Analytical Study of Series Solutions for the 'Nonlinear Mechanical System' with Stability Analysis

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Abstract

Any mechanical system can easily be analyzed with the help of proper designing or modeling. All kinematical constrains are also been considered at the time of modeling. Generally the traditional dynamical differential equations are solved to predict the nature of the system. In this work, some typical types of dynamical mechanical system in taken into consideration and further the dynamical equations are being solved using 'Series Solutions Technique' for the analysis of the nonlinear system. The solution is consisting of two sections. Using frequency response analysis via simulation, which section of the solution is more suitable or effective for the designing purpose has also been explained in detail. Finally, the local stability analysis of the entire system is studied in detail with proper numerical simulation.

Keywords: Nonlinearity, Series Solution, Stability Analysis, Phase Portrait, Frequency Response Function.

1. Nomenclature Section:

In this section the entire abbreviations and notation used throughout the article is tabulated in the following table 1. The variables used in the equations are not included in this table.

Table 1. Abbreviation of the used Notation in this article.

Notation	*Meaning*
NLS	Non Linear System
SS	Series Solution
SOS	Second Order System
DFA	Describinf Function Analysis
LCA	Limit Cycle Analysis

PTA	Phase Trajactory Analysis
DOF	Degree of Freedom
SFT	Sliding Filament Theory
STM	Springless Translational Mechanical Model
DTM	Damperless Translational Mechanical Model
TMM	Translational Mechanical Model
SMS	Suspended Mechanical System
MSDM	Mass- Spring-Damper Model
T.F	Transfer Function
FRF	Frequency Response Function
SSN	Stick Slip Nonlinearity
ZMNS	Zero Memory Nonlinear System
ODE	Ordinary Differential Equation

2. Introduction:

The rapid progress of the software based designing tool and hardware technologies always requires the proper mathematical analysis to make it effective and sophisticated. Mathematical analysis is not only important for the dynamical mechanical system but also important for the suspended mechanical system[1]. The key parameters for the modeling of any type of mechanical system are kinematical constrains. If the kinematical constrains of any mechanical system are varying with respect to time, then it is very difficult to study the nonlinear behavioral characteristics of the system, even abrupt types of discontinuity are observed in the motion of the system. This type of peculiar problems is not only found in the mechanical system, it is also found in the field of robotics. In some cases, the time varying topologies are taken into the consideration which is very much beneficial for the analysis of the system. This paper is mainly concerned with some analytical series solutions of the differential equation. Generally it is observed that, when we are using some topological based design, kinematical constrains causes into the change in the modeled dynamical equations whereas the series solutions are far better in the sense that, kinematical constrains never effects the output of the entire system. In order to study any nonlinear dynamical system, it is important to recognize the topological changes but it is not necessary if we are finding the solutions using series solution method[2]. Hence this research work makes a big impact when an object is continuously in motion.Actually for a nonlinear system, for a modeling purpose there are no such fixed method, but still there are few methods often used for finding the stability of a nonlinear system.Some of the popular methods are ‘DFA’ ,‘LCA’ , ‘PTA’ etc [3]. Here in this research work, we are analyzing the phase portrait of the typical modeled equation whose series solution is depicted in a systematic procedure. A ‘ZMN System’ can be defined by a nonlinear system whose output only depends on the present input. When the system goes under simulation, different sinusoidal frequencies are applied at different excitation level, means the nonlinearity can also be expressed as a function of frequency [4]. On the other hand in a vibrational mechanical system a special type of nonlinearity occurs called ‘Hysteretic type Nonlinearity’. It is very difficult to study these types of vibrational mechanical system with the help of conventional method. In these circumstances, ‘Truncated Series Solution’ is the best way to study the exact behavior of the system [5]. Apart from this, harmonic distorted output due to the inherent nonlinearity can also be detected using the above stated procedure. Anyway, in the next section, a detail literature review is depicted for finding the research gap in the domain of nonlinear modeling of vibrational mechanical system.

