ON ROUGH BI-SEMI GENERALIZED LOCALLY CLOSED SETS IN ROUGH SET BITOPOLOGICAL SPACES

J. SHEEBA PRIYADHARSHINI[1], K. BHUVANESWARI[2]

[1]Research Scholar, Department of Mathematics, Mother Teresa women's University, Kodaikanal.
[2]Associate Professor, Department of Mathematics, Mother Teresa women's University, Kodaikanal.

ABSTRACT

The purpose of this paper is to introduce a new class of sets called Rough bi-semi locally closed sets and Rough bi-semi generalized locally closed sets in Rough bitopological space and studied some of its properties.

Keywords: *Rough bitopology, Rough bi-closed Sets, Rough bi-sg closed Sets, Rough bi-SLC Sets, Rough bi-SLC* Sets,, Rough bi-SGLC Sets, Rough bi-SGLC* set, Rough bi-SGLC** set.*

AMS Subject Classification (2010): *54A05*

1. INTRODUCTION

Bourbaki [7] in 1966, defined a subset S of a space (X, τ) to be locally closed if it is the intersection of an open set and a closed set. Balachandran et.al, [2] introduced the concept of semi locally closed set and semi generalized locally closed set in Topology. The notion of Rough set theory was introduced by Pawlak [10] which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it. In 2017 Bhuvaneswari et.al, [5] introduced and studied Nano locally closed sets and Nano generalized locally closed sets in Nano topological spaces. In 1963 Kelly [9] initiated the systematic study of bitopological spaces. Baby Bhattacharya et al., [1] introduced the concept of (1,2)* - locally closed sets in bitopological space. Bhuvaneswari et.al, [6] introduced and studied Rough bitopological space. In this paper a new class of sets called Rough bi-semi locally closed sets and Rough bi-semi generalized locally closed sets are introduced and studied some of its properties.

2. PRELIMINARIES

Definition 2.1[10]: A subset A of a topological space (X,τ) is called a **semi open** set if $A \subseteq cl[Int(A)]$. The complement of a semi open set of a space X is called **semi closed** set in X.

Definition 2.2[3]: A subset A of (X,τ) is called a **semi generalized closed set** (briefly sg-closed) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open in X.

Definition 2.3[2]: A subset S of a space (X,τ), is called **semi locally closed** if $S = U \cap F$, where U is semi open and F is semi closed in (X,τ).

Definition 2.4[2]: A subset S of (X,τ), is called **semi generalized locally closed set** (briefly s*glc*) if $S = G \bigcap F$, where G is s*g*-open in (X,τ) and F is s*g*-closed in (X,τ). Every s*g*-closed set (resp. s*g*-open set) is s*glc*.

Definition 2.5[8]: For a subset S of (X,τ), $S \in SGLC^*(X,\tau)$ if there exist a s*g*-open set G and a closed set F of (X,τ), respectively, such that $S = G \bigcap F$.

Definition 2.6[8]: For a subset S of (X,τ), $S \in SGLC^{**}(X,\tau)$ if there exist a open set G and a s*g*-closed set F of (X,τ), respectively, such that $S = G \bigcap F$.

Definition 2.7[11]: Let U be the universe, R be an equivalence relation on U and the Rough topology $\tau_R(X) = \{U, \phi, \underline{R}(X), \overline{R}(X), BN_R(X)\}$, where $X \subseteq U$ which satisfies the following axioms:

(i). U and $\Phi \in \tau_R(X)$.

(ii).The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.

(iii). The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ is a topology on U called the **Nano/Rough Topology** on U with respect to X. $(U,\tau_R(X))$ is called the Rough topological space. The elements of $\tau_R(X)$ are known as Rough open sets in U.

Definition 2.8[11]: Let $(U,\tau_R(X))$ be a nano topological space and $A \subseteq U$. Then A is said to be

- **Nano semi open** if $A \subseteq Ncl[NInt(A)]$
- **Nano semi closed** if $NInt[Ncl(A)] \subseteq A$

Definition 2.9[11]: A subset A of $(U,\tau_R(X))$ is called **Nano semi generalized closed set** (*Nsg*-closed) if $Nscl(A) \subseteq V$ whenever $A \subseteq V$ and V is Nano semi open in $(U,\tau_R(X))$.

