SLIP EFFECTS ON MHD FREE CONVECTIVE FLOW OF A FLUID IN AN INCLINED CHANNEL

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Abstract :

In this research paper, analysis the flow and heat transfer aspects of conducting fluid in an inclined channel with constant a pressure gradient. The governing equations are solved analytically valid for small Prandtl number(Pr) and Eckert number (Ec). The expressions for the velocity and temperature are obtained analytically. The effects of various physical parameters on the velocity and the temperature are discussed in detail.

Keywords : *Magnetohydrodynamics, Heat transfer, Inclined Channel, Pressure gradient.*

1. Introduction

Channels are frequently used in various applications in designing ventilating and heating of buildings, cooling electronic components, drying several types of agriculture products grain and food, and packed bed thermal storage. Convective flows in channels driven by temperature differences of bounding walls have been studied and reported, extensively in literature. Free convection flows in vertical slots were discussed by Aung et al. (1972), Burch et al. (1985), Kim et al. (1990), Buhler (2003), Weidman (2006), Magyari (2007), Weidman and Medina (2008).

Past few decades, the study of magnetohydrodynamics flow of electrically conducting fluids in electric and magnetic fields are of considerable interest in modern metallurgical and metal working process. The Hartmann flow is a classical problem that has important applications in MHD power generators and pumps, accelerators, aerodynamic heating, electrostatic precipitation, polymer technology, the petroleum industry, purification of crude oil and design of various heat exchangers. Especially the flow of non-Newtonian fluids in channels is encountered in various engineering applications. For example, injection molding of plastic parts involves the flow of polymers inside channels. During the last few years the industrial importance of non-Newtonian fluids is widely known. Such fluids in the presence of a magnetic field have applications in the electromagnetic propulsion, the flow of nuclear fuel slurries and the flows of liquid state metals and alloys. Sarparkaya (1961) have presented the first study for MHD Bingham plastic and power law fluids. The effect of transverse magnetic field in physiological type of flow, through a uniform circular pipe was studied by Ramachandra Rao and Deshikachar (1988). Garandet et al. (1992) have discussed buoyancy driven convection in a rectangular enclosure with a transverse magnetic field. Chamkha (1999) have analyzed free convection effects on three-dimensional flow over a vertical stretching surface in the presence of a magnetic field. Malshetty et al. (2001) have studied the effects of heat transfer and MHD on the convective flow of two fluids in an inclined channel. Umapathi et al. (2010) have studied the effect of MHD on the convective couette flow of two fluids in an inclined channel.

In many applications the flow pattern corresponds to a slip flow, the fluid presents a loss of adhesion at the wetted wall making the fluid slide along the wall. When the molecular mean free path length of the fluid is comparable to the distance between the plates as in nano channels or micro channels, the fluid exhibits non-continuum effects such as slip-flow was demonstrated experimentally by Derek et al. (2002). Beavers and Joseph (1967) were the first to investigate the fluid flow at the interface between a porous medium and fluid layer in an experimental study and proposed a slip boundary conditions at the interface.

In view of these, we investigated the MHD free convective flow of a Newtonian fluid in an inclined channel with the effect of slip. The expressions for the velocity and temperature are obtained analytically. The effects of various physical parameters on the velocity and the temperature are discussed in detail.

2. Mathematical formulation

The physical configuration (Fig. 1) consists of two infinite inclined parallel plates extending in the z and x -directions, making an angle ϕ with the horizontal. The two plates are maintained at different constant temperatures T_1 and T_2 . A uniform magnetic field B_0 is applied normal to the plates. It is assumed that the magnetic Reynolds number is sufficiently small so that the induced magnetic field can be neglected, and the induced electric field is assumed to be negligible. The flow is assumed to be unidirectional, steady, laminar and fully developed. Under these assumptions, the governing equations of motion and energy are:

$$\mu\frac{d^2u}{dy^2}-\sigma B_0^2u+\rho g\beta\left(T-T_2\right)\sin\alpha=\frac{dp}{dx} \tag{2.1}$$

$$\frac{d^2T}{dy^2}+\frac{\mu}{k}\left(\frac{du}{dy}\right)^2+\frac{\sigma B_0^2}{k}u^2=0 \tag{2.2}$$