3.1. Dynamics of Mechanical System- *A literature Review*:

In the past decade, the researchers have come to study any nonlinear mechanical systems with very complicated nature, observing their basic characteristics with different types of experiments and computer models [6-9]. The main aim of these studies is that all the system must incorporates different types of complex nonlinearity. These dynamical systems have been selected from different sources. For example-Computer aided system, Basic experimental science, modeling issues of mechanical system, Mathematical formulation of any system [10–13]. Montassir, S. et.al.[14] describes the analysis of a mechanical system having cylindrical structure of in a beautiful way. The control system based modeling of a helicopter (including T.F) and proper numerical simulation was described by the researcher Tarsi, A. et.al.[15]. In a research work, cited in the reference [16], the researchers thinks about the reliability issues related to the simulation was studied in detail. Mechanical property of any nonlinear system was beautifully described by the authors Teodorescu, D.H et. al.[17], where a concept sandwich composite structure is discussed. Not only that, how this kind of structure causes nonlinearity is also delineated with the help of simulation. In the reference [18], Saviuc, A. et.al. examined the 'non-differentiability' type behavior of the mechanical system which is very much beneficial for the future research work. Not only that, the in phase coherency of the complex fluid dynamical structure has also been tested. In the recent years, the application of machine learning is growing exponentially. Derbeli,M. et.al.[19] had shown how to model a system with the help of predictive control along with machine learning. These types of modeling are able to track the system with higher efficiency. Uncertain turbulences in the domain of fluid dynamical system are a vast area of research related to mechanical engineering. There are so many computational tools are used to check the turbulence. Portal-Porras, K. et.al. [20] had described the modeling of turbulence with the help of very famous topic called artificial neural network. The experiential result of this research work explains the situation when uncertain turbulences come into the picture. Gálfi,B.-P et. al.[21] developed a model based approach of heat transfer mechanism of a nonlinear mechanical system with in depth knowledge. It helps to study how the heat is transferred from a cylindrical type of structure. This paper gives a basic idea related modeling which helps a newcomer in the field of nonlinear control system. Xu, B. et. al.[22] demonstrated thermo dynamical model with high temperature. The optimized relationship between power density and efficiency has been established in this paper. This article will provide a very good guidance to our research work. Bánó, G. et. al.[23] explained the process of development of the model in the domain of mechanics in a sophisticated way. Different types of computer simulation are delineated in a versatile way. This paper is very much helpful in the perspective of simulative aspect. Casas,P. et.al. [24] et.al. are described a digital twin approach towards the modeling issue. This model allows the efficient forecasting for different situation. The validity of the model is verified through simulation processes that are very interesting because all the simulation result is described with proper explanation.'Optimization Process' is very famous in the domain of 'Nonlinear Control System'. There are so many researchers are doing their research work in this section to get the desired output. Li,D. et.al. [25] developed a new algorithm related to the optimization which can be applied to our research article for future as the algorithm provides very low complexity and produces very close output. The researchers Zare,Y. et.al. [26] works on carbon nano tube which is not directly connected to our article, but still we have eyed through this paper as it gives a new idea related to the advance modeling technique. On the other hand Ogbonnaya, C. et.al. [27] demonstrated a beautiful way of solving the complex equation in computational approach. They also consider the multifunctional problem and multivariable structured model which is very much beneficial for the future research work. In the reference [28], the authors studied the behavior of the elasticity of a notch with proper mathematical equation and necessary simulation that is often used to design a linear or nonlinear mechanical system. However, any normal nonlinear system can be realized using some piecewise linear model which is discovered at least 70 years back. Lelkes, J. et al.[29] explained the entire operation of a typical nonlinearly coupled system using the concept of piece linear model. This paper basically tells that, whatever nay be the complexity of a system, if we blindly apply the rule of piecewise linear model, we can get the approximated result of a nonlinear system. Study of the nonlinear dynamics of any mechanical model sometime helps us to find the exact output for the system. In a research article, Freydin, M et. al. [30] studied the nonlinear structure of a 'Flutter of Plate' and corresponding response of the system under supersonic wind in detail. The frequency response function is used in this article to understand the internal state of the system. At the same time the approximate equation of the oscillation due to the vibration of a system is illustrated by the famous researcher Akhavan, H. et.al.[31]. Some time it is observed that a particular system cannot be explained by a single differential equation, rather coupled equation is necessary to represents the entire behavior of the system. Qu,Y. et. al. published a research article in to two parts related to the modeling of coupled vortex induced vibration which is very much related to our research work [32-33]. On the other hand, Han,P.et.al. [34] discussed the internal structure of the vortex induced vibration for a square element in detail with proper numerical simulation.Zulli,D. et.al.[35] had explained the effect of nonlinearity in case of mean wind force for the shallow cables that is very in term of inherent nonlinearity present with in a system. In this research article they also described the process of removing the effect of nonlinearity which makes a system more stable. The tuning process in the domain of control system is a very famous topic as we are getting more accurate output from a system. The famous rule related to the tuning is Z-N Tuning rule. D'Annibale,F et.al. [36] works on the linear damping but they described that how damping can affect the nonlinear Ziegler's column. Large deflection of the structurally damped system were studied by the researchers Balakrishna,A et. al.[37]. Similarly the 'Tracking modal' was developed by the researchers Habib,G. et.al. [38] that is linking the concept of energy with the nonlinear dynamical system. Anyway, we have eyed through so many research article. But few of them is given the literature review section as these are very much related and helpful to our research work.

