ON CLEAN NEUTROSOPHIC SEMI RINGS

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ABSTRACT:

A commutative semi ring is said to be clean if every element of the semi ring can be written as a sum of a unit and an idempotent. In this paper, we generalize this argument to structure of neutrosophic. We present the structure of clen neutrosophic semi ring. Some elementary properties of clean neutrosophic semi ring are also presented.

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INTRODUCTION:

We introduced the concept of clean neutrosophic semi ring are clean semi ring and neutrosophic element.By combining the two concepts we can define a new neutrosophic algebraic structure called a clean neutrosophic semi ring.

Let we have a commutative semi ring with a multiplicative identity.The semi ring is said to be clean if every element of this semi ring is clean.As we know an element of the semi ring is called a clean element if it can be expressed as the sum of an idempotent and a unit.

In this paper, we show that every clean semi ring is an exchange semi ring. And also prove that necessary and sufficient conditions for a semi ring with central idempotent is a clean semi ring that is, the semi ring is an exchange semi ring.

The neutrosophic theory, for the first time introduce by Smarandache in 1980.The theory has given way to the construction of the concept of new algebraic structures called the neutrosophic structure.The concept of neutrosophic algebraic structures introduced by Kandasamy and Smarandache.The study of neutrosophic rings was introduced by Kandasamy and Smarandache.

Motivated by the result of Anderson and Camilo on commutative clean ring. Nicholson on uniquely clean rings and Kandasamy and

Smarandache on neutrosophic rings .In this article we extend to study clean neutrosophic semi rings and clean elements according to some known results. Also, we give some elementary properties of a clean neutrosophic semi ring.

In this section ,we give several main definitions and results about neutrosophic semi rings.

Definition 2.1: Let (S, +, .) be a semi ring .Then the set of all elements $x=$

$p+$ q I with $p, q \in \mathbb{R}$ and I is an element such that $I^2=I$ denoted by $\langle S \cup I\rangle$ is the neutrosophic semi ring formed by S and I with respect to the addition and the multiplication on S.

Further more we define some algebraic aspects related to the neutrosophic semi ring

Definition 2.2: Let $\langle S \cup I\rangle$ be a neutrosophic semi ring and $e \in \langle S \cup I\rangle$.The element e is an idempotent for multiplicative operation on $\langle S \cup I\rangle$ $i.e, e^2=e.e=e$.

Definition 2.3: Let $\langle S \cup I\rangle$ be a neutrosophic semi ring and $e \in \langle S \cup I\rangle$ .The element $e=p+qI$ is a neutrosophic idempotent for multiplicative operation on $\langle S \cup I\rangle$ if $q \neq 0$ and $e^2=e$.

Note 2.4: Every neutrosophic idempotent element is an idempotent.

Definition 2.5: Let $\langle S \cup I\rangle$ be a neutrosophic semi ring with $1 \neq 0$ and let $u \in \langle S \cup I\rangle$ .The element u is a unit of $\langle S \cup I\rangle$ if there is an element $v \in \langle S \cup I\rangle$ and satisfies $u.v=1=v.u$

Definition 2.6: Let $\langle S \cup I\rangle$ be a neutrosophic semi ring with $1 \neq 0$ and let u $\in \langle S \cup I\rangle$. The element $u=p+qI \in \langle S \cup I\rangle$ with $q \neq 0$ is a neutrosophic unit of $\langle S \cup I\rangle$, if there is an element $v=r+sI \in \langle S \cup I\rangle$ with $s \neq 0$ and satisfies $.v=1=v.u$

3. Clean neutrosophic semi ring and their elementary properties

Definition 3.1 Assume that $\langle S \cup I\rangle$ is a neutrosophic semi ring and $x \in \langle S \cup I\rangle$ The element x is said to be a clean semi ring if $x = e + u$ where e is a idempotent element and u is a unit of $\langle S \cup I\rangle$.

Note 3.2 In this section, we use notation $\cup(\langle S \cup I\rangle)$ to express the set of all units in $\langle S \cup I\rangle$ and $Id(\langle S \cup I\rangle)$ to the set of all idempotent elements in $\langle S \cup I\rangle$.

