# Extended result on Dominator chromatic number of central graph of some individual graphs

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## *Abstract*

*Let G= (A, B) be a simple, connected, undirected and finite graph. In this paper, we obtain the dominator chromatic number of central graph of Dyck graph, Errera graph and wheel graph. A graph G has a dominator coloring if it has a proper coloring in which each vertex of the graph dominates every vertex of some color class. The minimum number of color classes needed for the dominator coloring of a graph G is the dominator chromatic number and is denoted by* $\chi_d(G)$.

**Keywords:** dominator chromatic number, dominator coloring, central graph.

**Mathematics Subject Classifications:05C69**

# 1.Introduction

This concept was introduced by Ralucca Michelle Gera in 2006. The notion of central graph and dominator coloring are reviewed in the following section. Let G be a graph such that A is the vertex set and B is the edge set. A dominating set V is a subset of the vertex set A of graph G such that every vertex in the graph either belongs to V or adjacent to V. Dominating sets are useful in routing problems, scheduling problems, assignment problems, computer communication networks, land surveying, coding theory etc.

A Central Graph C(G) is obtained by subdividing each edge of G exactly once and joining all the non-adjacent vertices of G. Let G be a simple and undirected graph and let its vertex set and edge set denoted by A(G) and B(G). The central graph of G, denoted by C(G) is obtained by subdividing each edge of G exactly one and joining all the non-adjacent vertices of G in C(G). **A** proper coloring of a graph G is an assignment of colors to the vertices of G in such a way that no two adjacent vertices receive the same color. The chromatic number $\chi(G)$, is the minimum number of colors required for a proper coloring of G. A color class is the set of all vertices, having the same color. The color class corresponding to the color i is denoted by $A_i$. A **dominator coloring** of a graph G is a proper coloring in which every vertex of G dominates every vertex of at least one color class. The convention is that if {a} is a color class, then a dominates the color class {a}. The **dominator chromatic number** $\chi_d(G)$ is the minimum number of colors required for a dominator coloring of G. [3] A subset D of the set of vertices A of a graph G is a dominating set G if every vertex A-D is adjacent to at least one vertex in D. [3] A dominator coloring of a graph G is a proper coloring in which every vertex of G dominates every vertex of at least one color class.

## 2. Preliminary results:

### Result 2.1

For any n, the dominator chromatic number of central graph of Cocktail Party graph $G=CP_x$ of x-vertices, where x=2n, then $\chi_d$[C(G)]=2n+1,n>2.

### Result 2.2

For any n, the dominator chromatic number of central graph of Antiprism graph $G=AP_x$ of x-vertices, where x=2n, then

$$\chi_d[C(G)] = \begin{cases} 5k+1, & n = 3k \\ 5k+3, & n = 3k+1, k = 1,2,\ldots \\ 5k+5, & n = 3k+2 \end{cases}$$

## Result 2.3

For any n, the dominator chromatic number of central graph of Musical graph $G=M_x$ of x-vertices, where x=2n, then

$$\chi_d[C(G)] = \begin{cases} 5k+1, & n = 3k \\ 5k+3, & n = 3k+1, k = 1,2,\ldots\ldots\ldots, n>3. \\ 5k+5, & n = 3k+2 \end{cases}$$

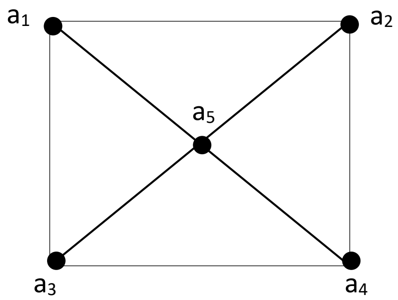
## 3. Dominator chromatic number of central graph of wheel graph

In this section, we determine the dominator chromatic number of central graph of the wheel graph.

## Definition 3.1

A wheel graph is a graph formed by connecting a single universal vertex to all vertices of a cycle. A wheel graph with n vertices can be defined as the 1-skelton of an (n-1) gonal pyramid.