3. Novelty of the work:

In this research work, we have proposed a very unique method to study the nonlinearity present within a typical type dynamical system that may arise in the time of modeling for an aircraft system. The unique method is nothing but a series solution of the nonlinear dynamical equation. It is found that, the final solution is consisting of two parts. Both the parts are truncated up to third order for simplicity. We are finding the best option from two part of the solution considering the frequency response function through simulation. To justify our proposed series solution, the stability of entire system has also been verified with the help of phase portrait analysis. Thus it can be concluded that, using this truncated series solution process we can study a nonlinear system very easily with respect to the other conventional method.

3.1 Dynamical Modeling of the some Standard Mechanical System:

The step by step designing method of any mechanical system is depicted in this section. It is quite obvious that any mechanical system is having a mass. If we want to design a series connection of two modules, then a damper is needed to represent it.

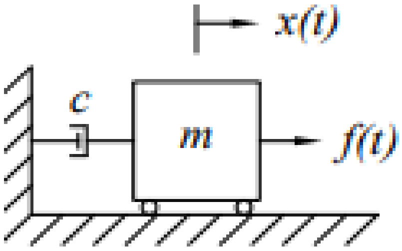

Figure 1. The ‘STM’ Model.

Again a spring is to be placed to represent the elasticity of the mechanical system. Figure 1 is describing the basic STM model. The dynamical equation of the above designed ‘STM’ model is given in the equation (1), where $'m'$ represent the mass of the mechanical system, $'c'$ is denoting the coefficient of damper and inherently generated force is given by $'f(t)'$. This inherent force is trying to move the body in the forward direction while the damper is compelled to back it in its original position.

$$m\ddot{x} + c.\dot{x} = f(t) \quad (1)$$

Taking the Laplace transformation of the equation (1), the T.F is formed and given in the equation (2).

$$\frac{X(s)}{F(s)} = \frac{1}{s(m.s+c)} \quad (2)$$

After developing the ‘STM’ Model, the ‘DTM Model’ of a standard mechanical system has been developed which is consist of mass and a linear/non-linear spring. The nature of the spring may be linear or may not be a linear function. Both the characteristics are given in the following figure 2. The left half of the section is representing the linear characteristics while the right half of the section is representing the non-linear characteristics of the spring.

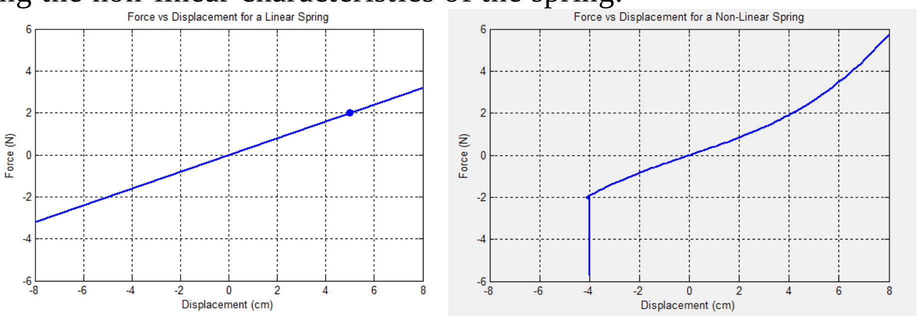

Figure 2. Characteristics of the Linear and Non-Linear spring.

From the above figure 2 it is observed that in case of linear spring force versus displacement is following a linear type of equation while in case of a non-linear spring, force versus displacement is not following a linear type of equation rather a dead zone is observed.

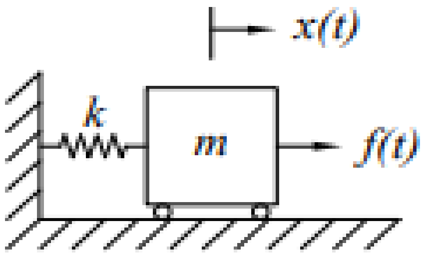

Figure 3. The 'DTM' Model.

Anyway, the above figure 3 is describing the 'DTM' model $'k'$ is denoting the spring constant.

For the above 'DTM model', the 'T.F' has been calculated and given in the equation (4).

$$m\ddot{x} + k.x = f(t) \tag{3}$$

$$\frac{X(s)}{F(s)} = \frac{1}{(m.s^2+k)} \tag{4}$$

Now both the influence of 'DTM model' and 'STM model' has been included to get a complete mechanical design of astandardmechanical model'. The following figure 4 given below is representing the Pictorial view of the 'MSD model'.

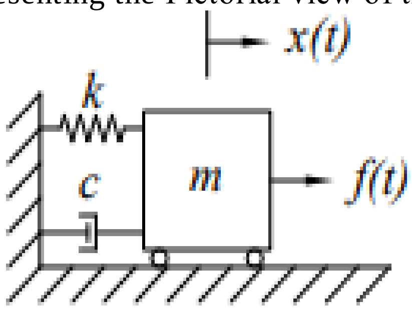

Figure 4. The 'MSD' Model.

