Modelling Bouc-Wen based MIMO Nano-Positioning System

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Highlights

- The static and dynamic behaviour of NPS is investigated using modified linearized Bouc-Wen (BW) hysteresis.
- The mathematical derivation comes by step to prevail the expression.
- The simulink model along with its output has been presented in this paper for the better understanding of the readers.

Abstract

*'Nano' has encroached into each and every application. Nano-*Positioning System (NPS) *is a mechanism where mechanical dynamics to be handled is in Nano scale. In this present work, NPS is designed using modified linearized Bouc-Wen (BW) hysteresis and its static and dynamic behaviour are studied. Previously , system was basically designed to handle conventional macro level dynamics and the same has been extended to handle Nano scale in this work. Kalman Filtering is included to achieve very sustainable results in NPS. The NPS is designed in Simulink with 3 types of electrical signals.*

Keywords: *Bouc-Wen, Kalman Filtering, MIMO, NPS*

I. INTRODUCTİON

Advancement in the nano-technology has encroached into every application where there is a need of high- speed high precision positioning systems like semiconductor and machining industries(ultra-precision) particularly the systems like Scanning Tunnelling Microscope (STM) [1], Atomic Force Microscope (AFM) etc. These systems are basically designed to measure the mechanical, electrical and structural characteristics of the surface at atomic level. Suppose in an example with STM, the distance between metallic surface and the conductive tip of STM is at nano-scale (i.e. $<10^{-9}$m) and when it is electrically biased (i.e. Voltage is applied) then transfer of electrons may take place between tip and surface. Due to this the intensity of tunnelling current is dependent on the surface gap. To procure electrical and topographical details the tunnelling current is maintained constant and tip is horizontally slide throughout the surface using piezoelectric actuators [2,3]. But in AFM, instead of tunnelling current sensor, position sensor is used which has extended its applications even for non-conductive surfaces [4]. This necessitates design of a highly precision positioning system which can handle different non- linear noises and dynamics in NPS particularly due to piezo-electrically actuated tunnelling tip as in STM and in micro-cantilever as in AFM. The designed control system should have high accuracy and bandwidth suitable at Nano scale. BW hysteresis model is integrated to handle the non-linearity [15]. The entire NPS is modelled using MATLAB/Simulink. Usually the linear systems are easy to design and handle. Also systems will become static with the linearity. Hence designed non-linear system is linearized. Furthermore, a capacitive sensor is used for detection [16] in actuators which imposes nonlinearity [17] and hence leads to low SNR value. To enhance SNR, filtering model is being furnished. Among filters, Kalman Filter shows self-adaptive nature which makes it a better precision filter to nullify the resonance noise caused due to inherent frequency of mechanical stage [18]. Even other noises like quantization noise, sampling noise and random noise etc. are also nullified.

This paper is organised into different sections. Section 1 gives an introduction about the NPS along with the need of each module in the design. Literature review on similar work is discussed in Section 2. The background details of hysteresis modelling are broadly described in Section 3. The design details of Kalman filter is explained in Section 4. Section 5 contains Simulink implementation of the entire system and finally Section VI has the summarised results and discussion.

II. RELATED WORK

The piezo-electric actuators pose high-precision and solid- state fast actuation which are mainly used in the various applications like sono-chemistry application domain [8], vibration control domain [5], hydraulic flow control domain [6], energy harvesting [7], and so on. Usually the designed systems have nonlinearity hysteresis which degrades the controlling nature of the system [9].

To analyze the system, mathematical model of the hysteresis is to be modeled and hysteresis compensation model needs to be enforced using inverse modeling. BW model [10] is one such hysteresis model. Paper [11] deals with One-DOF systems and multi degree freedom. Paper [12] designs Fourier Transform based Bouc-Wen and successive approximation. Paper [13] models hysteresis compensation using adaptive neural output feedback control.

Above mentioned papers predominantly designs the systems at macroscopic levels. The work [19] extends to design to nano-scale, in which maximum care has to be taken while designing the model as it involves the positioning which measures < 1 nm. The designed model is checked with 3 sorts of inputs like step input, sine input and pulse input. To have an improved output, Kalman filtering is added. Further in this work MIMO modelling is done for NPS.