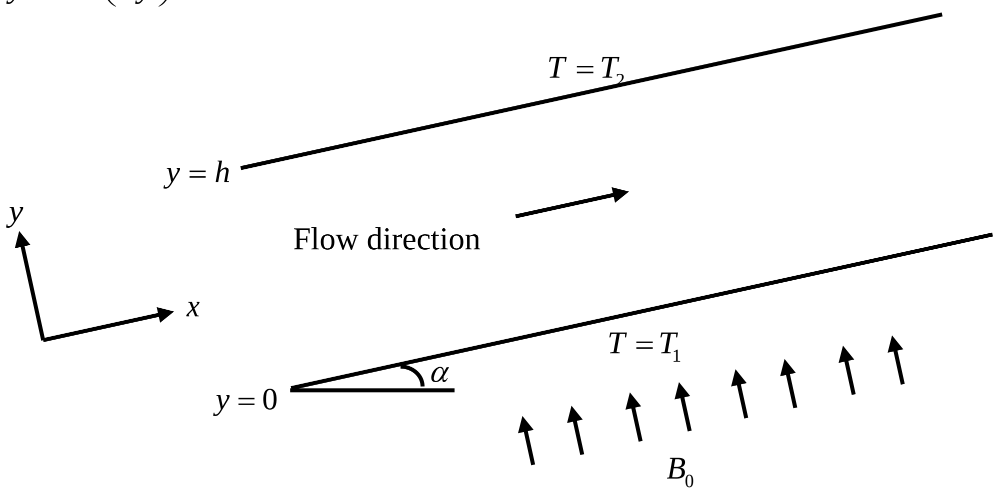

Fig. 1 The Physical model

The boundary conditions are

$$u=\gamma\frac{du}{dy}\ ,\quad T=T_1\quad \text{at}\quad y=0 \tag{2.3}$$

$$u=0\ ,\quad T=T_2\quad \text{at}\quad y=h \tag{2.4}$$

Introducing the following non-dimensional variables

$$\bar{u}=\frac{u}{U},\ \bar{y}=\frac{y}{h},\ \bar{\theta}=\frac{T-T_2}{T_1-T_2},\ \bar{\gamma}=\frac{\gamma}{h} \tag{2.5}$$

into Eqs. (2.1) and (2.2), we get (after dropping the bars)

$$\frac{d^2u}{dy^2}-M^2u=P-\left(\frac{Gr}{\mathrm{Re}}\sin\alpha\right)\theta \tag{2.6}$$

$$\frac{d^2\theta}{dy^2} + \Pr Ec\left(\frac{du}{dy}\right)^2 + \left(M^2 \Pr Ec\right)u^2 = 0 \quad (2.7)$$

here $P = \frac{h^2}{\mu U}\frac{dp}{dx}$, $M = B_0 h\sqrt{\frac{\sigma}{\mu}}$ is the Hartmann number, $Gr = \frac{g\beta h^3\left(T_1 - T_2\right)}{\nu^2}$ is the Grashof number, $\mathrm{Re} = \frac{hU}{\nu}$ is the Reynolds number, $\Pr = \frac{\mu c_p}{k}$ is the Prandtl number, $Ec = \frac{U^2}{c_p\left(T_1 - T_2\right)}$ is the Eckert number.