In the above figure, the spring is considered as linear nature. The dynamical equation and 'T.F' of the above diagram is given below.

$$m\ddot{x} + c.\dot{x} + k.x = f(t) \tag{5}$$

$$\frac{X(s)}{F(s)} = \frac{1}{(m.s^2+c.s+k)} \tag{6}$$

On the other hand, the 'Suspended Type Mechanical System' can be realized by the structure given in the following figure 5.

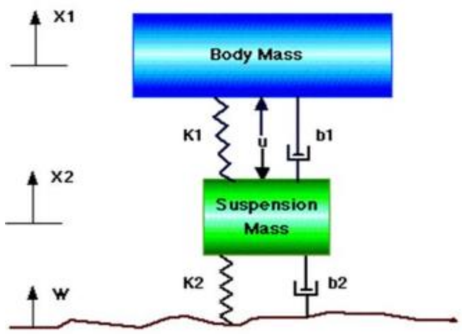

Figure 5. The series connection of 'SMS' Model

The dynamical equation using free body diagram for the above figure is constructed and corresponding transfer function is also developed which is given in the following equation (7) and(8). Here k_1, k_2 represents the linear spring stiffness of the body and x_1, x_2 represents the linear displacement of the body due to the inherent generated force. Similarly c_1, c_2 is denoting the coefficient of the dampers. On the other hand m_1, m_2 are representing the mass of the suspended body[39].

$$m_1\ddot{x}_1(t) + b_1[\dot{x}_1(t) - \dot{x}_2(t)] + k[\dot{x}_1(t) - \dot{x}_2(t)] = 0 \quad (7)$$

$$m_2\ddot{x}_2(t) + b_2[\dot{x}_2(t) - \dot{w}(t)] + k_2[x_2(t) - w(t)] + C_2\dot{y}_2(t) - b_1[\dot{x}_1(t) - \dot{x}_2(t)] - k_1[\dot{x}_1(t) - \dot{x}_2(t)] \;\; = 0 \quad (8)$$

$$T_u(s) = \frac{X_1(s)-X_2(s)}{U(s)} = \frac{(m_1+m_2).s^2+b_2.s+k_2}{(m_1s^2+b_1.s+k_1)+[m_2s^2+(b_1+b_2).s+(k_1+k_2]-(b_1.s+k_1)^2} \quad (9)$$

$$T_w(s) = \frac{X_1(s)-X_2(s)}{U(s)} = \frac{-b_1(b_2.s+k_2).s^2}{(m_1s^2+b_1.s+k_1)+[m_2s^2+(b_1+b_2).s+(k_1+k_2]-(b_1.s+k_1)^2} \quad (10)$$

Ifthe above mentioned concept is described in lateral direction, then a Two DOF system and Three DOF system can easily be realized using the following figures.

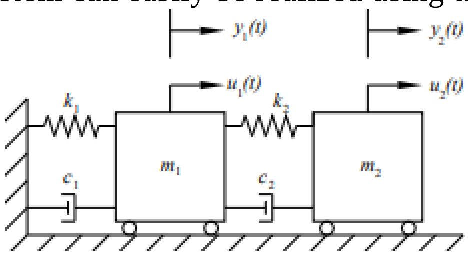

Figure 6. Design of MSDM considering 2 DOF

In the above figure 6, the MSDM is designed considering two degree of freedom whose dynamical equation is given below in the equation number 11.

$$\begin{cases} m_1\ddot{y}_1(t) + (c_1 + c_2)\dot{y}_1(t) + (k_1 + k_2)y_1(t) - c_2\,\dot{y}_2(t) - \;k_2\dot{y}_2(t) = u_1(t) \\ m_2\ddot{y}_2(t) + c_2\dot{y}_2(t) + k_2y_2(t) - c_2\,\dot{y}_1(t) - \;k_2y_1(t) = u_2(t) \end{cases} \quad (11)$$

The above equation is also called a coupled equation which is the key factor for this article because we are going to solve these types of equations is arising in the typical mechanical system. The above figure can be extended by realizing the series connection of three individual cells called MSMD design for 3 DOF system. We have also calculated all the necessary dynamical equations for the above system and given in the next section.