Definition 2.10[6]: Let U be the universe, R be an equivalence relation on U and $\tau_{R_{1,2}}(X) = \bigcup\{\tau_{R_1}(X), \tau_{R_2}(X)\}$ where $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ and $X \subseteq U$. Then $(U,\tau_{R_{1,2}}(X))$ is said to be **Rough/Nano bitopological space**. Elements of the Rough bitopology are known as Rough (1, 2)* open sets in U. Elements of $[\tau_{R_{1,2}}(X)]^c$ are called Rough (1, 2)* closed sets in $\tau_{R_{1,2}}(X)$.

Definition 2.11[6]: A subset A of $(U,\tau_{R_{1,2}}(X))$ is called **nano (1, 2)* semi open** set if

$A \subseteq N\tau_{1,2}cl[N\tau_{1,2}Int(A)]$. The complement of a nano (1, 2)* semi open set of a space U is called **nano (1, 2)* semi closed** set in $(U, \tau_{R_{1,2}}(X))$.

Definition 2.12[6]: A subset A of $(U, \tau_{R_{1,2}}(X))$ is called **Nano (1, 2)* semi generalized** closed set (briefly N (1, 2)*sg-closed) if $N\tau_{1,2}scl(A)) \subseteq V$ whenever $A \subseteq V$ and V is nano (1, 2)* semi open in $(U, \tau_{R_{1,2}}(X))$.

Remark 2.13[6]: Let A be any subset of $(U, \tau_{R_{bi}}(X))$

- *(i).* Let A be an $R_{bi} - sg$ closed set and suppose that F is a R_{bi} -closed set. Then $A \bigcap F$ is an $R_{bi} - sg$ closed set.
- *(ii).* Let A and B be subsets of a Rough bitopological space .Then A is $R_{bi} - sg$ open if it is R_{bi} -open and $A \bigcap B$ is $R_{bi} - sg$ open if both A and B are $R_{bi} - sg$ open.
- *(iii).* A is $R_{bi} - sg$ closed in $(U, \tau_{R_{bi}}(X))$ if and only if $A = R_{bi}sgcl(A)$
- *(iv).* $R_{bi}sgcl(A)$ is $R_{bi} - sg$ closed in $(U, \tau_{R_{bi}}(X))$
- *(v).* $x \in R_{bi}sgcl(A)$ if and only if $A \cap U \neq \phi$ for every $R_{bi} - sg$ open set U containing *x*.

3. ROUGH BI-GENERALIZED LOCALLY CLOSED SETS

Definition 3.1: A subset A of $(U, \tau_{R_{bi}}(X))$, is called **Rough bi- semi locally closed** (briefly R_{bi} -slc) if $A = G \bigcap F$, where G is R_{bi} -semi open and F is R_{bi} -semi closed in $(U, \tau_{R_{bi}}(X))$. The collection of all R_{bi} - semi locally closed sets of $(U, \tau_{R_{bi}}(X))$ will be denoted by $R_{bi}SLC(U, \tau_{R_{bi}}(X))$.

Definition 3.2: A subset A of $(U, \tau_{R_{bi}}(X))$, is called **Rough bi- semi locally* closed** (briefly R_{bi} -slc*) set, if there exist a R_{bi} -semi open set G and a R_{bi} - closed set F of $(U, \tau_{R_{bi}}(X))$, respectively such that $A = G \bigcap F$. The collection of all R_{bi} - semi locally closed sets of $(U, \tau_{R_{bi}}(X))$ will be denoted by $R_{bi}SLC*(U, \tau_{R_{bi}}(X))$.

Remark 3.3:

- *(i).* Every R_{bi} -semi open (resp. R_{bi} -semi closed) subset of U is R_{bi} -semi locally closed.
- *(ii).* The complement of a R_{bi} -semi locally closed set need not be R_{bi} -semi locally closed.
- *(iii).* Every R_{bi} -slc* set is R_{bi} -slc set
- *(iv).* If every subset of $(U, \tau_{R_{bi}}(X))$ is R_{bi} -locally closed, then it is R_{bi} -slc*

Theorem 3.4: For a subset A of $(U, \tau_{R_{bi}}(X))$, the following are equivalent