III. Hysteresis Modelling

This work models modified BW model by simulating the static hysteresis of piezoelectric stack. To cancel this hysteresis, inverse BW model is being designed in NPS. By linearizing the inverse model, the system is made static.

3.1.Mathematical model of BW

BW model is a non-linear hysteretic systems used to capture analytically the different shapes of hysteretic cycle and also matching behaviour of different hysterical systems. The motion equation with one-degree freedom is

$$m\frac{d^2x(t)}{dt^2} + c\frac{dx(t)}{dt} + F(t) = f(t)$$

Here, f(t) and F(t) are Excitation and Restoring force
Forces, m is Mass[Kg]and x(t) is the Displacement[m]
The Restoring force is given by,

$$F(t)=aK_i x(t)+(1-a)k_i z(t)$$

Here, z(t)→ hysteretic displacement , $a=\frac{\text{post- yield}(k_f)}{\text{pre-yield}(k_i)}$

With zero initial condition z(t) is represented as nonlinear differential equation:

$$\frac{dz(t)}{dt} = \frac{dx(t)}{dt}\left\{\alpha - \left[\beta \text{sign}\left(z(t)\frac{dx(t)}{dt}\right) + \gamma\right] |z(t)|^n\right\}$$

Where, α, β, γ, n→Dimentionless quantities which decides the hysteresis loop shape, sign()→signum function

In Piezo-electric actuator, f(t) is an input voltage, hysteretic state variable z(t) is length, and x(t) is an output positioning displacement. The motion equation and z(t) can be rewritten as,

$$m\frac{d^2x(t)}{dt^2} + c\frac{dx(t)}{dt} + kx(t) = k(d\,f(t) - z(t))$$

$$\frac{dz(t)}{dt} = \alpha d\frac{dx(t)}{dt} - \beta\left|\frac{df(t)}{dt}\right| - \gamma f(t)\left|\frac{df(t)}{dt}\right|$$

Here, d→effective piezoelectric coefficient (m/V)

3.2.Inverse BW model

This model nullifies the hysteresis imposed in the previous stage. The inverse model outputs a displacement:

$$f(t)=\left[\frac{m}{kd}\right]\frac{d^2x(t)}{dt^2} + \left[\frac{b}{kd}\right]\frac{dx(t)}{dt} + \left[\frac{1}{d} + \frac{k}{kd}\right]\frac{dx(t)}{dt} + \left[\frac{z(t)}{d}\right]$$

3.3.Linearising the non-linear hysteresis

The designed system poses non-linear hysteresis but as linear designs are easy, the designed system is linearized and made static by reducing the difference between estimated linear and non-linear values

$$\tilde{e} = \alpha d\frac{dx(t)}{dt} - \beta\left|\frac{dx(t)}{dt}\right| z(t) - \left(k_1\frac{dx(t)}{dt} + k_1 z(t)\right)$$

4. NPS model

The Fig 1(d) depicts BW model designed using Simulink model. Various considered parameters in the model is in the Table 1.

Table1.:BW model Parameters

Parameter	**Value**
α	0.36
d	$7x10^{-7}$m/V
β	0.03
k	$4.5x10^{5}$N/m
γ	0.02
c	$1.28x10^{3}$Ns/m
m	0.00156Kg

The overall transfer function G(s) of the PEA (piezo-electric actuator) is,

$$G(s)=\frac{7.2x10^{13}s^2+2.3x10^{16}s+3.2x10^{31}}{s^6+1.1x10^4s^5+9.5x10^7s^4+7x10^{13}s^3+2x10^{15}s^2+5.6x10^{18}s+10^{22}}$$

The complete MATLAB /Simulink model of NPS is as in Fig 1(b). Fig 1(c) shows the inverse hysteric model . Fig 1(d) shows the linearized model.

