Five Dimensional Kaluza-Klein Dark Energy Model with Strings in Brans-Dicke Theory of Gravity

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Abstract: *A five dimensional Kaluza-Klein space-time is considered in the framework of a Brans-Dicke (BD) scalar-tensor theory of gravitation when the source for energy-momentum tensor is a dark energy in the presence of one dimensional cosmic strings. A power law of the scale factor for the BD scalar field is assumed to get a determinate solution of the field equations. We discuss the evolution of the accelerated expansion of the universe through deceleration and equation of state (EoS) parameters for our model. The physical behavior of the model is also discussed.*

***Key words**: Higher dimensional, Kaluza-Klein, dark energy, Brans-Dicke theory, scalar-tensor theory.*

1 Introduction

Nowadays it is strongly believed that the universe is experiencing an accelerated expansion [1]-[2]. These observations lead to a matter called dark energy (DE) which has large negative pressure.There are a lot of DE models and modified gravity models have been put forward, among various DE models, we have taken the holographic dark energy (HDE) model. The HDE is arising from the holographic principle [3] which links the energy density of DE to the cosmic horizon, attempting to examine the nature of DE in the framework of quantum gravity. The DE can be explained in terms of cosmological constant, acts like a perfect fluid with an equation of state, satisfying the observational data so far. However, this involves the problems of fine tuning and cosmic coincidence. Many dynamical models like phantom [4], quintessence [5], quintom [6], etc. have been proposed to alleviate these problems.

In this article, among various DE models, we concentrate on the holographic dark energy (HDE) model. The HDE is arising from the holographic principle , which links the energy density of DE to the cosmic horizon, attempting to examine the nature of DE in the framework of quantum gravity. The energy density of HDE is defined as

$$\rho_h = 3C^2 M_p^2 L^{-2} \tag{1}$$

where M_p is the reduced Planck mass, $M_p^2 = (8\pi G)^{-1}$ C^2 is a dimensionless model parameter, L indicates the infrared (IR) cutoff radius [7,8]. Based on this principle, researches on theHDE have attracted so many scientists, and a lot of remarkableworks have been done in this field [9]. In order to examine the HDE, one should give a special form of the IR cutoff. Until now, there are many choices have been taken as the IR cutoff radius. Among several HDE models, Xu et al. [10] have considered generalized holographic and Ricci DE models. In their work, the energy densities of DE are given as $\rho_h = 3\alpha M_p^2 H^2 g(R/H^2)$ and $\rho_R = 3\alpha M_p^2 R f(H^2/R)$, where $f(x)$ and $g(y)$ are functions of the variables R/H^2 and H^2/R. Nowadays, various works show that the HDE model is in fairly good agreement with the observational data [11,12].

In order to explain late time acceleration, two different approaches have been advocated: one is to construct different dark energy candidates and the other is the modification of Einstein's theory of gravitation. Among the several modifications Brans-Dicke [13] and Saez-Ballester [14] scalar–tensor theories are significant.In BD theory a scalar field ϕ, in addition to the metric tensor field g_{ij}, has been introduced which has the dimension of the inverse of a gravitational constant and Einstein field equations have been modified. BD theory plays a vital role in modern cosmological applications [44, 45]. The latest inflationary models [15], possible graceful exit problem [16] and chaotic inflation [17] are based on BD scalar–tensor theory.

Higher dimensional cosmology is important because it has physical relevance to the early stages of evolution of the Universe before it has undergone compactification transitions. Hence several authors (Witten [18]; Applelquist et al. [19]) were attracted to the study of higher dimensional cosmology. Also, in the context of Kaluza–Klein and super string theories higher dimensions have recently acquired much significance. Several investigations have been made in higher dimensional cosmology in the framework of different scalar– tensor theories (Ref. [20-36]).