(i). $A \in R_{bi}SLC(U, \tau_{R_{bi}}(X))$

(ii). $A = P \bigcap (R_{bi}scl(A))$ for some R_{bi} -semi open set P

(iii). $R_{bi}scl(A) - A$ is R_{bi} -semi closed

(iv). $A \bigcup (U - R_{bi}scl(A))$ is R_{bi} -semi open

Proof: $(i) \Rightarrow (ii)$ Let $A \in R_{bi}SLC(U, \tau_{R_{bi}}(X))$. Then $A = P \bigcap F$ where P is R_{bi} -semi open and F is R_{bi} -semi closed. Since $A \subseteq P$ and $A \subseteq R_{bi}scl(A)$, $A \subseteq P \bigcap R_{bi}scl(A)$. Conversely, since $A \subseteq F$, $A \subseteq R_{bi}scl(A) \subseteq F$, $A = P \bigcap F \supseteq P \bigcap R_{bi}scl(A)$. That is $P \bigcap R_{bi}scl(A) \subseteq A$. Therefore $A = P \bigcap (R_{bi}scl(A))$

$(ii) \Rightarrow (i)$ Since P is R_{bi} -semi open and $R_{bi}scl(A)$ is R_{bi} -semi closed, $P \bigcap R_{bi}scl(A) \in R_{bi}SLC(U, \tau_{R_{bi}}(X))$ by Definition 3.1.

$(ii) \Rightarrow (iii)$ $A = P \bigcap (R_{bi}scl(A))$ implies that $R_{bi}scl(A) - A = R_{bi}scl(A) - (U - P) = R_{bi}scl(A) - P^C$ which is R_{bi} -semi closed, since P^C is R_{bi} -semi closed.

$(iii) \Rightarrow (ii)$ Let $P = (R_{bi}scl(A) - A)^C$. Then by assumption, P is R_{bi} -semi open in $(U, \tau_{R_{bi}}(X))$ and $A = P \bigcap (R_{bi}scl(A))$.

$(iii) \Rightarrow (iv)$ $A \bigcup (U - R_{bi}scl(A)) = A \bigcup (R_{bi}scl(A))^C = (R_{bi}scl(A) - A)^C$ and by assumption $(R_{bi}scl(A) - A)^C$ is R_{bi} -semi open and $A \bigcup (U - R_{bi}scl(A))$ is R_{bi} -semi open.

$(iv) \Rightarrow (iii)$ Let $P = A \bigcup (R_{bi}scl(A))^C$. Then P^C is R_{bi} -semi closed and $P^C = R_{bi}scl(A) - A$ and therefore $R_{bi}scl(A) - A$ is R_{bi} -semi closed.

Theorem 3.5: If $A \in (U, \tau_{R_{bi}}(X))$ is a R_{bi} -locally closed set, then A is R_{bi} -semi locally closed.

Proof: Let A be a R_{bi} -locally closed set. Then by definition of R_{bi} -locally closed set, A is the intersection of R_{bi} -open and R_{bi} -closed set. Since every R_{bi} -open set is R_{bi} -semi open and every R_{bi} -closed set is R_{bi} - semi closed, hence A is R_{bi} -semi locally closed set.

Example 3.6: Let $U = \{a, b, c, d\}$ with $U / R(X_1) = \{\{a\}, \{b\}, \{c, d\}\}$, $U / R(X_2) = \{\{b\}, \{d\}, \{a, c\}\}$ and $X_1 = \{a, c\}$, $X_2 = \{a, d\}$. Then $\tau_{R_{bi}}(X) = \{U, \phi, \{a\}, \{d\}, \{c, d\}, \{a, c\}, \{a, c, d\}\}$. The set $\{a, d\}$ is an R_{bi} -slc set but not R_{bi} -lc set.

Theorem 3.7: If $A \in (U, \tau_{R_{bi}}(X))$ is a R_{bi} -semi locally closed set and B is R_{bi} -semi open in $(U, \tau_{R_{bi}}(X))$, then $A \bigcap B$ is R_{bi} -semi locally closed.

Proof: Let A be a R_{bi} -semi locally closed set. Then by definition of R_{bi} -semi locally closed set, A is the intersection of R_{bi} -semi open and R_{bi} -semi closed set such that $A = G \cap F$. Now $A \cap B = (G \cap F) \cap B = (G \cap B) \cap F$, since $G \cap B$ is R_{bi} -semi open. Hence $A \cap B$ is R_{bi} -semi locally closed set in $(U, \tau_{R_{bi}}(X))$.