**Example 3.2** wheel graph $WG_k$ with k=5




## Theorem 3.3

For any n, the dominator chromatic number of central graph of wheel graph $G=WG_x$ of x vertices when x=n then $\chi_d[C(G)]$=n+1,n>2

## Proof:

Let G be a wheel graph with x vertices , where x=n, so by the definition of wheel graph formed by connecting a single universal vertex to all vertices of a cycle $A_1$. Let $A_1=\{a_1, a_2\ldots a_n\}$of

$A_1(G)$ by the definition of central graph each edge $b_{ij}$ for $1\leq i \leq n$ , $1\leq j \leq n$ is subdivided by a vertex $c_{ij}$ in $A_{1,}$ $A_{1,}$ $i\neq j$, C(G) and join all the non -adjacent vertices of G.

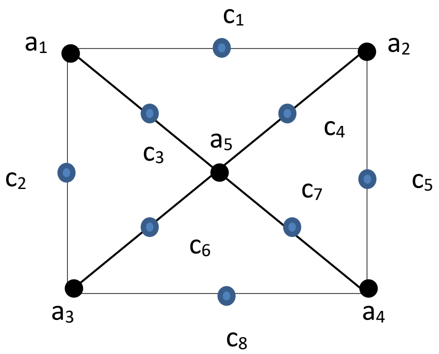
Let $A_1 =\{a_1, a_2 \ldots a_n\}$

$A_2=\{C_{ij}/1\leq i \leq n, 1 \leq j \leq n, i\neq j\}$ such that $A_1 \cup A_2$

The above procedure gives the dominator coloring of central graph. In $A_1$ cycle, we assign 1 to any one of the vertices and assign color for remaining vertices as 2,3…n-1.by the definition of dominator coloring all the vertices in the set V, contains all the vertices which dominates the color class V itself also all the color class are dominated by the dominating set V. Therefore the dominator chromatic number of central graph of wheel graph is n+1, n>2.

## Example 3.4

The central graph of $WG_5$ is depicted with a dominator coloring.




The color classes of $C(WG_5)$ are $A_1=\{a_1\}$, $A_2=\{a_2\}$, $A_3=\{a_3\}$, $A_4=\{a_4\}$, $A_5=\{a_5\}$, $A_6=\{b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8\}$.The dominator chromatic number is $\chi_d[C(WG_5)]=6$.

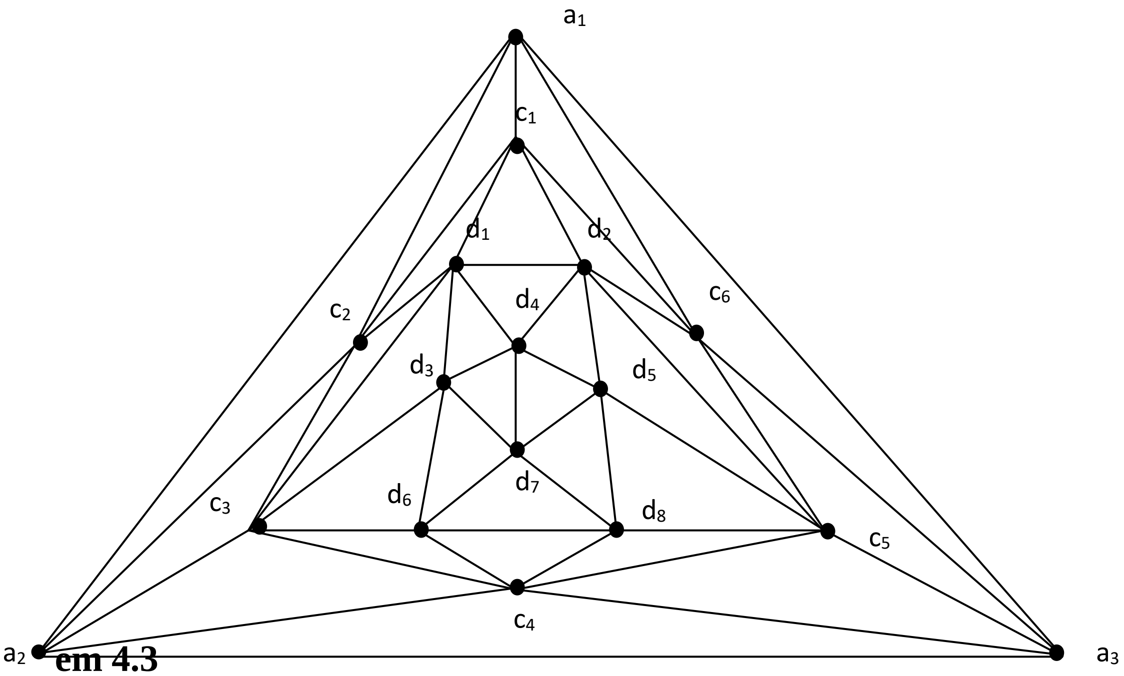
## 4. Dominator chromatic number of central graph of Errera graph