Motivated by the above authors, we proposed to investigate five dimensional Kaluza-Klein dark energy model is considered in the framework of a BD scalar-tensor theory of gravitation when the source for energy-momentum tensor as one dimensional cosmic strings. The organiation of paper is as follows: Sec. 2 contains model and their field equations. In Sec. 3, we obtain solution of the field equations. Sec. 4 contains some important properties of the model. Last section contains some conclusions of the model.

2 Model and field equations

We consider the five dimensional Friedmann-Robertson-Walker (FRW) metric in the form

$$ds^2 = dt^2 - D^2(t)(dx^2 + dy^2 + dz^2) - E^2(t)dm^2, \quad (2)$$

where D, E are functions of t only.

Several theories have been proposed as alternatives to Einstein's theory. Brans and Dicke [13] formulated a scalar-tensor theory of gravitation which is supposed to be the best alternative to Einstein's theory. We consider the universe filled with pressure less matter and dark energy (DE) fluid. In this case the field equations for the combined scalar and tenor fields given by Brans and Dicke are

$$R_{ij} - \frac{1}{2}Rg_{ij} = -\frac{8\pi}{\phi}(T_{ij} + \overline{T}_{ij}) - w\phi^{-2}\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) - \phi^{-1}\left(\phi_{i;j} - g_{ij}\phi^{;k}_{;k}\right), \quad (3)$$

$$\phi^{;k}_{;k} = \frac{8\pi}{(3+2w)}(T + \overline{T}) \quad (4)$$

and the energy conservation equation is

$$(T_{ij} + \overline{T}_{ij})_{;j} = 0, \quad (5)$$

which is a consequence of field equations (3) and (4).

Here R Ricci scalar, R_{ij} is Ricci tensor, w is a dimensionless coupling constant. T_{ij} and $\overline{T}_{ij}$ are energy-momentum tensors forcosmic string and RDE, respectively, which are defined as

$$T_{ij} = \rho u_i u_j + \lambda x_i x_j \quad (6)$$

$$\overline{T}_{ij} = (\rho_{de} + p_{de})u_i u_j - p_{de}g_{ij} \quad (7)$$

here p_{de} and ρ_{de} are the pressure and energy density of DE respectively, $\omega_{de} = p_{de}/\rho_{de}$ is the equation of state (EoS) parameter of DE. ρ is energy density of matter. The energy density of DE ρ_{de} was proposed by Granda and Oliveros [37] as

$$\rho_{de} = 3[\zeta H^2 + \eta\dot{H}] \quad (8)$$

where ζ , η are constants and H is a Hubble parameter which are must satisfy the restrictions imposed by the current observational data. $M_p^2 = \frac{1}{8\pi G}$ is the reduced plank mass and in Brans-Dicke theory $\phi \propto G^{-1}$.

By adopting comoving coordinates, the field equations (3) and (4) for the metric (2) using energy-momentum tensors (6) and (7) yield the following equations:

$$2\frac{\ddot{D}}{D} + \frac{\dot{D}^2}{D^2} + 2\frac{\dot{D}\dot{E}}{DE} + \frac{\ddot{E}}{E} + \frac{w}{2}\frac{\dot{\phi}^2}{\phi^2} + \left(2\frac{\dot{D}}{D} + \frac{\dot{E}}{E}\right)\frac{\dot{\phi}}{\phi} + \frac{\ddot{\phi}}{\phi} = -\frac{8\pi}{\phi}\omega_{de}\rho_{de} \tag{9}$$

$$3\frac{\dot{D}^2}{D^2} + 3\frac{\dot{D}\dot{E}}{DE} - \frac{w}{2}\frac{\dot{\phi}^2}{\phi^2} + \left(3\frac{\dot{D}}{D} + \frac{\dot{E}}{E}\right)\frac{\dot{\phi}}{\phi} = \frac{8\pi}{\phi}(\rho_{de} + \rho) \tag{10}$$

