost (c_{ij}^k), which includes distance (d_{ij}) and travel costs (f^k). For Tram or bus only trip's ticket is considered.

- **Time**

$$T_P = \sum_{[i,j]\in P} t_{ij}^k + W_P \quad (4)$$

$$t_{ij}^k = \frac{d_{ij}}{\pi_{ij}\times s_{ij}} + \Delta t_{ij}^k \quad (5)$$

$$\Delta t_{ij}^k = n_{ij}^k \times a^k \quad (6)$$

The time taken for travel depends on the total travel time (t_{ij}^k) and walking time (W_P) related to changing transport is given by (4). The travel time (t_{ij}^k), wich affected by several parameters such as the speed (s_{ij}) and the delay time using transportation means (Δt_{ij}^k) is given by formula (5). The speed (s_{ij}) can be reduced by the Congestion penalty (π_{ij}). The delay time using transportation means (Δt_{ij}^k) is affected by number of stops (n_{ij}^k) and average stop time (a^k) as indicate in (6).

- **Walk**

$$W_P = \sum_{k=0}^{|M|} N_P^k \times att^k + \sum_{x\in CN_P} \Delta w_x \quad (7)$$

Three parameters are considered to calculate the overall walking time (W_P) given by (7). The first is the average walking delay (Δw_x) linked to each correspondence's node. The second is the average waiting time (att^k) of the mean to be used at each correspondence. The third is the number of times that a trnportation mean is taken on P (N_P^k).

- **Quality**

$$Q_P = \frac{\sum_{[ij]\in P} q_{ij}^k}{e_p} \quad (8)$$

$$q_{ij}^k = \frac{m^k + r_{ij}}{2} \quad (9)$$

The quality of the path (Q_P) presents the average of all the overall qualities of each edge on P as given by (8). Overall quality (q_{ij}^k) is affected by the comfort of each mean of transportation (m^k) and the road condition (r_{ij}) as presented in formula (9).

4. PROPOSED APPROACH

The resolution strategy, based on mathematical morphology [28] using morphological filters [29], seeks global optimization in multidimensional space. OMF was proposed by Khelifa and Belmadani [30] and later used by Zaoui and Belmadani to solve engineering optimization problems [31] and combined economic and emission dispatch problems of power systems without penalty [32]. OMF identifies the smallest combination of pixel values using erosion process and structural elements to achieve optimal solutions [30].

4.1. General OMF

The OMF algorithm positions a set number of filters (NF) in a search space and calculates neighbors for each filter (NN). If a neighbor improves the useful function, it becomes the new filter; otherwise, the current filter's size (FS) is reduced, and the neighbors are recalculated. The process is repeated until the sum of filter sizes is less than a value ε close to zero.

4.2. Proposed adaptation of OMF to the problem of the MCSP (MCSP-OMF)

The adaptation of OMF to the problem of the MCSP is described in this section. We propose number of iteration as the stopping criterion. Two neighborhood calculation systems are proposed: *Neighborhood in depth* and *Neighborhood in width.* The objective of our proposed MCSP-OMF is to find a set of Non-Dominated solutions that represent the Pareto front. The MCSP-OMF's parameters are:

- ***NF*** **(Number of filters)**: each filter represents a path.
- ***NN*** **(Number of neighbors)**: neighbors are determined using a neighborhood calculation system on each filter. They represent paths that have the same starting and finishing nodes as the filter.
- ***IT*** **(Number of iterations)**: the stopping criterion.
- ***Neighborhood in depth***: the path represented by the filter is kept and the means of transportation on the edges are redefined to calculate the neighbors.
- ***Neighborhood in width***: new random paths are calculated with the same starting and finishing nodes.

A probability R is defined to choose which neighborhood calculation system to use. It favors either the width (Algorithm 1) or depth (Algorithm 2) of the neighborhood. Algorithm 1 is also used to generate new filters.