In this section, we determine the dominator chromatic number of central graph of the Errera graph.

## Definition 4.1

The Errera graph is a graph with 17 vertices and 45 edges. Alfred Errera published it in 1921 as a counterexample to kempe's erroneous proof of the four color theorem, it was named after Errera by Hutchinson & Wagon (1998).

**Example 4.2** Errera graph $EG_x$ with k=17




**T em 4.3**

The dominator chromatic number of central graph of Errera graph G = $EG_x$ of the 17 vertices then $\chi_d$[C(G)] = 4n+3.

## Proof:

Let G be a Errera graph with 17 vertices. By definition of Errera graph is constructed by joining three cycles of $C_n$.

Let $A_1$={$a_1$, $a_2$,…$a_n$}be the vertex set of outer cycle $C_n$ and $A_2$={$c_1$, $c_2$,...,$c_n$}be the vertex set of inner(1) cycle $C_n$ and $A_3$={$d_1$,$d_2$,...,$d_n$} be the vertex set of inner(2) cycle $C_n$.

By the definition of certral graph C(G) each $b_{ij}$ for $1\leq i \leq n$ , $1\leq j \leq n$ is subdivided by a vertex $e_{ij}$ in $A_1$, $f_{ij}$ in $A_2$, $g_{ij}$ in $A_3$,$dfg_{ij}$ in $A_1 \cup A_2 \cup A_3$, i≠j and join the all non adjacent vertices.

Let $A_1$={$a_1$, $a_2$,…$a_n$}, $A_2$={$c_1$, $c_2$,...,$c_n$}, $A_3$={$d_1$,$d_2$,..., $d_n$}, $A_4$={$e_{ij}/1\leq i \leq n$ , $1\leq j \leq n, i \neq$j},

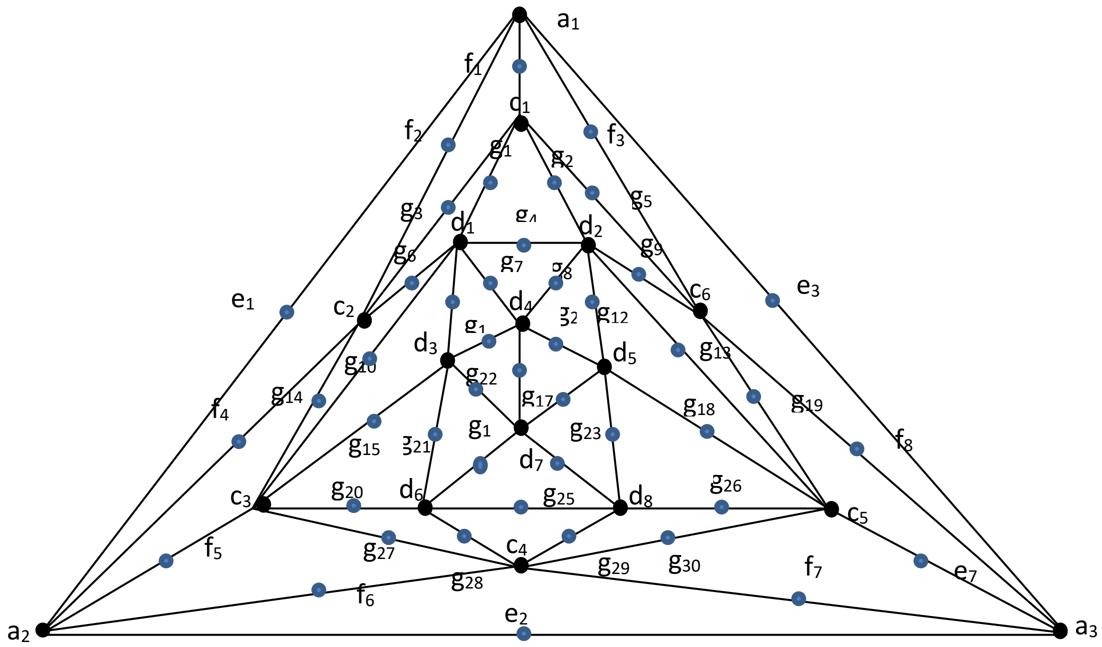
$A_5$={$f_{ij}/1\leq i \leq n$ , $1\leq j \leq n, i \neq$j}, $A_6$={$g_{ij}/1\leq i \leq n$ , $1\leq j \leq n, i \neq$j}, $A_7$={$ef_{ij}/1\leq i \leq n$ , $1\leq j \leq n, i \neq$j}, $A_8$={$eg_{ij}/1\leq i \leq n$ , $1\leq j \leq n, i \neq$j}, $A_9$={$fg_{ij}/1\leq i \leq n$ , $1\leq j \leq n, i \neq$j},