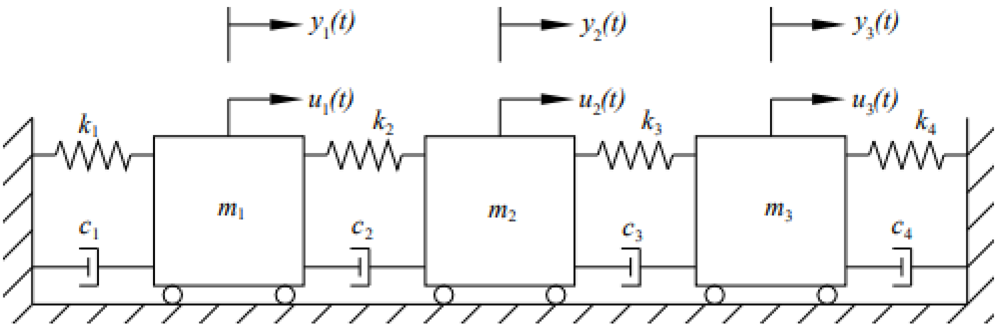

Figure 7. Design of MSDM considering 3 DOF

Similar to the figure 6, the above figure 7 is describing the design of MSMD which is constructed considering three degree of freedom whose dynamical equation is given below in the equation number 12.

$$\begin{cases} m_1\ddot{y}_1(t) + (c_1 + c_2)\dot{y}_1(t) + (k_1 + k_2)y_1(t) - c_2\,\dot{y}_2(t) - \;k_2\dot{y}_2(t) = u_1(t) \\ m_2\ddot{y}_2(t) + (c_1 + c_3)\dot{y}_2(t) + (k_1 + k_3)y_2(t) - c_2\,\dot{y}_1(t) - \;k_2y_1(t) - c_3\dot{y}_3(t) - k_3y_3(t) = u_2(t) \\ m_3\ddot{y}_3(t) + (c_3 + c_4)\dot{y}_3(t) + (k_3 + k_4)y_3(t) - c_3\,\dot{y}_2(t) - \;k_3\dot{y}_2(t) = u_3(t) \end{cases} \quad (12)$$

Now we are taking a typical type of mechanical system or complex type of mechanical system for internal analysis regarding nonlinearity [40]. This type of situation can be arises in the designing of a part of ‘Aircraft System’which is given in the figure 8.

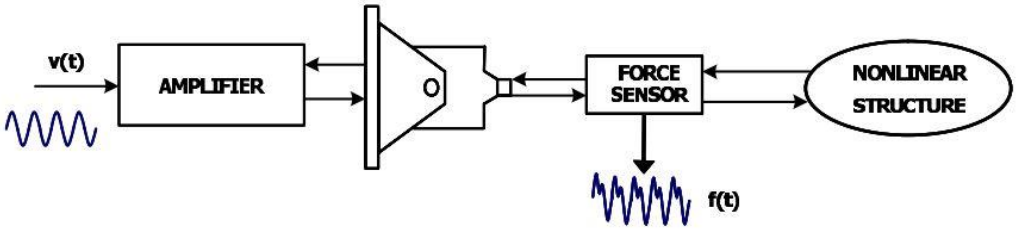

Figure 8. Nonlinear Structure for Aircraft System

The dynamical equation considering approximationfor the above figure is given the following equation (13).

$$\frac{d^2v}{dx^2} + 2bxv + \omega^2 v = 0 \quad (13)$$

3.2Series Solution of the Nonlinear Dynamical Equation:

This section is basically the core part of this research article where, the series solutions of the typical mechanical system are depicted in step by step. The above equation (13) is taken for the analysis. Equation (13) is rearranged as follows.

$$\frac{d^2v}{dx^2} + (2bx + \omega^2)v = 0 \quad (14)$$

The above equation is representing a second order homogeneous linear O.D.E. It is observed that, zero is an ordinary point of (14), so that the solutions in the power of $'x'$ actually do exist. Thus we denote such solution by the following equation [40-43].

$$v = \sum_{n=0}^{\propto} c_n x^n \quad (15)$$

Taking the first derivative of the above equation, the following equation is obtained.

$$\frac{dv}{dx} = \sum_{n=1}^{\propto} nc_n x^{n-1} \quad (16)$$

Taking the first derivative of the above equation, the following equation comes.

$$\frac{d^2v}{dx} = \sum_{n=2}^{\propto} n(n-1)c_n x^{n-2} \quad (17)$$

Putting the value of equation (15) & (16) into equation (14), we get the following result.

$$\sum_{n=2}^{\propto} n(n-1)c_n x^{n-2} + (2bx + \omega^2)\sum_{n=0}^{\propto} c_n x^n = 0 \ldots.(17)$$

$$2c_2 + 6c_3 x + \sum_{n=2}^{\propto}(n-2)(n-1)c_{n+2}x^n + 2bc_0 x + 2b\sum_{n=0}^{\propto} c_{n-1}x^n + \omega^2 c_0 + \omega^2 c_1 x + \omega^2 \sum_{n=0}^{\propto} c_n x^n = 0$$

$$(\omega^2 c_0 + 2c_2) + (2bc_0 + \omega^2 c_1 + 6c_3)x + \sum_{n=2}^{\propto}[(n+2)(n+1)c_{n+2} + 2bc_{n-1} + \omega^2 c_n]x^n = 0 \quad (18)$$