Previously, the energy dissipation method was formulated with Bouc-Wen eventually proceeding on again to modify that in 1976. But since then, that final information, designated as the Bouc-Wen method, has been commonly applied to analyze energy absorption tools and devices. That solution was essentially a new non-linear mathematical function. This has an energy absorption relationship between the data (deflection) and the outcome (improving current). Because of the set of input data, this calculation changes from one element to another. This is conceivable because we can simulate this model's reaction with actual saturation magnetization through selecting the most suitable subset of features. This optimization of properties for both individual aspects and elements has become an important component of controller design. While m would be the piezoactuator's measurement help (kg), b would be the functional absorption (N-s/m), k would be the structural flexibility (N/m), and d would be the piezoelectric ratio (m/V). V (volts) would be the control signal towards the piezoelectric transducer, x would be the piezoelectric actuator's movement (m), while his is the energy absorption constant variable (in m). Elements determined the volume and structure of an energy absorption circuit, as well as this "n" option regulates the efficiency of the transfer from the fluid towards the deformation; therefore, considering that its piezoelectric activator exhibits elastic response, n was specified as constant at 1. The figure below shows the simulation element block representing this Bouc-Wen system.

Inverse block diagram Bouc-Wen Model displays various simulation component topologies for such an Inverted Bouc-Wen system and also the general simulation model. These three components of the Bouc-Wen model for APA120S were computed using the MATLAB Transfer Function Capabilities. These measurements, received first from the amplification piezostack actuator's operational assessment, were employed. These sources are really the movements collected during the experiment, as well as the outcomes, which voltage ratios were ranging between 0 and 160 V. These estimates of a component were determined using the symmetric methodology and also the accepted reflection nonlinear estimation methodology.

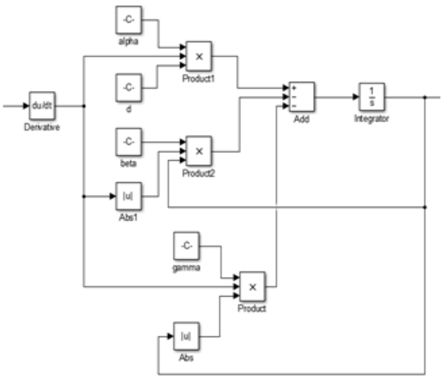

Fig.1(a): Simulink model of BW model

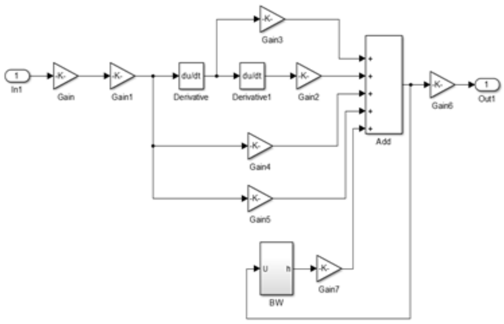

Fig.1(c): Simulink model of inverse system

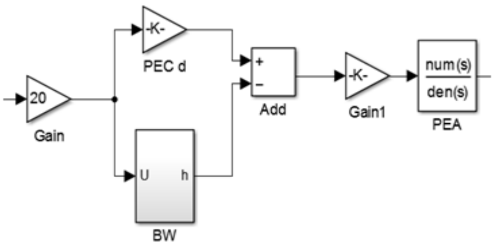

Fig.1(b): Simulink model of Complete NPS

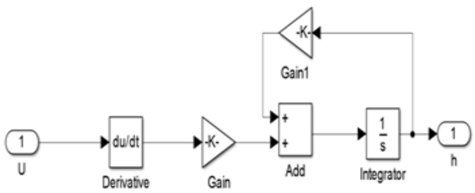

Fig.1(d): Simulink model of linearized system

5. Design of Kalman Filter

Kalman filtering or Linear Quadratic Estimation has huge variety of applications wherever there is need of more accurate filtered results. The filtering method used here is an optimal linear recursive filtering which has simple mathematical design and structure. During the filter operation, the probable outputs are estimated and are compared with the measured values. If required the self-adaptive corrections are made for future estimations. In order to further simplify the design for nano stage filtering.