$$3\frac{\ddot{D}}{D} + 3\frac{\dot{D}^2}{D^2} + \frac{w}{2}\frac{\dot{\phi}^2}{\phi^2} + 3\frac{\dot{D}}{D}\frac{\dot{\phi}}{\phi} + \frac{\ddot{\phi}}{\phi} = -\frac{8\pi}{\phi}(\omega_{de}\rho_{de} + \lambda) \tag{11}$$

$$\left(3\frac{\dot{D}}{D} + \frac{\dot{E}}{E}\right)\dot{\phi} + \ddot{\phi} = \frac{8\pi}{(3+2w)}(\rho - \lambda + (1 - 4\omega_{de})\rho_{de}) \tag{12}$$

and the energy conservation equation (5), leads to

$$\dot{\rho}_{de} + \dot{\rho} + \left(3\frac{\dot{D}}{D} + \frac{\dot{E}}{E}\right)(\rho_{de} + \rho - 4p_{de}) + \frac{\dot{E}}{E}\lambda = 0 \tag{13}$$

where overhead dot denotes ordinary differentiation with respect to time t.

3 Solution of field equations

We can observe that the field equations (9)-(12) is a system of four independent equations with seven unknown parameters D,E p_{de}, ρ_{de}, ρ, λ and ϕ. Hence in order to solve this inconsistent system we need three additional constraints. We consider that shear scalar (σ) is proportional to the expansion scalar (θ). This leads to a relation between the metric potentials (Collins et al. [38]) as

$$E = D^K \tag{14}$$

In literature it is also common to use a power-law relation between Brans-Dicke scalar field ϕ and average scale factor a of the form (Johri and Sudharsan [39]; Johri and Desikan [40]).

$$\phi(t) = \phi_0[a(t)]^l, \ where \quad \phi_0 \neq 0. \tag{15}$$

where ϕ_0 is a constant and l is a power index.

To get the metric potential, we assume that the string tension $\lambda = \lambda_0(k-1)D^{k_1-1}$,where λ_0 is an arbitrary constant. Clearly, for $k = 1$ the model becomes isotropic and also we have $\lambda = 0$. This shows that string does not survive in isotropic model, thus our assumption for is physically valid. Here $k_1 = \frac{(k+3)l}{3}$

From equ.(9),(11) and (14), we get

$$D(t) = \frac{1}{4c_2}[(c_2t + c_2c_3)^2 - 4c_1] \tag{16}$$

$$E(t) = \{\frac{1}{4c_2}[(c_2t + c_2c_3)^2 - 4c_1]\}^{\frac{1}{k}} \tag{17}$$

where $c_2 = 16(\phi_0(2(k + k_1) + 5))^{-1}$ and c_1 and c_2 are arbitrary constants.

From equ.(16) and (17), now the metric (2) can be written as

$$ds^2 = dt^2 - \{\frac{1}{4c_2}[(c_2t + c_2c_3)^2 - 4c_1]\}^2(dx^2 + dy^2 + dz^2)$$
$$-\{\frac{1}{4c_2}[(c_2t + c_2c_3)^2 - 4c_1]\}^{\frac{2}{k}}dm^2 \tag{18}$$

From equ.15,(16) and (17), we get the Scalar field as

$$\phi = \phi_0\{\frac{1}{4c_2}[(c_2t + c_2c_3)^2 - 4c_1]\}^{k_1} \tag{19}$$

We get the sting tension as

$$\lambda = \lambda_0(k-1)\{\frac{1}{4c_2}[(c_2t + c_2c_3)^2 - 4c_1]\}^{k_1-1} \tag{20}$$

4 Some physical properties

Avarage scale factor

$$a(t) = \{\frac{1}{4c_2}[(c_2t + c_2c_3)^2 - 4c_1]\}^{\frac{k+3}{3}} \tag{21}$$

Hubble parmeter

$$H = \frac{2c_2(k+3)(c_2t+c_2c_3)}{3}\{[(c_2t + c_2c_3)^2 - 4c_1]\}^{-1} \tag{22}$$

$$V = a^4 = \{\frac{1}{4c_2}[(c_2t + c_2c_3)^2 - 4c_1]\}^{\frac{4(k+3)}{3}} \tag{23}$$

$$\theta = 4H = 4\frac{\dot{a}}{a} = \frac{8c_2(k+3)(c_2t+c_2c_3)}{3}\{[(c_2t + c_2c_3)^2 - 4c_1]\}^{-1} \tag{24}$$