The corresponding dimensionless boundary conditions

$$u = \gamma\frac{du}{dy} \ , \quad \theta = 1 \quad \text{at} \quad y = 0 \quad (2.8)$$

$$u = 0 \ , \quad \theta = 0 \quad \text{at} \quad y = 1 \quad (2.9)$$

3. Perturbation Solution

Eq. (2.7) is non-linear and it is difficult to get a closed form solution. However for vanishing $\varepsilon\left(= Ec\Pr\right)$, the boundary value problem is agreeable to an easy analytical solution. In this case the equation becomes linear and can be solved. Nevertheless, small ε suggests the use of perturbation technique to solve the non-linear problem. Accordingly, we write

$$u = u_0 + \varepsilon u_1 + \mathrm{O}\left(\varepsilon^2\right) \quad (3.1)$$

$$\theta = \theta_0 + \varepsilon\theta_1 + \mathrm{O}\left(\varepsilon^2\right) \quad (3.2)$$

Substituting equations (3.1) and (3.2) into Eqs. (2.6) and (2.7) and boundary conditions (2.8) and (2.9) and then equating the like powers of ε, we obtain

3.1 System of order zero

$$\frac{d^2u_0}{dy^2} - M^2u_0 = P - \left(\frac{Gr}{\mathrm{Re}}\sin\alpha\right)\theta_0 \quad (3.3)$$

$$\frac{d^2\theta_0}{dy^2} = 0 \quad (3.4)$$

Together with the boundary conditions

$$u_0 = \gamma\frac{du_0}{dy} \ , \quad \theta_0 = 1 \quad \text{at} \quad y = 0 \quad (3.5)$$

$$u_0 = 0 \ , \quad \theta_0 = 0 \quad \text{at} \quad y = 1 \quad (3.6)$$

3.2 System of order one

$$\frac{d^2u_1}{dy^2} - M^2u_1 = -\left(\frac{Gr}{\mathrm{Re}}\sin\alpha\right)\theta_1 \quad (3.7)$$

$$\frac{d^2\theta_1}{dy^2} + \left(\frac{du_0}{dy}\right)^2 + M^2u_0^2 = 0 \quad (3.8)$$

Together with the boundary conditions

$$u_1 = \gamma\frac{du_1}{dy} \ , \quad \theta_1 = 0 \quad \text{at} \quad y = 0 \quad (3.9)$$

$$u_1 = 0 \ , \quad \theta_1 = 0 \quad \text{at} \quad y = 1 \quad (3.10)$$

3.3 Solution of order zero

Solving Eqs. (3.3) and (3.4) using the boundary conditions (3.5) and (3.6), we get

$$\theta_0 = 1 - y \tag{3.11}$$

$$u_0 = c_1 \cosh My + c_2 \sinh My - \frac{P}{M^2} + \frac{Gr \sin\alpha}{\mathrm{Re}\, M^2}(1-y) \tag{3.12}$$

where $c_1 = \dfrac{\gamma P - MA_1 \sinh M}{M\left(\sinh M + \gamma M \cosh M\right)}$, $c_2 = \dfrac{M^2 A_1 \cosh M + P}{M^2\left(\sinh M + \gamma M \cosh M\right)}$ and

$A_1 = -\dfrac{P}{M^2} + \dfrac{Gr \sin\alpha}{\mathrm{Re}\, M^2}(1+\gamma)$.

3.4 Solution of order one

Solving Eqs. (3.7) and (3.8) using the boundary conditions (3.9) and (3.10), we get

$$\theta_1 = \begin{pmatrix} c_4 + c_3 y + A_{13} y^2 + A_{14} y^3 - A_{15} y^4 - A_{16} \cosh 2My - A_{17} \sinh 2My \\ + A_{18} \sinh My + A_{19} \cosh My + A_{20} y \cosh My + A_{21} y \sinh My \end{pmatrix} \tag{3.13}$$

$$u_1 = \left\{ \begin{array}{l} c_5 \cosh My + c_6 \sinh My \\ -\dfrac{Gr}{\mathrm{Re}} \sin\alpha \begin{pmatrix} -\dfrac{1}{M^2}\left[A_{22} + A_{23} y + A_{24} y^2 + A_{14} y^3 - A_{15} y^4\right] \\ -\dfrac{A_{16}}{3M^2} \cosh 2My - \dfrac{A_{17}}{3M^2} \sinh 2My + \dfrac{A_{18}}{2M} y \cosh My \\ + \dfrac{A_{19}}{2M} y \sinh My + A_{20}\left[\dfrac{y^2}{4M} \sinh My - \dfrac{y}{4M^2} \cosh My\right] \\ + A_{21}\left[\dfrac{y^2}{4M} \cosh My - \dfrac{y}{4M^2} \sinh My\right] \end{pmatrix} \end{array} \right\} \tag{3.14}$$