$A_{10}$={$efg_{ij}/1\leq i \leq n$ , $1\leq j \leq n, i \neq$j},such that $A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5 \cup A_6 \cup A_7 \cup A_8 \cup A_9 \cup A_{10}$

Let V={$d_i$, $f_j$, $g_i$, $efg_{ij}$, for i=1,2…n,j=1,2…n} be the dominating set of Errera graph.

The above procedure gives the dominator coloring of central graph. A graph G is contains 3 cycles, so we assign 1 color to any one of the vertices of $A_1$,$A_2$,$A_3$and assign color for remaining vertices as 2,3…n-1. By the definition of dominator coloring all the vertices in the set V, contains all the vertices which dominates the color class of V itself also all the color classes are dominated by the dominating set V, therefore, the dominator chromatic number of central graph of Errera graph is 4n+3,where n=3.

**Example 4.4** The central graph of $EG_{17}$ is depicted with a dominator coloring




The color classes of $C(EG_{17})$ are $A_1=\{a_1, c_1, d_1\}$, $A_2=\{a_2\}$, $A_3=\{a_3\}$, $A_4=\{c_2\}$, $A_5=\{c_3\}$, $A_6=\{c_4\}$, $A_7=\{c_5\}$, $A_8=\{c_6\}$, $A_9=\{d_2\}$, $A_{10}=\{d_3\}$, $A_{11}=\{d_4\}$, $A_{12}=\{d_5\}$, $A_{13}=\{d_6\}$, $A_{14}=\{d_7\}$, $A_{15}=\{d_8\}$, $A_{16}=\{e_1, e_2, e_3, f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8, g_1, g_2, g_3, g_4, g_5, g_6, g_7, g_8, g_9, g_{10}, g_{11}, g_{12}, g_{13}, g_{14}, g_{15}, g_{16}, g_{17}, g_{18}, g_{19}, g_{20}, g_{21}, g_{22}, g_{23}, g_{24}, g_{25}, g_{26}, g_{27}, g_{28}, g_{29}, g_{30}\}$.The dominator chromatic number is $\chi_d[C(EG_{17})]=16$.