Decelaration parameter

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = \frac{3}{2(k+3)}(1 + 4(c_2t + c_2c_3)^{-2}c_1) - 1 \tag{25}$$

Density of DE

$$\rho_{de} = \{\frac{2c_2(k+3)}{\sqrt{3}((c_2t+c_2c_3)^2-4c_1)}\}^2\{\zeta(c_2t + c_2c_3)^2 + \eta[c_2\frac{(c_2t+c_2c_3)^2+4c_1}{(c_2t+c_2c_3)^2-4c_1}]^2\} \tag{26}$$

Energy density of matter

$$\rho = \frac{\phi_0\left(\frac{1}{4c_2}((c_2t+c_2c_3)^2-4c_1)\right)^{k_1}(3(k+1)+(k+3-w)k_1)}{8\pi}\{\frac{2c_2[c_2t+c_2c_3]}{[c_2t+c_2c_3]^2-c_1}\}^2$$
$$-\frac{2c_2^2(k+3)}{9}(c_2t + c_2c_3 - 4c_1)^{-2}\{2\zeta(k+3)(c_2t + c_2c_3)^2 - 3\eta c_1\} \tag{27}$$

Pressure of matter

$$p_{de} = \frac{(3+2w)k_1\phi_0(4c_2)^{2-k_1}}{256}\{[(c_2t + c_2c_3)^2 - 4c_1]^{k_1-2}(c_2t + c_2c_3)^2\}$$
$$\{8c_2(3+k) + 2(k_1 - 1) + (c_2t + c_2c_3)^{-1} - 4(c_2t + c_2c_3)^{-1}\} \tag{28}$$

$$\omega_{de} = \{\frac{9(3+2w)k_1\phi_0(4c_2)^{2-k_1}\left((c_2t+c_2c_3)^2-4c_1\right)^{k_1-2}(c_2t+c_2c_3)^2}{512c_2^2(k+3)(c_2t+c_2c_3-4c_1)^{-2}(2\zeta(k+3)(c_2t+c_2c_3)^2-3\eta c_1)}\}$$
$$\{8c_2(3+k) + 2(k_1 - 1) + (c_2t + c_2c_3)^{-1} - 4(c_2t + c_2c_3)^{-1}\} \tag{29}$$