Kalman filter is also modelled using state equation model which has two equations,

1. State Equation: $X(k) = AX(k-1) + BW(k-1)$

2. Measurement equation: $Y(k)=CX(k)+V(k)$

Here, X(k) and $X(k-1)$ indicates Present and Previous State values respectively, and Y(k) is Present observed Output, W(k-1) is a Process noise , A,B, C are State Transformation matrices defining previous state , noise part and amount of input respectively and V(k)→ is an Observation noise.

State Equation defines the present state calculated using a linear stochastic equation which is a linear combination of control signal, process noise and previous value. W(k) and V(k) are statistically independent AWGN with the covariance of A and B respectively.
Calculation part of Kalman Filter takes place with 2 simultaneous stages

- Time update stage:
  - Propagate State:

x^{*}_{k+1}=f($\widehat{x_k}$, u_k,0)=A$\hat{x}^{-}_{k-1}$+Bu_k

  - Propagate Error Covariances:

$P^{*}_{k+1} = A\widehat{P}^{-}_{k-1}\ A^{T}$+Q =$A_{k+1}P_kA^{T}_{k+1}$+ $W_{k+1}Q_kW^{T}_{k+1}$

- Measurement stage:
  - Kalman Gain: K_{k+1} = $\widehat{P}^{-}_{k}\ H^{T}(H\widehat{P}^{-}_{k}\ H^{T}+R)^{-1}$

$= P^{*}_{k+1}C^{T}_{k+1}(C_{k+1}P^{*}_{k+1}C^{T}_{k+1} + V_{k+1}\ R_{k+1}V^{T}_{k+1})^{-1}$

  - Estimate State: $\widehat{x_{k+1}}$ =$\hat{x}^{-}_{k}$+$K_k(z_k - H\hat{x}^{-}_{k})$= x^{*}_{k+1} + K_{k+1}

  - Estimate Error Covariances: P_{k+1}=(I-K_kH) P^{-}_{k} =(I-$K_{k+1}C_{k+1}$)

The values evaluated in the first stage are used as an input to second stage. These are retained and used in future iterations. The Kalman Filtering process is shown in Fig 2.

The following design specifications are used to model NPS [25], A, B and C are $\begin{bmatrix}1 & T\\0 & 1\end{bmatrix}$, $\begin{bmatrix}0\\1\end{bmatrix}$, and $\begin{bmatrix}1 & 0\end{bmatrix}$ respectively, T is 100nS, Q and R are 0.01 and $4x10^{-9}$ respectively.

6. MIMO Modeling

The NPS model designed in the previous section is an example of SISO. This SISO is used as subsystem in MIMO NPS. In SISO the movement is along one axis. MIMO system can control the movements along three axes. The MIMO is designed by decomposing the MIMO system into several SISO-like subsystems. Therefore here 3 voltage levels acts as multiple inputs and 3-axial displacements forms multiple output. Along with this if any noise is found in output , it is rectified using Kalman filter. Fig 3 is Simulink model of SISO like model of MIMO.

IV. RESULT

The MIMO NPS is designed and modeled in SIMULINK as shown in previous sections. In order to check the output, sine input, step input and pulse inputs are considered. The displacement output is taken along single axis in SISO and along three axes in MIMO NPS. The Output of Simulink scope for all three inputs are depicted in Fig 4,5 and 6. The some samples of the output are taken in Table 2. One can observe the stability in the output with the linearization.

Table.2: Simulation summary

Overall module with	**Displacement ($x10^{-9}$)**
BW model	420 to 540
BW along with Gain	45 to 520
Linearized BW along with Gain	120 to 200