where $A_2 = \dfrac{Gr}{\mathrm{Re}} \dfrac{\sin\alpha}{M^2}$, $A_3 = \dfrac{P^2}{M^4} + A_2^2$, $A_4 = M^2\left(c_1^2 + c_2^2\right)$, $A_5 = 2c_1 c_2 M^2$,

$A_6 = 2\left(c_1 M A_2^2 + P c_2\right)$, $A_7 = 2\left(c_2 M A_2^2 + P c_1\right)$, $A_8 = A_2^2 M^2$, $A_9 = 2PA_2$, $A_{10} = 2c_1 A_2 M^2$,

$A_{11} = 2A_2 c_2$, $A_{12} = \begin{pmatrix} -\dfrac{A_3}{2} - \dfrac{A_4 \cosh 2M}{4M^2} - \dfrac{A_5 \sinh 2M}{4M^2} + \dfrac{A_6 \sinh M}{M^2} + \dfrac{A_7 \cosh M}{M^2} \\ -\dfrac{A_8}{4} + \dfrac{A_9}{3} - 2A_{10} \dfrac{\sinh M}{M^3} - 2A_{11} \dfrac{\cosh M}{M^3} \end{pmatrix}$,

$A_{13} = \dfrac{-A_3 - A_8 + A_9}{2}$, $A_{14} = \dfrac{A_8}{3} - \dfrac{A_9}{6}$, $A_{15} = \dfrac{A_8}{12}$, $A_{16} = \dfrac{A_4}{4M^2}$, $A_{17} = \dfrac{A_5}{4M^2}$,

$A_{18} = \dfrac{A_6}{M^2} - \dfrac{2A_{10}}{M^3} - \dfrac{A_{11}}{M^2}$, $A_{19} = \dfrac{A_7}{M^2} - \dfrac{A_{10}}{M^2} - \dfrac{2A_{11}}{M^3}$, $A_{20} = \dfrac{A_{10}}{M^2}$, $A_{21} = \dfrac{A_{11}}{M^2}$,

$A_{22} = c_4 + \dfrac{2A_{13}}{M^2} - \dfrac{24A_{15}}{M^4}$, $A_{23} = c_3 + 6\dfrac{A_{14}}{M^2}$, $A_{24} = A_{13} - 12\dfrac{A_{15}}{M^2}$,

$A_{25} = \dfrac{Gr}{\mathrm{Re}} \sin\alpha \left(\dfrac{A_{22}}{M^2} + \dfrac{A_{16}}{3M^2} + \gamma\left[-\dfrac{A_{23}}{M^2} - \dfrac{2A_{17}}{3M} + \dfrac{A_{18}}{2M} - \dfrac{A_{20}}{4M^2}\right]\right)$,

$$A_{26} = \frac{Gr}{\mathrm{Re}}\sin\alpha \left\{ \begin{array}{l} -\frac{1}{M^2}\left(A_{22}+A_{23}+A_{24}+A_{14}-A_{15}\right)-\frac{A_{16}}{3M^2}\cosh 2M \\ -\frac{A_{17}}{3M^2}\sinh 2M+\left(\frac{A_{18}}{2M}+\frac{A_{21}}{4M}-\frac{A_{20}}{4M^2}\right)\cosh M \\ +\left(\frac{A_{19}}{2M}+\frac{A_{20}}{4M}-\frac{A_{21}}{4M^2}\right)\cosh M \end{array} \right\},$$

$$c_3 = -c_4 - A_{12}\ ,\ c_4 = \frac{A_4}{4M^2}-\frac{A_7}{M^2}+\frac{A_{10}}{M^2}+2\frac{A_{11}}{M^3},\ c_5 = \frac{\gamma M A_{26} - A_{25}\sinh M}{\sinh M + \gamma M\cosh M}$$

and $c_6 = \dfrac{A_{25}\cosh M + A_{26}}{\sinh M + \gamma M\cosh M}$.