## 5. Dominator Chromatic Number of Central Graph of Dyck Graph

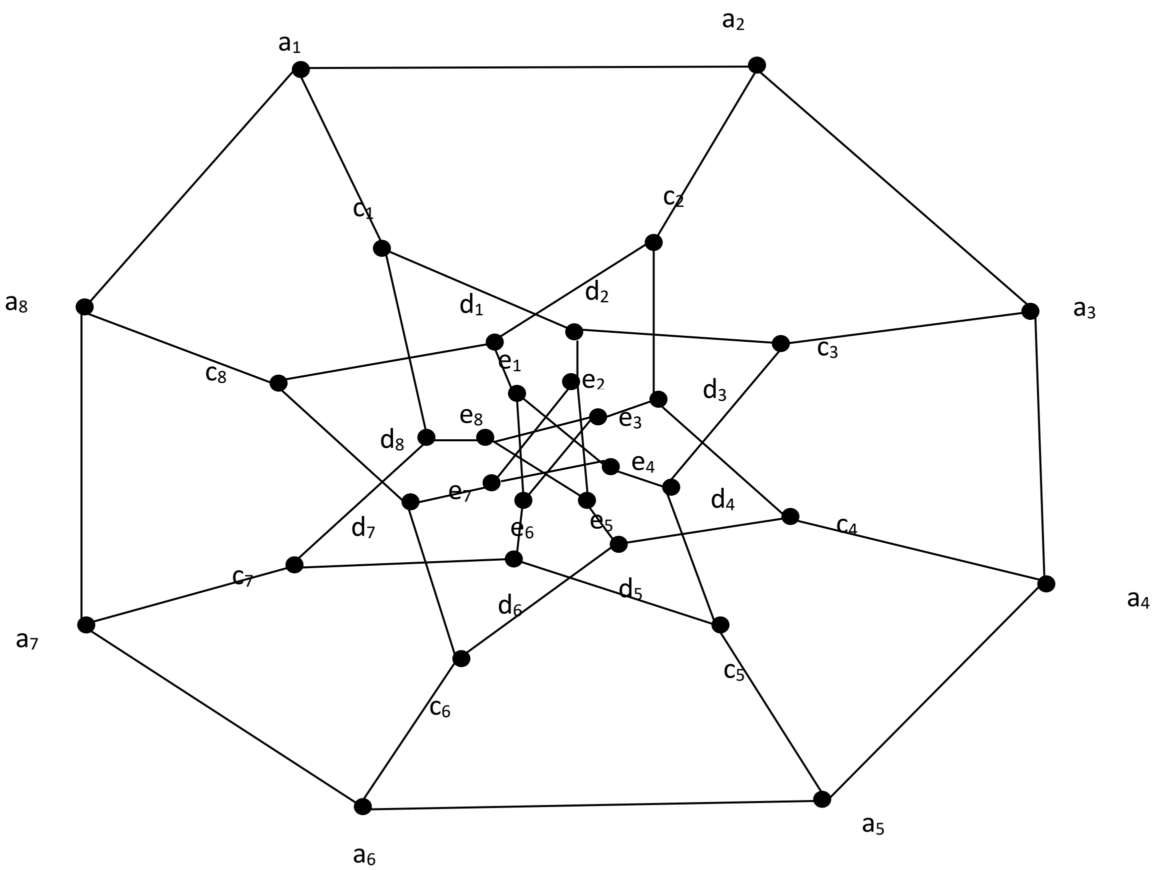
In this section , we determine the dominator chromatic number of central graph of the Dyck graph.

### Definition 5.1

The Dyck graph is a 3 regular graph with 32 vertices and 48 edges, named after Walther von Dyck. It is Hamilton with 120 distinct Hamilton cycles.

### Example 5.2

Dyck graph $DG_x$ with k=32




### Theorem 5.3

The dominator chromatic number of central graph of Dyck Graph $G=DG_x$ of 32 vertices where x=4n then $\chi_d[C(G)]=3n+6$.

### Proof:

Let G be a Dyck graph with 32 vertices, where x=4n. By definition of Dyck Graph is constructed by joining four parallel copies of the cycle $C_n$. Let $A_1=\{a_1, a_2,\ldots a_n\}$be the vertex set of outer cycle $C_n$ and $A_2=\{c_1, c_2,...,c_n\}$be the vertex set of inner(1) cycle $C_n$ and $A_3=\{d_1,$