Algorithm 1. Random path generation (neighborhood in width)

input Graph G, starting node S and ending node E
initialize : path $P = \{S\}$, *visited* $= \{S\}$
repeat
    initialize *neighbor* $\leftarrow \emptyset$, $N_{curr} \leftarrow tail(P)$
    for *each neighbor* V_i *of* N_{curr} do
        if $V_i \notin visited$ then
            add V_i to *neighbor*
        end if
    end for
    if *neighbor* $\neq \emptyset$ then
        $V \leftarrow$ random element from *neighbor*
        add V to P
        add V to *visited*
    else
        delete N_{curr} from P
    end if
until $E \in P$
return P

Algorithm 2. Changing means of Transportation (neighborhood in depth)

```
input path P
for each edge e on P do
    if 2 and 3 are available on e then
        k ← random value in {1, 2, 3}
    else
        if 2 is available on e then
            k ← random value in {1, 2}
        else
            if 3 is available on e then
                k ← random value in {1, 3}
            end if
        end if
    end if
end for
return P
```

The adapted MCSP-OMF algorithm is given as follows:

Algorithm 3. MCSP-OMF

```
input Graph G, start node S, end node E
initialize NF, NN, R, IT, ND= ∅ // Set of Non-Dominated solutions
for i = 0 to NF do
    Fi ← random path using Algorithm 1 (G,S,E)
    Add Fi to ND
end for
repeat
    for i = 0 to NF do
        for j = 0 to NN do
            if R< random value in [0,1] then
            calculate neighbor Vij using Algorithm 1(G,S,E) // in width
            else
            calculate neighbor Vij using Algorithm 2(Fi) // in depth
            end if
            if (Vij Dominate Fi) then
                Fi ← Vij
                Remove Fi from ND
            end if
            if Vij ∉ ND then
                Add Vij to ND
            end if
```

22: end for
23: end for
24: *IT* ← *IT - 1*
25: until *IT = 0*
26: return *ND*

As indicated before X is the set of criteria, with $X = \{D, C, T, W, Q\}$. The dominance relationship is used to compare two solutions *P1* and *P2*. We say that *P1* dominate *P2* if formula (10) is verified:

$$\forall I \in X \begin{cases} I_{P2} \geq I_{P1} \; if I \neq Q \\ I_{P2} \leq I_{P1} \; if I = Q \end{cases} \exists J \in X \begin{cases} J_{P2} > J_{P1} \; if J \neq Q \\ J_{P2} < J_{P1} \; if J = Q \end{cases} \quad (10)$$

Figures 2 to 5 illustrate an example of the optimization process. In Figure 2, the initial graph is presented, comprising 9 nodes, 12 arcs, the departure node S, and the destination node E. In Figure 3, the calculation of the filters and their neighbors is shown. The next step involves testing the dominance between the filter and its neighbors. If a neighbor dominates the filter, it becomes the new filter, as demonstrated in Figure 4. Finally, the resulting non-dominated paths, which represent the Pareto front, are shown in Figure 5.