Finally, the perturbation solutions up to first order for θ and u are given by

$$\theta = \theta_0 + \varepsilon\theta_1 \tag{3.15}$$

and

$$u = u_0 + \varepsilon u_1 \tag{3.16}$$

4. Discussion of the Result

In order to see the effects of various parameters like $\gamma, \varepsilon, M, \alpha, Gr$ and Re on the velocity we have plotted the Figs. 2-7. From Fig. 2, it is found that the velocity u increases with an increase in slip parameter γ. From Fig. 3, it is seen that the velocity u increases with increasing ε. From Fig. 4, it is observed that the velocity u decreases with increasing Hartmann number M. From Fig. 5, it is noticed that the velocity u increases with increasing α. From Fig. 6, it is found that the velocity u increases with increasing Grashof number Gr. From Fig. 7, it is observed that the velocity u decreases with increasing Reynolds number Re.

In order to see the effects of various parameters like $\varepsilon, M, \alpha, Gr$ and Re on the temperature we have plotted the Figs. 8-12. . From Fig. 8, it is found that the temperature θ increases with an increase in ε. From Fig. 9, it is seen that the temperature θ decreases with increasing M. From Fig. 10, it is observed that the temperature θ increases with increasing α. From Fig. 11, it is noticed that the temperature θ decreases with increasing Gr. From Fig. 12, it is found that the temperature θ increases with increasing Reynolds number Re. From Table-1, it is observed that, the temperature θ first decreases and then increases with increasing slip parameter γ.

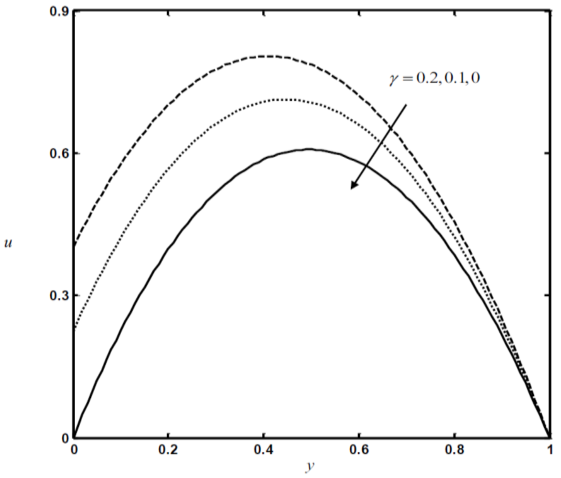

Fig. 2 The effect of slip parameter γ on the velocity u for $\mathrm{Re}=1$, $Gr=1$, $P=-5$, $M=1$, $\alpha=\frac{\pi}{4}$ and $\varepsilon=0.1$.

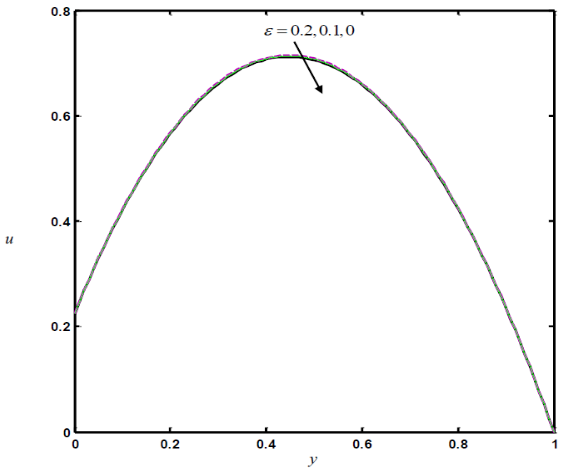

Fig. 3 The effect of $\varepsilon(=Ec\,\mathrm{Pr})$ on the velocity u for $\mathrm{Re}=1$, $Gr=1$, $P=-5$, $M=1$, $\alpha=\frac{\pi}{4}$ and $\gamma=0.1$.