$d_2,...,d_n\}$ be the vertex set of inner(2) cycle $C_n$. $A_4=\{e_1, e_2,...,e_n\}$ be the vertex set of inner(3) cycle $C_n$. By the definition of central graph C(G), each edge $b_{ij}$ for $1\leq i \leq n$ , $1\leq j \leq n$ is subdivided by a vertex $f_{ij}$ in $A_1$, $g_{ij}$ in $A_2$, $h_{ij}$ in $A_3$, $k_{ij}$ in $A_4$,$fghk_{ij}$ in $A_1 \cup A_2 \cup A_3 \cup A_4$ $i\neq j$ and join the all non- adjacent vertices.
Let $A_1=\{a_1, a_2,…a_n\}$, $A_2=\{c_1, c_2,...,c_n\}$, $A_3=\{d_1, d_2,...,d_n\}$,$A_4=\{e_1, e_2,…e_n\}$$A_5=\{f_{ij}/1\leq i \leq n$ , $1\leq j \leq n, i \neq j\}$, $A_6=\{g_{ij}/1\leq i \leq n$ , $1\leq j \leq n, i \neq j\}$, $A_7=\{h_{ij}/1\leq i \leq n$ , $1\leq j \leq n, i \neq j\}$, $A_8=\{k_{ij}/1\leq i \leq n$ , $1\leq j \leq n, i \neq j\}$, $A_9=\{fg_{ij}/1\leq i \leq n$ , $1\leq j \leq n, i \neq j\}$,

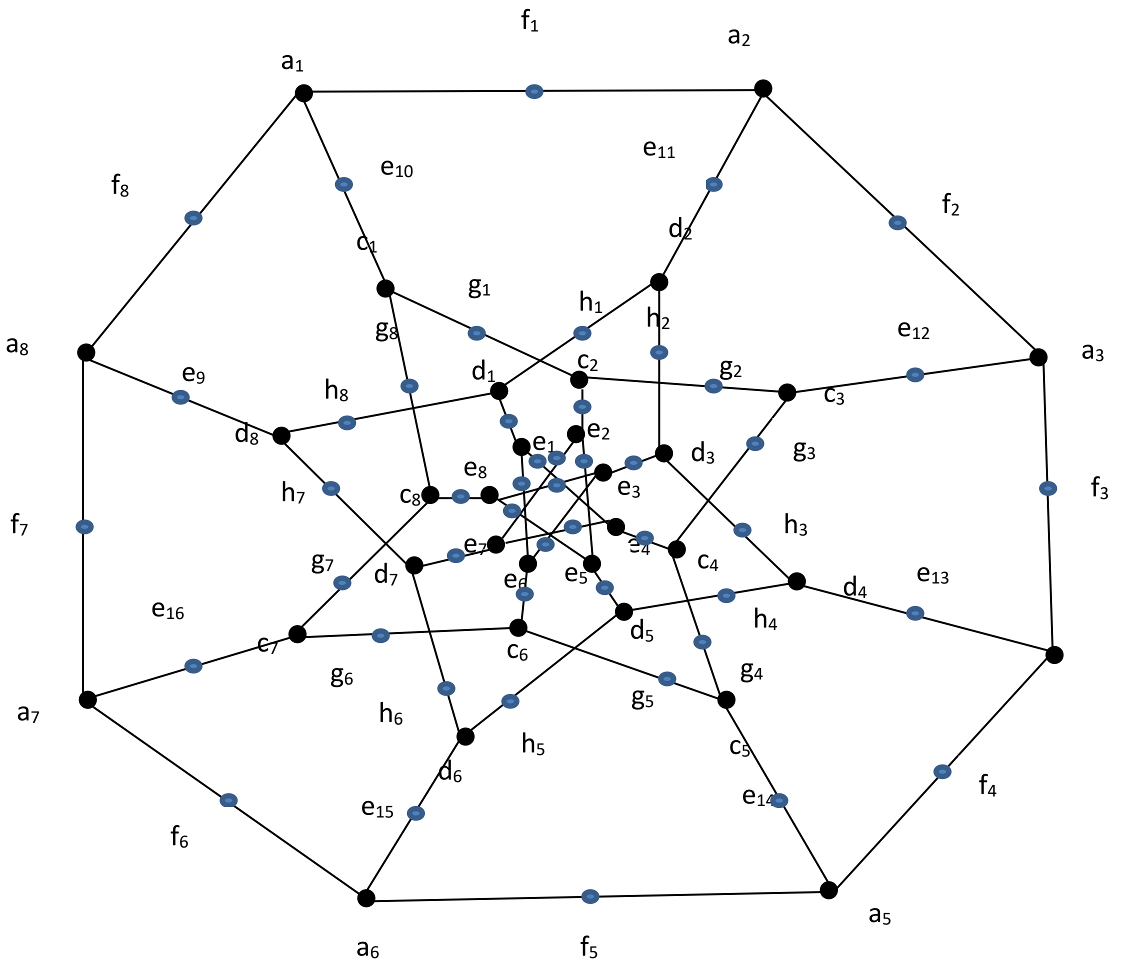
$A_{10}=\{fh_{ij}/1\leq i \leq n$ , $1\leq j \leq n, i \neq j\}$, $A_{11}=\{fk_{ij}/1\leq i \leq n$ , $1\leq j \leq n, i \neq j\}$, $A_{12}=\{gh_{ij}/1\leq i \leq n$ , $1\leq j \leq n, i \neq j\}$, $A_{13}=\{gk_{ij}/1\leq i \leq n$ , $1\leq j \leq n, i \neq j\}$, $A_{14}=\{hk_{ij}/1\leq i \leq n$ , $1\leq j \leq n, i \neq j\}$, $A_{15}=\{fg_{ij}/1\leq i \leq n$ , $1\leq j \leq n, i \neq j\}$, $A_{16}=\{fgk_{ij}/1\leq i \leq n$ , $1\leq j \leq n, i \neq j\}$,