Definition 3.8: A subset A of $(U, \tau_{R_{bi}}(X))$, is called **Rough bi-semi generalized locally closed set** (briefly $R_{bi} - sglc$) if $A = G \cap F$, where G is $R_{bi} - sg$ open $(U, \tau_{R_{bi}}(X))$ and F is $R_{bi} - sg$ closed in $(U, \tau_{R_{bi}}(X))$.

The collection of all R_{bi} -semi generalized locally closed sets of $(U, \tau_{R_{bi}}(X))$ is denoted by $R_{bi}SGLC(U, \tau_{R_{bi}}(X))$.

Theorem 3.9: If $A \in (U, \tau_{R_{bi}}(X))$ is a R_{bi} -locally closed set, then A is $R_{bi} - sg$ locally closed set but not conversely.

Proof: Let A be a R_{bi} -locally closed set. Then by definition of R_{bi} -locally closed set, A is the intersection of R_{bi} -open and R_{bi} -closed set. Since every R_{bi} -open set is $R_{bi} - sg$ open and every R_{bi} -closed set is $R_{bi} - sg$ closed, A is $R_{bi} - sg$ locally closed set.

Example 3.10: In the example 3.7 $R_{bi} - sglc$ sets are *P(U)* and hence the converse does not hold.

Theorem 3.11: If $A \in (U, \tau_{R_{bi}}(X))$ is a R_{bi} -semi locally closed set, then A is $R_{bi} - sg$ locally closed set but not conversely.

Proof: Let A be a R_{bi} -semi locally closed set. Then by definition of R_{bi} -semi locally closed set, A is the intersection of R_{bi} -semi open and R_{bi} -semi closed set. Since every R_{bi} -semi open set is $R_{bi} - sg$ open and every R_{bi} -semi closed set is $R_{bi} - sg$ closed, hence A is $R_{bi} - sg$ locally closed set.

Example 3.12: Let $U = \{a, b, c, d\}$ with $U / R(X_1) = \{\{a\}, \{d\}, \{b, c\}\}$, $U / R(X_2) = \{\{a\}, \{c\}, \{b, d\}\}$ and $X_1 = \{a, c\}$, $X_2 = \{c, d\}$. Then $\tau_{R_{bi}}(X) = \{U, \phi, \{a\}, \{c\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{b, c, d\}\}$. The set $\{c, d\}$ is an R_{bi} -sglc set but not R_{bi} -slc set.

Hence

R_{bi}-lc $\rightarrow$ / $\not\leftarrow$ R_{bi}-slc $\rightarrow$ / $\not\leftarrow$ R_{bi}-sglc

The following two collections of subsets of $(U, \tau_{R_{bi}}(X))$ that is, $R_{bi}SGLC^*(U, \tau_{R_{bi}}(X))$ and $R_{bi}SGLC^{**}(U, \tau_{R_{bi}}(X))$ are defined as follows.

Definition 3.13: A subset A of $(U,\tau_{R_{bi}}(X))$, is called **Rough bi-semi generalized locally* closed set**, (briefly R_{bi} -sglc*) if there exist an $R_{bi}-sg$ open set G and a R_{bi} -closed set F of $(U,\tau_{R_{bi}}(X))$ respectively, such that $A=G\bigcap F$.

Definition 3.14: A subset A of $(U,\tau_{R_{bi}}(X))$, is called **Rough bi-semi generalized locally** closed set**, if there exist an R_{bi} -open set G and a $R_{bi}-sg$ closed set F of $(U,\tau_{R_{bi}}(X))$, respectively, such that $A=G\bigcap F$.

Remark 3.15:

(i). If A is a $R_{bi}-sg$ closed set of $(U,\tau_{R_{bi}}(X))$ then A is $R_{bi}-sglc$ set.

(ii). Every R_{bi} -sg open (resp.- R_{bi} -sg closed) subset of U is R_{bi} -sg locally closed.

(iii). Every R_{bi} -sglc* set is R_{bi} -sglc set and every R_{bi} -sglc** set is R_{bi} -sglc set

(iv). If every subset of $(U,\tau_{R_{bi}}(X))$ is R_{bi} -locally closed, then it is R_{bi} -sglc* and R_{bi} -sglc** in $(U,\tau_{R_{bi}}(X))$.