From the above equation (18), it can be concluded that, coefficient of each power of $'x'$, must be equal to zero. In order to do this, we are led to the relations.

$$\begin{cases} \omega^2 c_0 + 2c_2 = 0 \\ 2bc_0 + \omega^2 c_1 + 6c_3 = 0 \end{cases} (19)$$

$$[(n+2)(n+1)c_{n+2} + 2bc_{n-1} + \omega^2 c_n = 0 \quad (20)$$

The above equation is valid for only when $n \geq 2$.Again from the first part of the equation (19), we can write the following equation.

$$c_2 = \frac{\omega^2 c_0}{2} \quad (21)$$

Similarly, from the second part of the equation (19), we can write the following equation.

$$6c_3 = -2bc_0 - \omega^2 c_1$$

$c_3 = -\frac{1}{3}bc_0 - \frac{\omega^2}{6}c_1$ (22)

From the equation (20), the following equation has been derived.

$c_{n+2} = -\frac{2bc_{n-1}+\omega^2 c_n}{(n+1)(n+2)}$, for $n \geq 2$.

Putting n = 2, in the above equation, equation (23) is obtained and given below.

$$c_4 = -\frac{2bc_1 + \omega^2 c_2}{12}$$

$c_4 = \frac{\omega^4}{24}c_0 - \frac{b}{6}c_1$ (23)

Similarly, the coefficients c_5, c_6 has been calculated and given as follows.

$$c_5 = -\frac{2bc_2 + \omega^2 c_3}{12}$$

Substituting the values of (c_2, c_3) the following equation is obtained.

$c_5 = \frac{\omega^4}{120}c_1 + \frac{b\omega^2}{15}c_0$ (24)

$$c_6 = -\frac{2bc_3 + \omega^2 c_4}{30}$$

$c_6 = (-\frac{\omega^6}{720} + \frac{b^2}{45})c_0 + \frac{b\omega^2}{60}c_1$ (25)

Similarly the coefficient c_7 has also been calculated and given below.

$$c_7 = -\frac{2bc_4 + \omega^2 c_5}{42}$$

$c_7 = \frac{b\omega^4}{2520}c_0 - \left(\frac{\omega^6}{5040} + \frac{b^2}{126}\right)c_1$ (26)

Now the final equation of $'v'$ in terms of the all coefficient is expressed by the following equation (27).

$v = (c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 + c_5 x^5 + c_6 x^6 + c_7 x^7 + \cdots)$ (27)

$$v = [\, c_0 + c_1 x - \frac{\omega^2}{2}c_0 x^2 + \left(-\frac{\omega^2}{6}c_1 - \frac{b}{3}c_0\right)x^3 + \left(\frac{\omega^4}{24}c_0 - \frac{b}{6}c_1\right)x^4 + \left(\frac{\omega^4}{120}c_1 + \frac{b\omega^2}{15}c_0\right)x^5 + \left\{\left(-\frac{\omega^6}{720} + \frac{b^2}{45}\right)c_0 + \frac{b\omega^2}{60}c_1\right\}x^6 + \left\{\frac{b\omega^4}{2520}c_0 - \left(\frac{\omega^6}{5040} + \frac{b^2}{126}\right)c_1\right\}x^7 + \cdots]$$

$v = c_0\left\{1 - \frac{\omega^2}{2}x^2 - \frac{b}{3}x^3 + \frac{\omega^4}{24}x^4 + \frac{b\omega^2}{15}x^5 + \left(-\frac{\omega^6}{720} + \frac{b^2}{45}\right)x^6 + \frac{b\omega^4}{2520}c_0 x^7 + \cdots \ldots \ldots \ldots\right\} + c_1\left\{x - \frac{\omega^2}{6}x^3 - \frac{b}{6}x^4 + \frac{\omega^4}{120}x^5 + \frac{b\omega^2}{60}x^6 - \left(\frac{\omega^6}{5040} + \frac{b^2}{126}\right)x^7 + \cdots \ldots \ldots\right\}$ (28)

The two series in the parentheses of the above equation (28), are nothing but the power series expansion of two linearly independent solutions of equation (13) and the coefficients are called arbitrary coefficient. In the next section we will be verifying the validity of the two linearly independent solutions of equation(28) via simulation.

4. Experimental Result and Stability Analysis:

It is generally observed that, almost all nonlinear system contains additional frequency component. Now we are going to simulate the proposed system based on the FRF. Before going to simulate the proposed system, the SSN is tested at different excitation level which is given in the figure 9. In this section all the simulations are done with the help of MATLAB 17.0 Software.