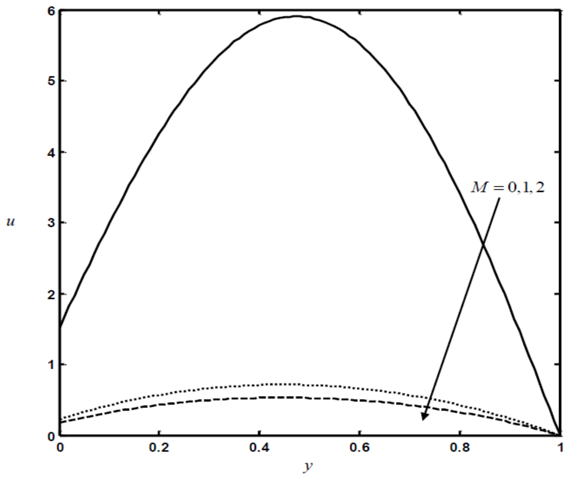

Fig. 4 The effect of Hartmann number M on the velocity u for $\mathrm{Re}=1$, $Gr=1$, $P=-5$, $\gamma=0.1$, $\alpha=\frac{\pi}{4}$ and $\varepsilon=0.1$.

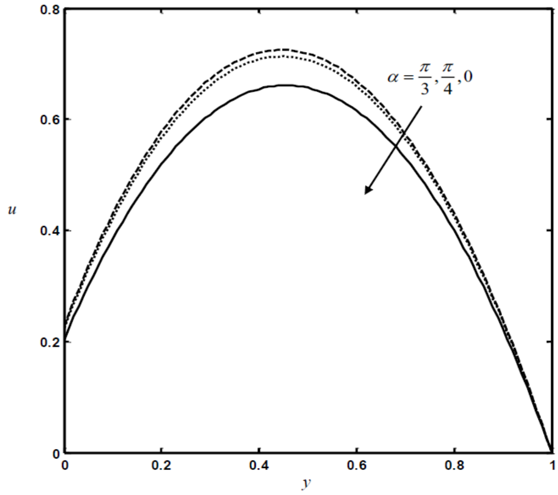

Fig. 5 The effect of inclination angle α on the velocity u for $\mathrm{Re}=1$, $Gr=1$, $P=-5$, $M=1$, $\gamma=0.1$ and $\varepsilon=0.1$.

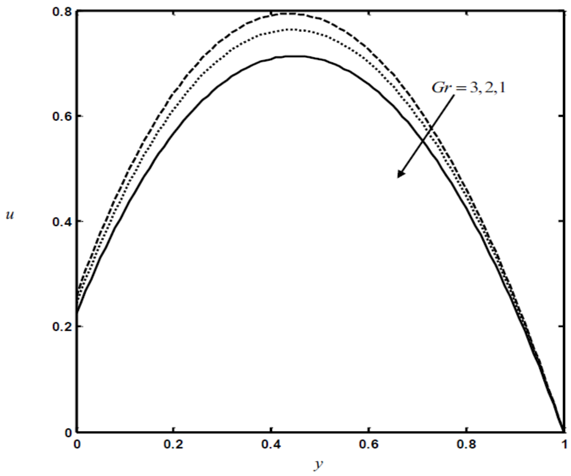

Fig. 6 The effect of Grashof number Gr on the velocity u for $\mathrm{Re}=1$, $\gamma=0.1$, $P=-5$, $M=1$, $\alpha=\frac{\pi}{4}$ and $\varepsilon=0.1$.