The following results are characterizations of $R_{bi}SGLC$ set, $R_{bi}SGLC*$ set and $R_{bi}SGLC**$ set.

Theorem 3.16: For a subset A of $(U,\tau_{R_{bi}}(X))$, the following statements are equivalent.

(i). $A\in R_{bi}SGLC(U,\tau_{R_{bi}}(X))$

(ii). $A=P\bigcap(R_{bi}sgcl(A))$ for some $R_{bi}-sg$ open set P

(iii). $R_{bi}sgcl(A)-A$ is $R_{bi}-sg$ closed

(iv). $A\bigcup(R_{bi}sgcl(A))^C$ is $R_{bi}-sg$ open.

(v). $A\subseteq R_{bi}sg\operatorname{int}(A\bigcup(R_{bi}sgcl(A))^C)$

Proof: $(i)\Rightarrow(ii)$ Let $A\in R_{bi}SGLC(U,\tau_{R_{bi}}(X))$. Then $A=P\bigcap F$ where P is $R_{bi}-sg$ open and F is $R_{bi}-sg$ closed. Since $A\subseteq P$ and $A\subseteq R_{bi}sgcl(A)$ implies $A\subseteq P\bigcap R_{bi}sgcl(A)$ conversely, by Remark 2.13*(iv)* $R_{bi}sgcl(A)\subseteq F$ and hence $P\bigcap R_{bi}sgcl(A)\subseteq P\cap F=A$. Therefore, $A=P\bigcap(R_{bi}sgcl(A))$.

$(ii)\Rightarrow(iii)$ $A=P\bigcap(R_{bi}sgcl(A))$ implies $R_{bi}sgcl(A)-A=R_{bi}sgcl(A)\bigcap P^C$ which is $R_{bi}-sg$ closed since P^C is $R_{bi}-sg$ closed.

$(iii)\Rightarrow(iv)$ $A\bigcup(R_{bi}sgcl(A))^C=(R_{bi}sgcl(A)-A)^C$ and by assumption $(R_{bi}sgcl(A)-A)^C$ is $R_{bi}-sg$ open and therefore $A\bigcup(R_{bi}sgcl(A))^C$.

$(iv)\Rightarrow(v)$ By assumption, $A\bigcup(R_{bi}sgcl(A))^C = R_{bi}sg\operatorname{int}(A\bigcup(R_{bi}sgcl(A))^C)$ and hence $A\subseteq R_{bi}sg\operatorname{int}(A\bigcup(R_{bi}sgcl(A))^C)$.

$(v)\Rightarrow(i)$ By assumption and since $A\subseteq R_{bi}sgcl(A)$, $A=R_{bi}sg\operatorname{int}(A\bigcup(R_{bi}sgcl(A))^C)\bigcap R_{bi}sgcl(A)$ $\in R_{bi}SGLC(U,\tau_{R_{bi}}(X))$.

Theorem 3.17: For a subset A of $(U,\tau_{R_{bi}}(X))$, the following statements are equivalent.

(i). $A\in R_{bi}SGLC^*(U,\tau_{R_{bi}}(X))$

(ii). $A=P\bigcap(R_{bi}cl(A))$ for some $R_{bi}-sg$ open set P

(iii). $R_{bi}cl(A)-A$ is $R_{bi}-sg$ closed

(iv). $A\bigcup(U-R_{bi}cl(A))$ is $R_{bi}-sg$ open.

Proof : $(i)\Rightarrow(ii)$ Let $A\in R_{bi}SGLC^*(U,\tau_{R_{bi}}(X))$. Then $A=P\bigcap F$ where P is $R_{bi}-sg$ open and F is R_{bi} -closed. Since $A\subseteq P$ and $A\subseteq R_{bi}cl(A)$, $A\subseteq P\bigcap R_{bi}cl(A)$. Conversely, since $A\subseteq F$, $R_{bi}cl(A)\subseteq F$, $A=P\bigcap F\supseteq P\bigcap R_{bi}cl(A)$. That is $P\bigcap R_{bi}cl(A)\subseteq A$. Therefore $A=P\bigcap(R_{bi}cl(A))$
$(ii)\Rightarrow(i)$ Since P is $R_{bi}-sg$ open and $R_{bi}cl(A)$ is R_{bi} -closed, $P\bigcap R_{bi}cl(A)\in R_{bi}SGLC^*(U,\tau_{R_{bi}}(X))$ by Definition of $R_{bi}SGLC^*(U,\tau_{R_{bi}}(X))$