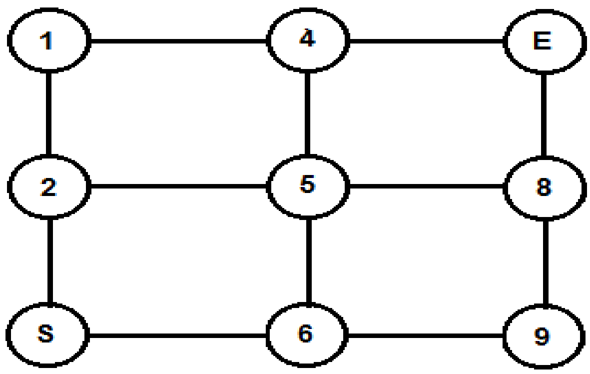

Figure 2. Initial graph

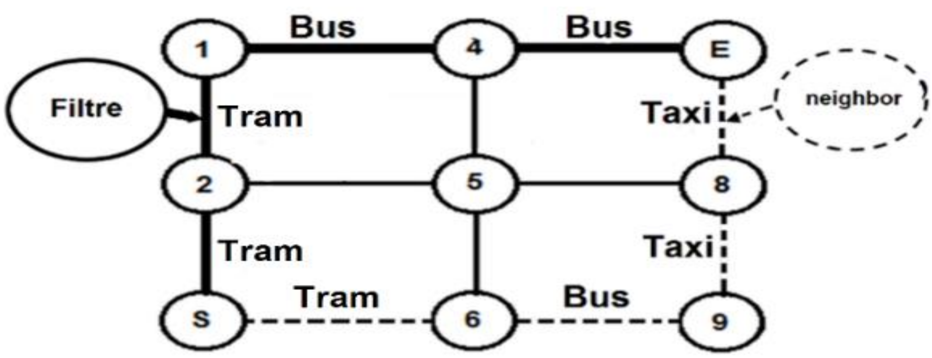

Figure 3. Calculations of filter and neighbor

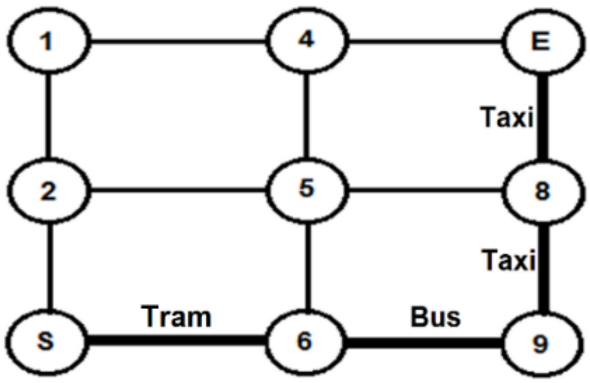

Figure 4. The movement of the filter

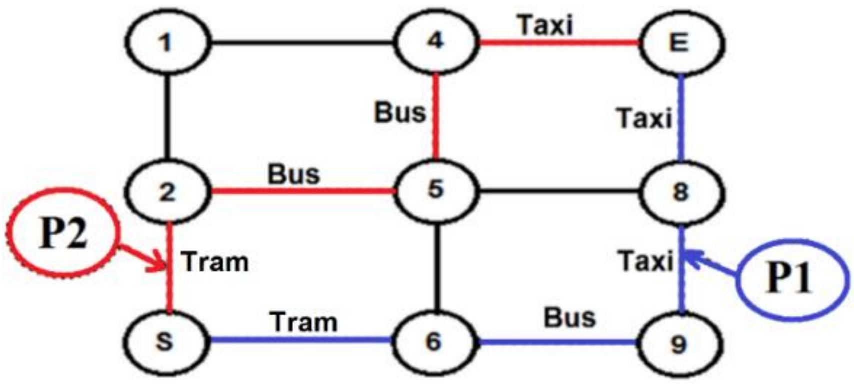

Figure 5. Non-Dominated solutions

Using a network of 9 nodes as an example, presented by the graph in Figure 2. The objective is to find the non-dominated paths between the departure node S and the arrival node E that build the Pareto front. We put P = [D, C, T, W, Q] to indicate the values of the criteria associated to the path P. For exemple if we have : Filtre(S-2-1-4-E) = [24, 242, 31, 12, 75] and Neighbor(S-6-9-8-E) = [19, 215, 30, 10, 78] calculated in Figure 3, according to formula (10) Neighbor dominate Filter, so Filter is mouved to Neighbor as shown in Figure 4. When the stopping criterion is validated after a specific number of iterations, the algorithm returns the set of Non-Dominated solutions, for examples we obtain two solutions, as shown in Figure 5, {P1(S-6-9-8-E) = [19, 215, 30, 10, 78], P2(S-2-5-4-E) = [18, 225, 26, 9, 72]}.

5. SIMULATION AND RESULTS

The MCSP-OMF algorithm is compared with NSGA-II [24]. Figure 3 shows the geographical studied area that represents a section of the transport network of Oran, Algeria as shown in. OpenStreetMap and ArcGIS are used to create a geographical database and collect road network data. The study aimed to optimize multi-criteria transportation problems. Graph Stream, a Java library for dynamic graph analysis, was used to build a network model based on the data collected. The study aimed to export the cartography of the studied area and perform multi-criteria optimization to improve transportation.

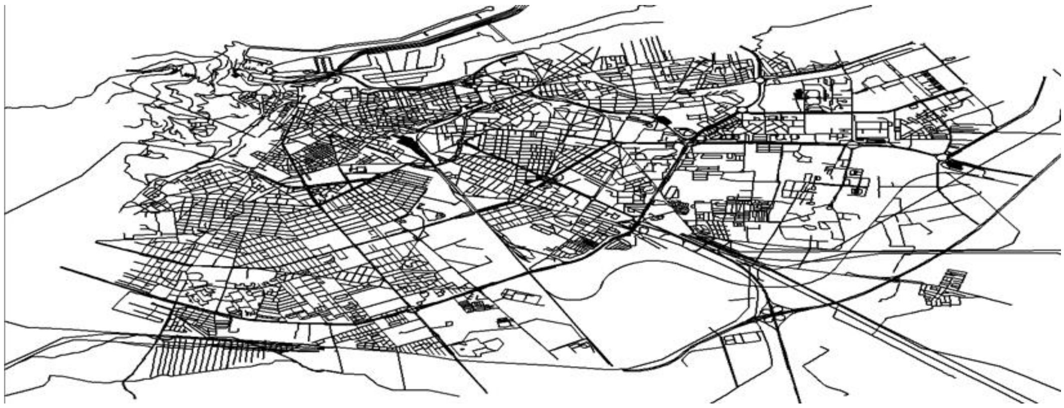

Figure 6. Part of the geographical studied area (Oran, Algeria)

