Marshall-Olkin Stereographic Circular Logistic Distribution

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Abstract

Marshall and Olkin (1997) proposed an interesting method of adding a new parameter to the existing distributions. The resulting distributions are called the Marshall-Olkin distributions, these distributions include the original distributions as a special case and are more flexible and represent a wide range of behavior than the original distributions. In this paper, a new class of asymmetric stereographic circular logistic distribution is introduced by using Marshall-Olkin transformation on stereographic circular logistic distribution (Dattatreyarao et al (2016)), named as Marshall-Olkin Stereographic Circular Logistic Distribution. The proposed model admits closed form density and distribution functions, generalizes the stereographic circular logistic model and is more flexible to model various types of data (symmetric and skew-symmetric circular data).

Keywords:*Characteristics****,*** *Stereographic circular logistic distribution, circular data, Marshall-Olkin transformation, l-axial data.*

1. Introduction

Directions in two-dimensions can be represented as points on the circumference of a unit circle and models for representing such data are called circular distributions. There are many areas that deals with directional/circular data such as animal, birds navigation, sound waves in physics, wind direction in meteorology, image analysis in computer science and vector cardiology in medical science, to name some of them. For modeling circular data, Dattatreyarao et al (2007), Jammalamadaka and SenGupta (2001)etc derived many models for such data. Most of these models are typically symmetric around some center and very few asymmetricaldistributions are available for describing circular data. In this paper, we describe a broad class of model for the asymmetric case and discuss anapplication.

Phani (2013) constructed new circular models, Stereographic Circular Logistic, Stereographic Extreme-Value, Stereographic Double Exponential and some Stereographic Semicircular models by applying Inverse Stereographic Projection on linear models.Adding parameters to a well-established distribution is a time honored device for obtaining more flexible new families of distributions. Marshall and Olkin(1997) proposed an interesting method of adding a new parameter to the existing distributions. The resulting distributions, called the Marshall-Olkin (MO) distributions, which includes the parent distributions as a special case and gives more flexible to model various types of data. In this paper, we use the Marshall-Olkin transformation to derive a new flexible circular model, so called The Marshall-Olkin Stereographic Circular Logistic Distribution, which generalizes the baseline distribution (i.e., Stereographic Circular Logistic Distribution) for modeling circular data.

2. Methodology of Marshall-Olkin Transformation

An ingenious general method of adding a parameter to a family of distribution is introduced by Marshall-Olkin (1997). Here we adapt the idea of Marshall-Olkin to circular case for deriving more flexible circular models. For a circular random variable θ with a distribution function $F(\theta)$, we can obtain a new family $G(\theta)$ which contains one more parameter is given by

$$G(\theta)=\frac{F(\theta)}{\alpha+(1-\alpha)F(\theta)},\ \theta\in[-\pi,\pi)\ , \tag{1}$$

Where $F(\theta)$ is a distribution function and $\alpha>0$ .If $\alpha=1$ then we have $G(\theta)=F(\theta)$ and the probability density function corresponding to (2.1), say $g(\theta)$ takes the form

$$g(\theta)=\frac{\alpha f(\theta)}{\left(1-(1-\alpha)(1-F(\theta))\right)^2},\ -\pi\le\theta<\pi. \tag{2}$$

Where $F^{'}(\theta)=g(\theta)$ is the baseline density function.

3. Marshall–Olkin Stereographic Circular Logistic Distribution

Here probability density and distribution functions of Stereographic Circular Logistic Distribution are revisited Phani (2013).

Stereographic Circular Logistic Distribution

A random variable θ on unit circle is said to have stereographic circular logistic distribution with location parameter μ and scale parameter $\sigma>0$ denoted by SCLG(μ,σ) , if the probability density and cumulative distribution functions are respectively given by

$$f(\theta)=\frac{1}{2\sigma}\sec^2\left(\frac{\theta}{2}\right)\left[1+\exp\left(\frac{-\left(\tan\left(\frac{\theta}{2}\right)-\mu\right)}{\sigma}\right)\right]^{-2}\exp\left(-\left(\frac{\tan\left(\frac{\theta}{2}\right)-\mu}{\sigma}\right)\right),$$

Where $\sigma>0$ and $-\pi\le\theta,\mu<\pi$

$$F(\theta)=\left[1+\exp\left(\frac{-\left(\tan\left(\frac{\theta}{2}\right)-\mu\right)}{\sigma}\right)\right]^{-1},\text{where } \sigma>0,\ -\pi\le\theta<\pi.$$

Then by applying Marshall-Olkin transformation on stereographic circular logistic distribution, we obtain more flexible asymmetric circular distribution, we call it as Marshall-Olkin Stereographic Circular Logistic Distribution.