$(ii)\Rightarrow(iii)$ $A=P\bigcap(R_{bi}cl(A))$ implies that $R_{bi}cl(A)-A=R_{bi}cl(A)\cap P^C$ which is $R_{bi}-sg$ closed, since P^C is $R_{bi}-sg$ closed.
$(iii)\Rightarrow(ii)$ Let $P=(R_{bi}cl(A)-A)^C$. Then by assumption, P is $R_{bi}-sg$ open in $(U,\tau_{R_{bi}}(X))$ and $A=P\bigcap(R_{bi}cl(A))$.

$(iii)\Rightarrow(iv)$ $A\bigcup(U-R_{bi}cl(A))=A\bigcup(R_{bi}cl(A))^C=(R_{bi}cl(A)-A)^C$ and by assumption $(R_{bi}cl(A)-A)^C$ is $R_{bi}-sg$ open and $A\bigcup(U-R_{bi}cl(A))$ is $R_{bi}-sg$ open.
$(iv)\Rightarrow(iii)$ Let $P=A\bigcup(R_{bi}cl(A))^C$. Then P^C is $R_{bi}-sg$ closed and $P^C=R_{bi}cl(A)-A$ and therefore $R_{bi}cl(A)-A$ is $R_{bi}-sg$ closed.

Theorem 3.18: Let A be a subset of $(U,\tau_{R_{bi}}(X))$. Then $A\in R_{bi}SGLC^{**}(U,\tau_{R_{bi}}(X))$ if and only if $A=P\bigcap(R_{bi}sgcl(A))$ for some R_{bi} -open set P.

Proof: Let $A\in R_{bi}SGLC^{**}(U,\tau_{R_{bi}}(X))$. Then $A=P\bigcap F$ where P is R_{bi} -open and F is $R_{bi}-sg$ closed. Since $A\subseteq F$, $R_{bi}sgcl(A)\subseteq F$. Now $A=A\bigcap R_{bi}sgcl(A)=P\bigcap F\bigcap R_{bi}sgcl(A)=$ $P\bigcap R_{bi}sgcl(A)$.

Corollary 3.19: Let A be a subset of $(U, \tau_{R_{bi}}(X))$. If $A \in R_{bi}SGLC^{**}(U, \tau_{R_{bi}}(X))$, then $R_{bi}sgcl(A) - A$ is $R_{bi} - sg$ closed and $A \bigcup (R_{bi}sgcl(A))^C$ is $R_{bi} - sg$ open.

Proof: Let $A \in R_{bi}SGLC^{**}(U, \tau_{R_{bi}}(X))$ .Then by Theorem 3.12, $A = P \bigcap R_{bi}sgcl(A)$ for some R_{bi} -open set P and $R_{bi}sgcl(A) - A = R_{bi}sgcl(A) \bigcap P^C$ is $R_{bi} - sg$ closed in $(U, \tau_{R_{bi}}(X))$. If $F = R_{bi}sgcl(A) - A$, then $F^C = A \bigcup (R_{bi}sgcl(A))^C$ and F^C is $R_{bi} - sg$ open and therefore $A \bigcup (R_{bi}sgcl(A))^C$ is $R_{bi} - sg$ open.

Theorem 3.20: Let A and B be subsets of $(U, \tau_{R_{bi}}(X))$.

(i). If $A \in R_{bi}SGLC^*(U, \tau_{R_{bi}}(X))$ and $B \in R_{bi}SGLC^*(U, \tau_{R_{bi}}(X))$, then $A \bigcap B \in R_{bi}SGLC^*(U, \tau_{R_{bi}}(X))$

(ii). If $A \in R_{bi}SGLC^{**}(U, \tau_{R_{bi}}(X))$ and B is R_{bi} -closed or R_{bi} -open , then $A \bigcap B \in R_{bi}SGLC^{**}(U, \tau_{R_{bi}}(X))$

(iii). If $A \in R_{bi}SGLC(U, \tau_{R_{bi}}(X))$ and B is $R_{bi} - sg$ open or R_{bi} -closed, then $A \bigcap B \in R_{bi}SGLC(U, \tau_{R_{bi}}(X))$

Proof: *(i)* It follows from the Theorem 3.19 *(ii)*, that there exists $R_{bi} - sg$ open sets P and Q such that $A = P \bigcap R_{bi}cl(A)$ and $B = Q \bigcap R_{bi}cl(B)$. Then $A \bigcap B \in R_{bi}SGLC^*(U, \tau_{R_{bi}}(X))$ since $P \bigcap Q$ is $R_{bi} - sg$ open and $R_{bi}cl(A) \bigcap R_{bi}cl(B)$ is R_{bi} -closed.