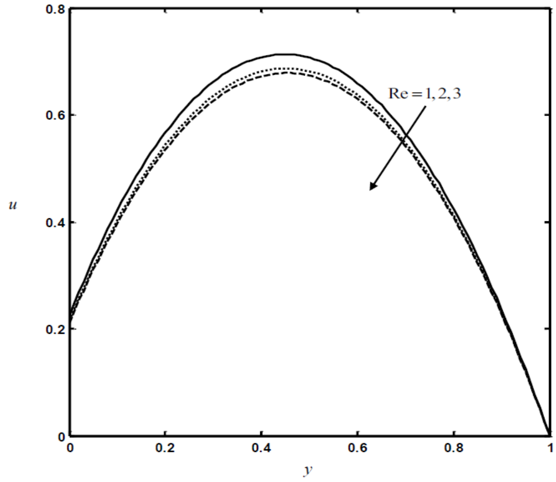

Fig. 7 The effect of Reynolds number Re on the velocity u for $\gamma=0.1$, $Gr=1$, $P=-5$, $M=1$, $\alpha=\frac{\pi}{4}$ and $\varepsilon=0.1$.

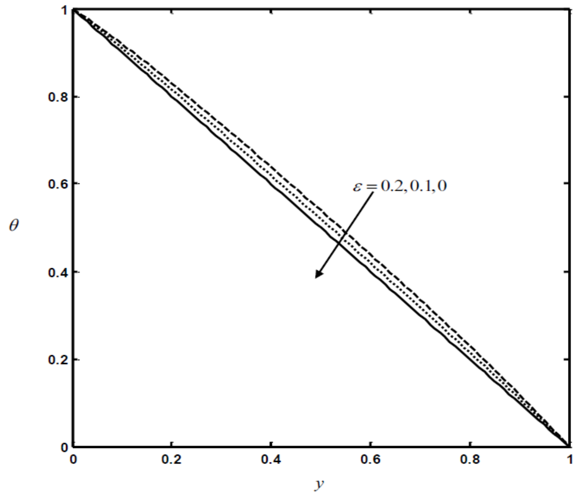

Fig. 8 The effect of ε on the temperature θ for $\mathrm{Re}=1$, $Gr=1$, $P=-5$, $M=1$, $\alpha=\frac{\pi}{4}$ and $\gamma=0.1$.

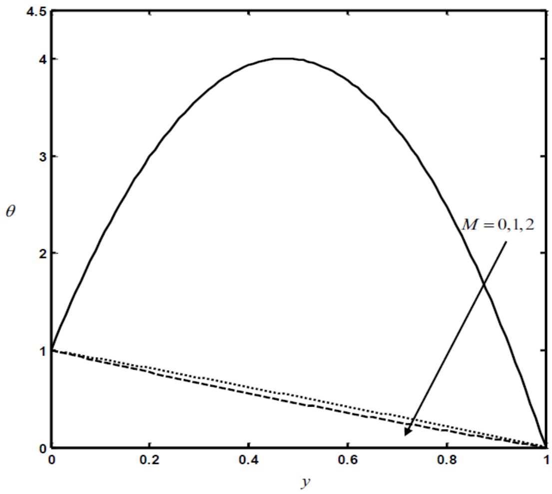

Fig. 9 The effect of Hartmann number M on the temperature θ for $\mathrm{Re}=1$, $Gr=1$, $P=-5$, $\varepsilon=0.1$, $\alpha=\frac{\pi}{4}$ and $\gamma=0.1$.

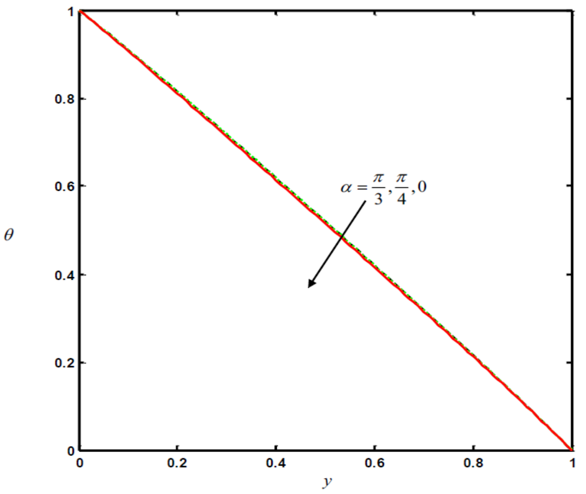

Fig. 10 The effect of inclination angle α on the temperature θ for $\mathrm{Re}=1$, $Gr=1$, $P=-5$, $M=1$, $\varepsilon=0.1$ and $\gamma=0.1$.