Definition:

A random variable θ on unit circle is said to have Marshall-Olkin stereographic circular logistic distribution with location parameter μ, scale parameter $\sigma > 0$ and tilt parameter $\alpha > 0$ denoted by MOESCLG(μ, σ, α), if the probability density and cumulative distribution functions are respectively given by

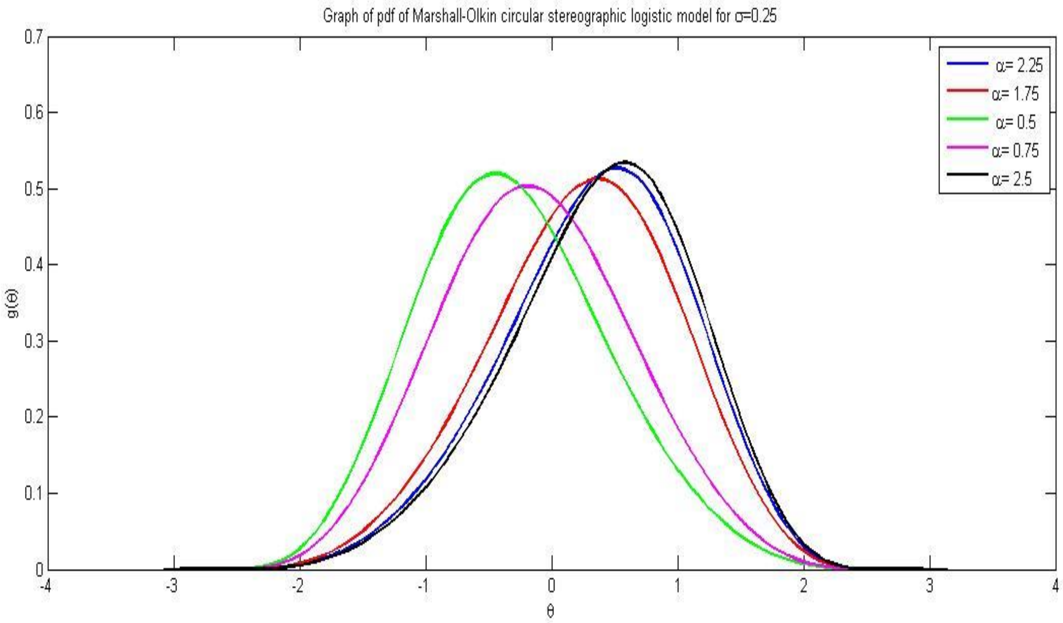
$$g(\theta) = \frac{\alpha}{2\sigma}\sec^2\left(\frac{\theta}{2}\right)\left[1+\alpha\exp\left(\frac{-\left(\tan\left(\frac{\theta}{2}\right)-\mu\right)}{\sigma}\right)\right]^{-2}\exp\left(-\left(\frac{\tan\left(\frac{\theta}{2}\right)-\mu}{\sigma}\right)\right), \qquad (3)$$