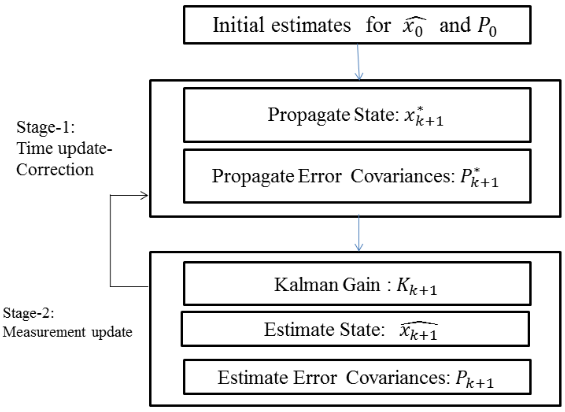

Fig.2: Flowchart of Kalman Filtering

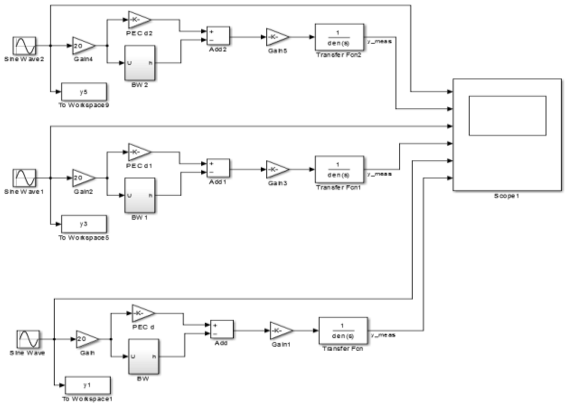

Fig.3: Simulink model of MIMO model

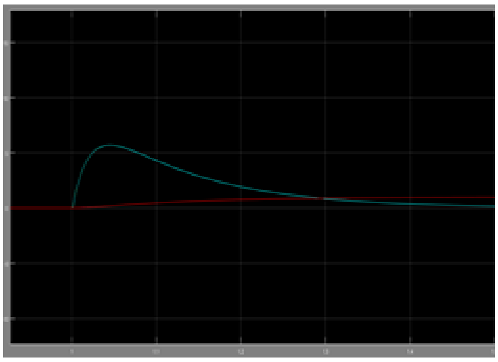

Fig 4. Displacement output of Simulink

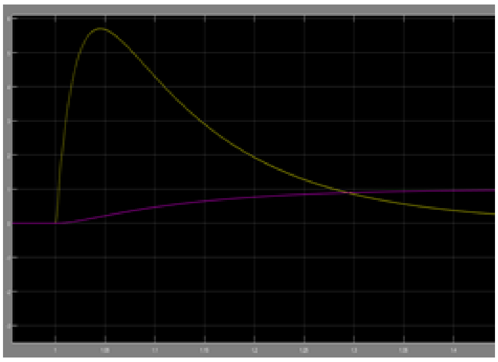

Fig 5. Current output of Simulink

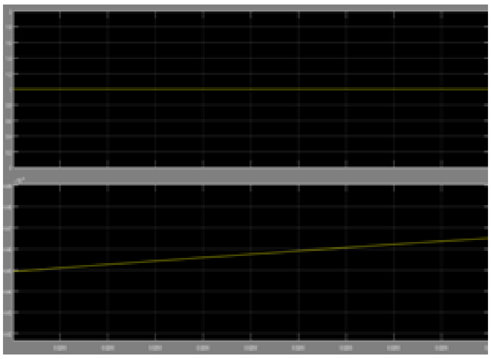

Fig 6(a). Simulink output for sine input

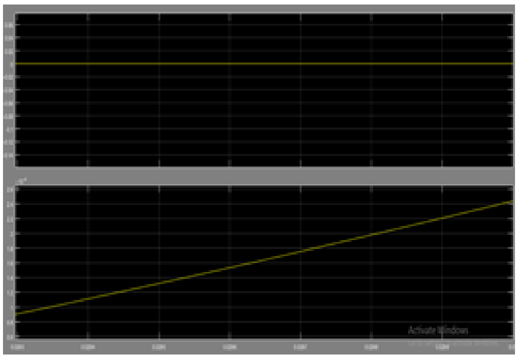

Fig 6(b). Simulink scope output for step input

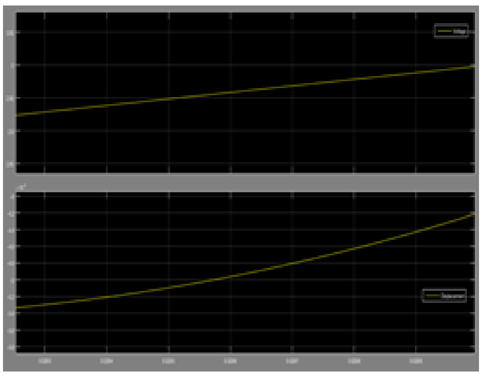

Fig 6(c). Simulink scope of Fig 1(b)