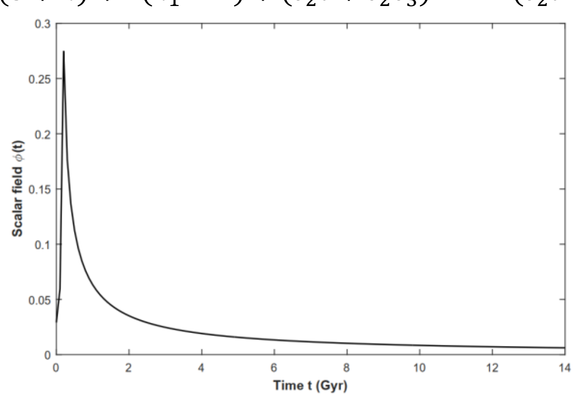

Fig. 1; Plot of scalar field versus cosmic time t for

$\phi_0 = 0.45, c_1 = 0.9, \quad l = -0.35, k = 0.98 \ \ and \ \ c_3 = 0.2.$

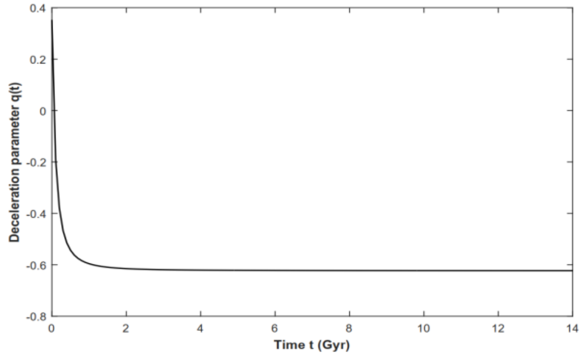

Fig. 2: Plot of deceleration parameter versus cosmic time t for

$\phi_0 = 0.45, c_1 = 0.9, l = -0.35, k = 0.98 \;\; and \;\; c_3 = 0.2.$

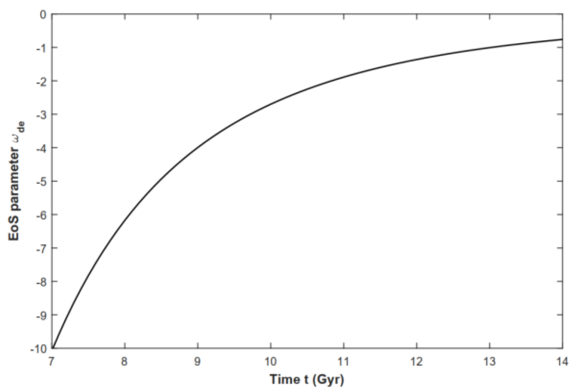

Fig. 3: Plot of EoS parameter versus cosmic time t for
$\phi_0 = 0.45, c_1 = 0.9, l = -0.35, k = 0.98, c_3 = 0.2, w = 100000, \zeta = 0.01 \;\; and \;\; \xi = 0.001.$

In Fig. 1, we have plotted the behaviour for BD scalar field in terms of cosmic time. It can be seen from Fig. 1 that the BD scalar field is positive and decreasing function of cosmic time. This behavior is quite similar to the scalar field and string models in modified theories of gravitation (Aditya and Reddy [30]; Naidu et al. [31]; Raju et al. [32]; Bhaskara Rao et al. [23]).

We study the behavior of deceleration parameter in terms of cosmic time in Fig. 2. It exhibits a signature change from positive to negative. Hence, our model shows a smooth transition from early deceleration era to the present acceleration era of the universe, which agrees with the observations from various schemes.

From Fig. 3, we observe that the model starts from aggressive phantom region $\omega_{de} << -1$ crosses phantom divided line (ω_{de} = -1) and finally attains a constant value at DE-dominated era.

5 Conclusions

There has been a growing interest in the universe's accelerated expansion phenomenon. Many different dynamical DE models and modified/extra-dimensional theories of gravity have been used to explain this occurrence up until this point. The work in this paper is devoted to the discussion of a five dimensional Kaluza-Klein space-time is considered in the framework of a Brans-Dicke scalar-tensor theory of gravitation when the source for energy-momentum tensor is a dark energy in the presence of one dimensional cosmic strings. The physical behavior of the obtained model is discussed at early and late times can be summarized as follows:

The volume of the model allows constant value at $t = 0$ and starts increasing with cosmic time approaching to very large as $t \to \infty$. Thus, the space-time does not show any type of initial singularity in our DE model. The expansion scalar and the mean Hubble parameter are constant at initial epoch indicating homogeneous expansion of the universe and approaches to zero at late times. Since the scalar field decreases with cosmic time the corresponding kinetic energy of the model increases with the passage of time (Fig. 1). The behavior of scalar field is in agreement with the recent scalar field models in modified theories of gravitation explored in the literature (Aditya and Reddy [30]; Naidu et al. [31]; Raju et al. [32]; Bhaskara Rao et al. [23]). The EoS parameter of our DE model may be playing an important role to represent the dark energy universe since our model starts from aggressive phantom region, crosses the phantom devided line and finally approaches a constant value in DE dominated era (Fig. 3). Also, our model exhibits a smooth transition from early deceleration to late time acceleration (Fig. 2). Hence the results obtained in this work provides consistent behavior with the present day observations.

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