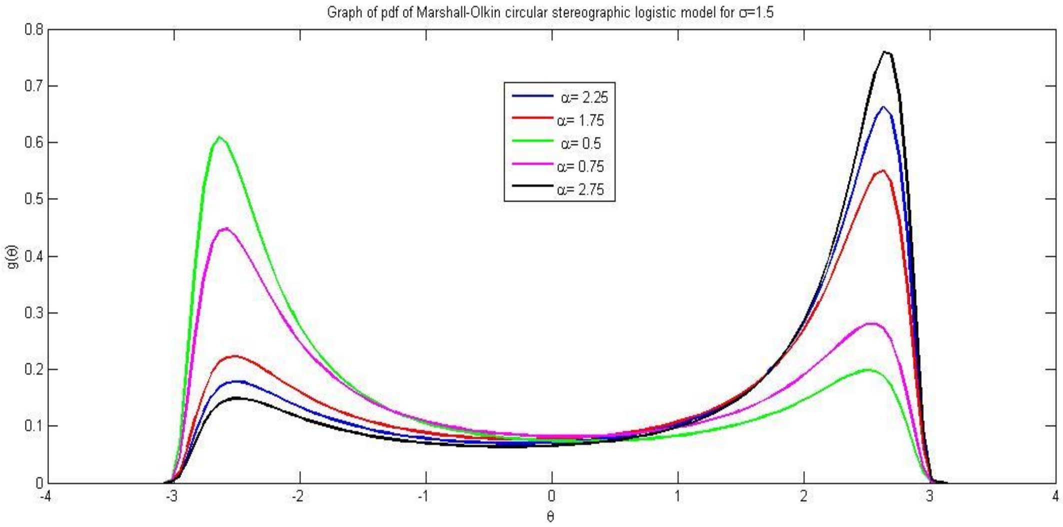
Where $\sigma,\ \alpha > 0$ and $-\pi \le \theta, \mu < \pi$

$$G(\theta) = \left[1+\alpha\exp\left(\frac{-\left(\tan\left(\frac{\theta}{2}\right)-\mu\right)}{\sigma}\right)\right]^{-1}, \text{where } \sigma, \alpha > 0,\ -\pi \le \theta < \pi. \qquad (4)$$

This distribution is asymmetric for $\alpha \neq 1$ and symmetric for $\alpha = 1$. Marshall-Olkin Stereographic logistic distribution is **unimodal** if $\sigma < 0.5$ and **bimodal** if $\sigma > 0.5$

Graphs of probability density function of Marshall-Olkin Stereographic Circular Logistic Distribution for various values of σ, α and $\mu = 0$ are presented here.





4. Extension to l-axial Marshall-Olkin Stereographic Circular Logistic Distribution

We extend the proposed model to the l -axial distribution, which is applicable to any arc of arbitrary length say π/l for $l=1,2,...,$ so it is desirable to extend the Marshall-Olkin stereographic circular logistic distribution. To construct the l -axial Marshall-Olkin stereographic circular logistic distribution, we consider the density function of Marshall-Olkin stereographic circular logistic distribution and use the transformation $\phi=\theta/l$, $l=1,2,...,$. The probability density function of ϕ is given by

$$g(\phi)=\frac{\alpha l}{2\sigma}\sec^2\left(\frac{l\phi}{2}\right)\left[1+\alpha e^{-\frac{\tan\left(\frac{l\phi}{2}\right)}{\sigma}}\right]^{-2} e^{-\frac{\tan\left(\frac{l\phi}{2}\right)}{\sigma}}, \text{where } -\frac{\pi}{l}<\phi<\frac{\pi}{l} \quad (5)$$

Case (1) when $l=1$, the probability density function (6.1) is the same as that of Marshall-Olkin Stereographic Circular Logistic Distribution.

$$g(\phi)=\frac{\alpha}{2\sigma}\sec^2\left(\frac{\phi}{2}\right)\left[1+\alpha e^{-\frac{\tan\left(\frac{\phi}{2}\right)}{\sigma}}\right]^{-2} e^{-\frac{\tan\left(\frac{\phi}{2}\right)}{\sigma}}, \text{ where } -\pi<\phi<\pi \quad (6)$$

Case (2) When $l=2$, the probability density function (6.1), the probability density function is

$$g(\phi)=\frac{\alpha}{\sigma}\sec^2(\phi)\left[1+\alpha e^{-\frac{\tan(\phi)}{\sigma}}\right]^{-2} e^{-\frac{\tan(\phi)}{\sigma}}, \text{ where } -\frac{\pi}{2}<\phi<\frac{\pi}{2} \quad (7)$$

It is called as Marshall-Olkin Stereographic Semicircular Logistic Distribution.

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