Definition 3.3 A neutrosophic semiring in which all elements are clean,then the semi ring is called a clean neutrosophic semi ring .

Furthermore if each element of the neutrosophic semi ring is uniquely clean,then the semi ring is called a uniquely clean neutrosophic semi ring.

Example 3.4 Clearly $\langle Z_2 \cup I\rangle$ is clean neutrosophic semi ring .Since every element of $\langle Z_2 \cup I\rangle$, ts presentation as sum o f an idempotent and unit is unique,so the semi ring $\langle Z_2 \cup I\rangle$ to be uniquely clean semi ring.

Example 3.5 We have $(\langle Z_3 \cup I\rangle) = \{\bar{0}, \bar{1}, I, \bar{1} + \bar{2}\ I\}$

$\cup(\langle Z_3 \cup I\rangle) = \{\bar{1}, \bar{2}, \bar{1} + I, \bar{2} + \bar{2} + I\}$ .All elements of $\langle Z_3 \cup I\rangle$ are clean elements. Take $\bar{2} +$ in $\langle Z_3 \cup I\rangle$, Clearly $\bar{2} + I = I + \bar{2}$,and also we have $\bar{2} + I = \bar{1} + (\bar{1} + I)$.So, $\langle Z_3 \cup I\rangle$ is not uniquely clean neutrosophic semi ring .It is only a clean neutrosophic semi ring.

Lemma 3.6 Suppose $\langle S \cup I\rangle$ is a neutrosophic semi ring with 1.If

$e \in (\langle S \cup I\rangle)$,then $1 - e \in Id(\langle S \cup I\rangle)$ where 1 is unity element in $\langle S \cup I\rangle$.
Proof : Let $e^2 = e \in \langle S \cup I\rangle$ and $f = 1 - e$

We have $f^2 = (1 - e)^2 = (1 - e)(1 - e) = (1 - e) - (e - e^2) = (1 - e) - (e - e) =$

$1 - e = f$

Hence $f = 1 - e \in (\langle S \cup I\rangle)$

Definition 3.7 Let $\langle S \cup I\rangle$ be a neutrosophic semi ring and $e \in \langle S \cup I\rangle$ is an idempotent element. The idempotent e is a central if $e.x = x.e$ for every x $\in \langle S \cup I\rangle$ .The set of all central idempotent of $\langle S \cup I\rangle$ is denoted by $(\langle S \cup I\rangle)$

Lemma 3.8 Let $\langle S \cup I\rangle$ be a neutrosophic semi ring with the identity 1 .If

$e \in (\langle S \cup I\rangle)$ then $1 - e \in (\langle S \cup I\rangle)$ where $e \in (\langle S \cup I\rangle)$

Proof: Let $e \in (\langle S \cup I\rangle)$ and $f = 1 - e$.

If $x \in \langle S \cup I\rangle$ then $fx = (1 - e)$

$= 1.x - (e.x)$

$= x.1 - (x.e)$

$= (1 - e)$

$= x.f$

This prove that $f = 1 - e \in (\langle S \cup I \rangle)$

Theorem 3.9 In any neutrosophic semi ring ,a central idempotent is a uniquely clean element

Proof : Let $e^2 = e$. We have $e = (1 - e) + (2e - 1)$

Suppose that $e = f + u$, $f^2 = f$ *where* $u \in \cup(\langle S \cup I \rangle)$

If $eu = ue$ we obtain $f + u = (f + u)^2 = f^2 + 2fu + u^2$

So, $= 1 - 2f$

Hence $f = 1 - e$

Lemma 3.10 Every Idempotent element is a uniquely clean neutrosophic semi ring is central

Proof : Let 'e' is an idempotent element in $\langle S \cup I \rangle$

If $x \in \langle S \cup I \rangle$ then $e + (ex - exe)$ is an Idempotent and $1 + (ex - exe)$ is a unit.So, the fact that $[e + (ex - exe)] + 1 = e + [1 + (ex - exe)]$, implies that $e + (ex - exe) = e$ because $\langle S \cup I \rangle$ is uniquely neutrosophic semi ring.
Hence $ex = exe$ *and* $xe = exe$.

So, $ex = xe$ as required.

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