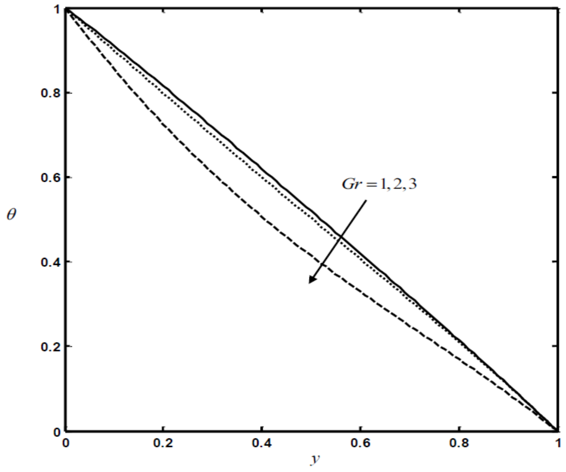

Fig. 11 The effect of Grashof number Gr on the temperature θ for $\mathrm{Re}=1$, $\varepsilon=0.1$, $P=-5$, $M=1$, $\alpha=\frac{\pi}{4}$ and $\gamma=0.1$.

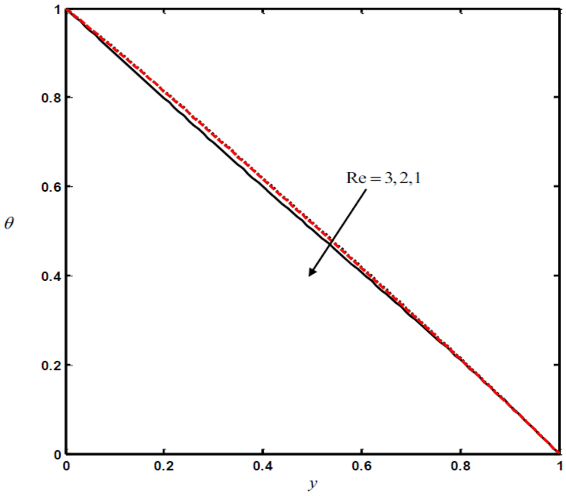

Fig. 12 The effect of Reynolds number Re on the temperature θ for $\varepsilon=0.1$, $Gr=1$, $P=-5$, $M=1$, $\alpha=\frac{\pi}{4}$ and $\gamma=0.1$.

Table-1: The effect of slip parameter γ on the temperature θ for $\varepsilon=0.1$, $Gr=1$, $P=-5$, $M=1$, $\alpha=\frac{\pi}{4}$ and $\mathrm{Re}=1$.

y	$\gamma=0$	$\gamma=0.1$	$\gamma=0.2$
0	1	1	1
0.1	0.9108	0.9096	0.9091
0.2	0.8165	0.8150	0.8147
0.3	0.7190	0.7179	0.7181
0.4	0.6196	0.6192	0.6200
0.5	0.5191	0.5196	0.5210
0.6	0.4180	0.4192	0.4211
0.7	0.3161	0.3178	0.3199
0.8	0.2132	0.2149	0.2168
0.9	0.1082	0.1094	0.1107

5. Conclusions :

In this paper, we investigated the flow and heat transfer aspects of conducting fluid in an inclined channel with constant a pressure gradient. The governing equations are solved analytically valid for small Pr *Ec*.

- It is found that the velocity u increases with increasing γ, ε, Gr and α , while it decreases with increasing M and Re, and
- the temperature θ increases with increasing ε, α and Re, while it decreases with increasing γ, M and Gr.

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