(ii) From definition of $R_{bi}SGLC^{**}(U, \tau_{R_{bi}}(X))$, there exist a R_{bi} -open set G and $R_{bi} - sg$ closed set F such that $A \bigcap B = G \bigcap F \bigcap B$. Suppose that B is R_{bi} -open. Then $A \bigcap B \in R_{bi}SGLC^{**}(U, \tau_{R_{bi}}(X))$ next, suppose that B is R_{bi} -closed. Then by the Remark 2.13 (ii), it is proved that $F \bigcap B$ is $R_{bi} - sg$ closed and therefore $A \bigcap B \in R_{bi}SGLC^{**}(U, \tau_{R_{bi}}(X))$

(iii) From definition, there exist a $R_{bi} - sg$ open set G and $R_{bi} - sg$ closed set F such that $A \bigcap B = G \bigcap F \bigcap B$. First suppose that B is $R_{bi} - sg$ open. Then by Remark 2.14 (ii), $G \bigcap B$ is $R_{bi} - sg$ open. So $A \bigcap B \in R_{bi}SGLC(U, \tau_{R_{bi}}(X))$.

Next suppose that B is R_{bi} -closed. Then by Remark 2.14 (i), it is proved that $F \bigcap B$ is $R_{bi} - sg$ closed and therefore $A \bigcap B \in R_{bi}SGLC(U, \tau_{R_{bi}}(X))$.

REFERENCES

[1]. Baby Bhattacharya, Arnab Paul and Sudip Debnath, Some Properties of (1,2)*-Locally Closed Sets, Hindawi Publishing Corporation International Journal of Analysis, Vol.2014.

[2]. Balachandran.K, Gnanambal.Y and Sundaram.P, Far East J. Math. Sci. Special Volume Part II 1997, Pp:189-200.

[3]. Bhattacharyya.P. and Lahiri.B.K., (1987), "Semi-generalized Closed Sets in Topology", Indian J. Pure Appl. Math., Vol.29, Pp: 375 – 382.

[4]. Bhuvaneswari.K and Ezhilarasi.A., (2014), "On Nano Semi-Generalized and Nano Generalized-Semi Closed Sets in Nano Topological Spaces" International Journal of Mathematics and Computer Applications Research (IJMCAR), Vol.4, Issue 3, Pp: 117-124.

[5]. Bhuvaneswari.K and Mythili Gnanapriya.K, Nano Generalized Locally Closed Sets and NGLC-Continuous Functions in Nano Topological Spaces, International Journal of Mathematics and its Applications, Vol.4, Issue 1-A (2116), 101-106.

[6]. Bhuvaneswari.K, and Sheeba Priyadharshini.J, On Nano (1,2)* Semi-Generalized Closed Sets in Nano Bitopological Spaces, International Journal of Applied Mathematics and Statistical Sciences, Vol. 5, Issue 3, Apr - May 2016, Pp:19-30.

[7]. Bourbaki.N, General Topology, Part I, Addison-Wesley, Reading, Mass 1966.

[8]. Jin Han Park and Jin Keun Park, On Semi Generalized Locally Closed Sets and SGLC-Continuous Functions, Indian J. Pure appl. Math., 31(9):1103-1112, September 2000.

[9]. Kelly.J.C, Bitopological spaces, Proceedings of the London Mathematical Society, Vol. 13, Pp:71–89, 1963.

[10]. Levine.N (1963), "Semi-open Sets and Semi-continuity in Topological Spaces", Amer. Math. Monthly, 70(1), Pp: 36 - 41.

[11]. LellisThivagar.M and Carmel Richard, On Nano forms of weakly open sets, International Journal of Mathematics and Statistics Invention, 1(1)(2013), Pp:31-37.

[12]. Zdzistaw Pawlak, Rough Sets, International Journal of Computer and Information Sciences, Vol. 11, No. 5, (1982), Pp: 341-356.