$A_{17}=\{ghk_{ij}/1\leq i \leq n$ , $1\leq j \leq n, i \neq j\}$, $A_{10}=\{fhk_{ij}/1\leq i \leq n$ , $1\leq j \leq n, i \neq j\}$, $A_{10}=\{fhgk_{ij}/1\leq i \leq n$ , $1\leq j \leq n, i \neq j\}$, such that $A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5 \cup A_6 \cup A_7 \cup A_8 \cup A_9 \cup A_{10}$ $A_{11} \cup A_{12} \cup A_{13} \cup A_{14} \cup A_{15} \cup A_{16} \cup A_{17} \cup A_{18} \cup A_{19}$.

Let $V=\{f_i, g_j, h_i, k_j, fghk_{ij}$, for $i=1,2…n, j=1,2…n\}$ be the dominating set of Dyck graph.

The above procedure gives the dominator coloring of central graph. A Graph G contains 4 cycles, so we assign the color 1 to any one of the vertices of $A_1$, $A_2$, $A_3$, $A_4$ and assign color for the remaining vertices as 2,3,…n-1. By the definition dominator coloring all the vertices in the set V, contains all the vertices which dominates the color class of V itself also all the color classes are dominated by the dominating set V. Therefore, the dominator chromatic number of central graph of Dyck graph is 3n+6 , where n=8.

**Example 5.4** The central graph of $DG_{32}$ is depicted with a dominator coloring




The color classes of $C(DG_{17})$ are $A_1=\{a_1, c_1, d_1, e_1\}$, $A_2=\{a_2\}$, $A_3=\{a_3\}$, $A_4=\{a_4\}$, $A_5=\{a_5\}$, $A_6=\{a_6\}$, $A_7=\{a_7\}$, $A_8=\{a_8\}$, $A_9=\{c_2\}$, $A_{10}=\{c_3\}$, $A_{11}=\{c_4\}$, $A_{12}=\{c_5\}$, $A_{13}=\{c_6\}$, $A_{14}=\{c_7\}$, $A_{15}=\{c_8\}$, $A_{16}=\{d_2\}$, $A_{17}=\{d_3\}$, $A_{18}=\{d_4\}$, $A_{19}=\{d_5\}$, $A_{20}=\{d_6\}$, $A_{21}=\{d_7\}$, $A_{22}=\{d_8\}$, $A_{23}=\{e_2\}$,

$A_{24}=\{e_3\}$, $A_{25}=\{e_4\}$, $A_{26}=\{e_5\}$, $A_{27}=\{e_6\}$, $A_{28}=\{e_7\}$, $A_{29}=\{e_8\}$, $A_{30}=\{f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8, g_1, g_2, g_3, g_4, g_5, g_6, g_7, g_8, h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8, k_1, k_2, k_3, k_4, k_5, k_6, k_7, k_8, k_9, k_{10}, k_{11}, k_{12}, k_{13}, k_{14}, k_{15}, k_{16}\}$ .The dominator chromatic number is $\chi_d[C(DG_{32})]=30$.

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