The created graph has 2600 nodes and 10100 edges. The MCSP-OMF algorithm's parameters include NF, NN, IT, and probability R fixed at 0.7, while NSGA-II's parameters include population size, number of generations, mutation rate, and crossover rate, fixed at 0.1 and 0.9 respectively. The study aimed to compare the quality of the optimal solution found and the processor execution time (*CPU*) between the two algorithms, with a focus on non-dominated solutions. The distance between a solution produced on the Pareto front and the ideal point was measured. The ideal point is serving as a useful reference for the optimal solution's distance as indicated in Figure 4, it is determinated by formula (11). *ND* is the set of non-dominated solutions.

$$I_{IP} = \begin{Bmatrix} min(I_P)\ if\ I \neq Q \\ max(I_P)\ if\ I = Q \end{Bmatrix} P \in ND, I \in X \tag{11}$$

The best solution is the one with the smallest distance *Dist(IP, P)* between a solution P and the Ideal Point *Dist(IP, P)* that is given by the formula (12).

$$Dist(IP, P) = \sqrt{\sum_{I \in X}(I_{IP} - I_P)^2} \tag{12}$$

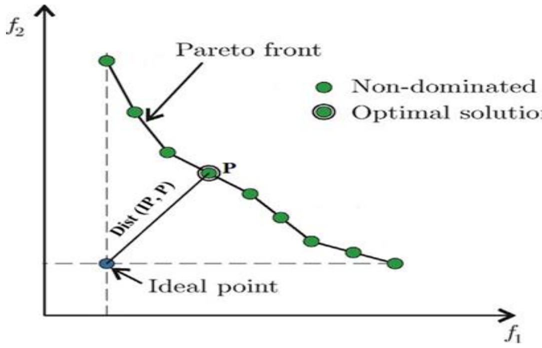

Figure 6. Determination of the optimal solution on a Pareto front

The tests are performed to identify an optimal path between two nodes of the network. The ideal point is identified following a series of tests as follows: *IP* = [15, 20, 17, 1, 83]. The results of each test are provided after dozens of evaluations for each algorithm.

Table 3. MCSP-OMF's test results

Parameters			*NND*	*Optimal Solution O*					*Dist(IP,O)*	*CPU (s)*
NF	*NN*	*IT*		*D*	*C*	*T*	*W*	*Q*		
1	2	5	24	26	82	39	4	54	72,79	0,4
2	3	5	69	32	56	46	2	56	56,18	1,1
2	5	7	138	26	44	39	2	55	44,34	2,4
3	5	10	367	22	41	29	2	60	34,12	6,1
4	5	15	791	16	40	17	1	79	20,42	16,2

Table 4. NSGA-II's test results

Parameters		*NND*	*Optimal Solution O*					*Dist(IP,O)*	*CPU (s)*
PS	*NG*		*D*	*C*	*T*	*W*	*Q*		
3	2	33	30	88	39	3	57	77,54	2,1
5	3	83	23	77	29	3	67	60,97	7,6
10	5	226	25	45	34	2	55	42,41	15,9
15	7	496	24	45	24	2	64	33,42	37,3
20	10	886	17	32	27	2	70	20,45	93,8

Tables 3 and 4 show that MCSP-OMF provides better results in terms of CPU time. Even while NSGA-II provides a number of non-dominated solutions slightly greater than MCSP-OMF, both of them provide an optimal solution with nearly equal quality. The MCSP-OMF is based on the hybridization of local search which is ensured by the neighborhood in depth and a global one which is ensured by the neighborhood in width. This approach offers practically identical efficiency to that of NSGA-II with a significant time saving. Moreover, Table 4 show the influence of the parameters on the behavior of MCSP-OMF. It is clear that increasing NF, NN or IT brings us closer to the optimal solution but at the expense of significant execution processor time.

8. CONCLUSION

In this paper, a new model for the Multi-Modal Transport Network is proposed with the Multi-Criteria Shortest Path as an investigated topic. It should be emphasized that metaheuristics are used to solve this type of problem, since there are no other exact methods for doing so. The suggested algorithm adapts Optimization by Morphological Filters through with the aspect of the neighborhood in width and the neighborhood in depth. Therefore, the execution time to reach the optimum solution and exploring the search space has been improved. The performance of the proposed approach is investigated by comparing its results with those reported in a research work and according to the preliminary experiments, we confirmed that the proposed approach offers similar results to NSGA-II with an important gain of CPU time.

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