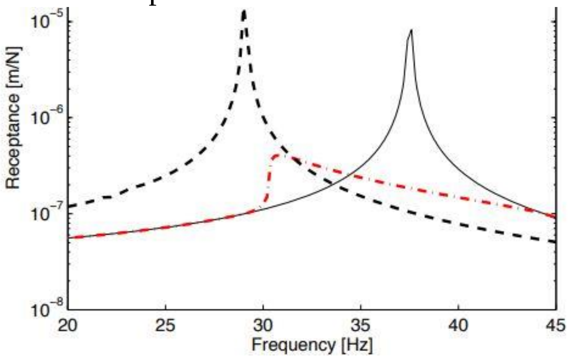

Figure 9. Effect of 'SSN' for different excitation level.

From the above simulation it is observed that, there are two nonlinear regions exists which implies two series solutions of the system taken into consideration. Anyway, at the low level of input frequency, the never goes into the slip region but the stiffness is maximum. Similarly at the high level of input frequency, the system shows the maximum displacement but the stiffness gets lowered. In the middle region, the system shows an uncertain nonlinearity that cannot be explained by our series solution or we can say that, this is the limitation of our work. Now the nonlinear coefficient are studied with respect to the FRF and given in the Figure 10.

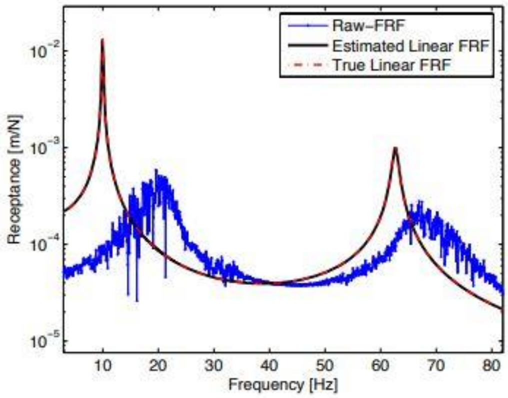

Figure 10. Plot of 'Receptance of the system' versus frequency

In the above figure 10, Raw FRF, Estimated FRF and purely linear FRF are plotted at a time considering the entire series solution. It is observed from the figure that, the system estimated or expected FRF is almost equal to the linear FRF whereas the raw FRF is very uncertain in nature that implies the inherent nonlinearity presents within the system. Let us suppose P_1 and P_2 are representing nonlinear coefficients for the two parts of the series solution, then we can simulates it individually and given in the following figure 11.

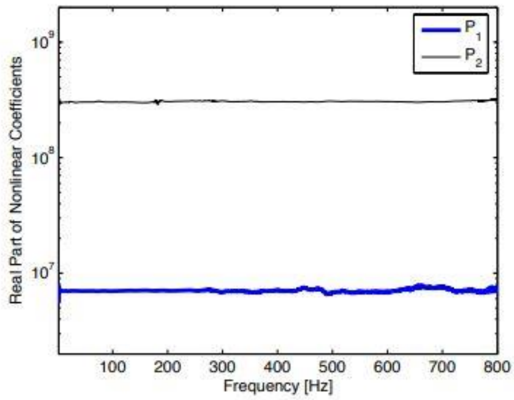

Figure 11. Real part of nonlinear coefficient versus Frequency.

From the above figure it is found that, nonlinear truncated coefficient for both the section of series solution are constant which implies a good frequency response of the entire design. Now we are interested to observe the coherency or the similarity of the nonlinear coefficient which is given in the figure 12 below.

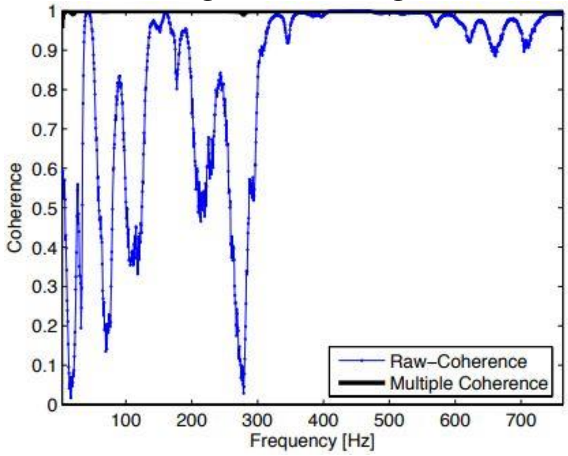

Figure 12. Coherence versus frequency plot

Figure 12 is describing the coherency of the nonlinear coefficient with respect to the frequency. From the simulation it can be concluded that raw coherency between input and output of the aircraft system taken into consideration is not good which implies another limitation of the series solution technique. But one interesting thing can be concluded that the multiple coherencies is almost equals to the unity for all frequencies that is the another aspect of our proposed method. Finally the stability analysis of the entire system is done using nonlinear simulation process and depicted in the following figure 13(a) and 13(b).