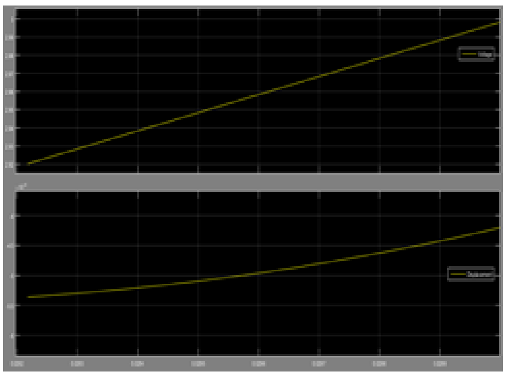

Fig 6(d). Simulink Scope of Fig 3

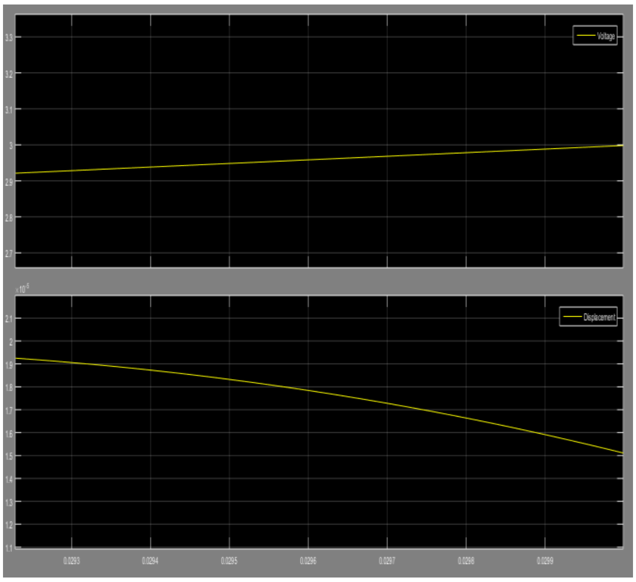

Fig 6(e). Simulink Scope of Fig 5 , 6(a)

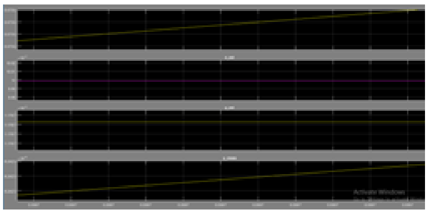

Fig 7(a). Simulink Scope of Kalman Filtering

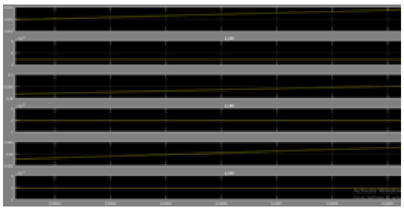

Fig 7(b). Simulink Scope of Filtered MIMO

Fig 6 details about the Simulink scope results with various input voltage range from 2.93 to 2.99 V. In Table 3 one can observe the further stability enhancement in the output with the Kalman filtering. Results of NPS system with and without Kalman filtering are summarized.

Table.3. Filtering Comparision Summary

Input	Without Kalman	With Kalman(nm)
Sine (0.73V)	8.2 to 8.4	9.9
Step (3V)	7.7 to 7.9	7.5

Table 4 summarises the output of MIMO with sine input given to all 3 inout with variety voltage range.

Table.4. MIMO output with Filtering (Sine Input)

Axis	Input(5Hz)	Output (in nm)
X	0.25V	5
Y	1V	200
Z	1.5V	290

V. CONCLUSION

- Summarization of the work indicates that BW model is more efficient among many non-linear models especially for positioning system.
- By inhibiting the error occurred between expected linear result and non-linear obtained result, even the non-linear system can be linearized
- More stabilized displacement movement can be observed with linearized BW model
- The system has been extended to MIMO to handle 3- axial displacement and also offer low noise using filtering.

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