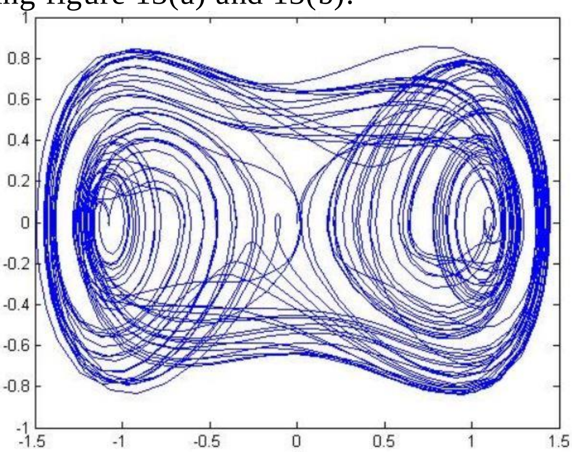

Figure 13(a). Phase portrait of the first part of the series solution

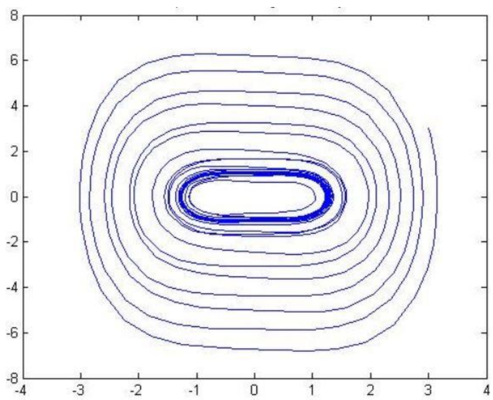

Figure 13(b). Phase portrait of the Second part of the series solution

Both the figure 13(a) and 13(b) are representing the phase portrait of the series solution. The first part of the series solution is simulated and given in the 13(a) while the first part of the series solution is simulated and given in the 13(b). The axes of the phase portrait are delineating the state variables corresponding to the equation. From the figure 13(a), it is observed that, the trajectory is not converging and hence no stable limit cycle is found. Though the trajectory is not extended beyond a limit, still it is better to avoid the first part of the solution .This may happen due to the abrupt truncation of the series. Anyway if we look about the phase portrait of the second part of the series solution, a stable limit cycle is found as the trajectory is not diverging rather converging which implies the stability for any nonlinear system. Thus it can be concluded that the second part of the series solution gives more accurate or desired result with respect to the first part of the solution.

5. Conclusion:

There are so many conclusions can be made from this research article. As almost any physical system working in the field of instrumentation is nonlinear by nature, hence the study of the nonlinear analysis is very essential to find the exact output of the system. Generally the analysis of nonlinear dynamical equation of any system is executed with the help of some often used traditional method. For example: Very popular ‘Describing Function Method’, ‘Isocline Method’, ‘Jacobian formulation’ etc. But here we are applying the concept of ‘Truncated Series Solution’ method on the nonlinear dynamical equation that helps to study the system in another aspect means the we are also able to observe the frequency response function of the system at a time. Not only that, using this process, the nonlinear coefficient of the system using the ‘FR Function’ can easily been observed. On the other hand, the coherency exist between the nonlinear coefficients of the system can also be analyzed that is really difficult if we gone through the conventional methods. From the frequency response plot we can select the proper series solution among all solutions for the typical dynamical system. Applying the proposed method, multiple coherencies between the nonlinear coefficient can be visualized which is another benefit of using this type of procedure. Any proposed method regarding nonlinearity must have an ability to check the stability of the system. Here also the stability analysis is done via numerical simulation (Phase Portrait) and it is found the system is asymptotically stable due to its converging limit cycle. There are so many solutions can be obtained from the series solution procedure. The beauty of the proposed technique is that, the perfect solution can also be opted after observing the simulated output related to stability which is the uniqueness of this research article.

Acknowledgments

Not all professionals do their work by themselves. Although they can be as prolific or as adept in their respective fields, they will still need assistance one way or another. First and foremost, praise and thanks to the God, the Almighty for his showers of blessing throughout my research work to complete this research work successfully. I would like to express my special thanks to our Principal (Dr. Jinia Datta) for constant inspiration, one of our research team member (Ms. Payel Chakraborty) for hard working mentality and my guide (Dr. Biswarup Neogi) to give me the idea to do this beautiful research work. As a result I came to know about so many new thinks. I am really grateful to them. Any attempt at any level cannot be satisfactory completed without the support and guidance of the team member..

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