

Theory of Runs Sampling Plan through Maximum Allowable Percent Defectives

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Abstract

In this paper, Statistical Quality Control of acceptance sampling plan approaches and their applications-oriented articles are considered. In this study on a new design of the theory of runs sampling plan concerning a single sampling plan through maximum allowable percent, defective is used. This plan is used to introduce a new product in the first trial item inspection. The product trial is very important for the sustainability of the product in the long run. This design is used to inspect each item one by one, and fraction defectives have to be much less than or equal to 1%. Then the product is get accepted and the process is continued. The main aim of this design is to make the producer gives the product and the consumer buy it with maximum quality and a minimum of risk. Product quality and consumer satisfaction are vital as far as a product is concerned. So, it helps to save money and time, and the item level will be increased. Producer risk level and consumer risk level will be minimized.

Keywords: *Theory of Runs, Maximum Allowable Percent Defective (MAPD), Poisson distribution, producer and consumer risk.*

1. Introduction

Statistical Quality Control is the use of statistical methods in monitoring and maintaining the quality of products and services. The first method, referred to as acceptance sampling, can be used when a decision must be made to accept or reject a group of parts or items based on the quality found in a sample. A second method, referred to as statistical process control, uses graphical displays known as control charts to determine whether a process should be continued or should be adjusted to achieve the desired quality. Assume that a consumer receives a shipment of parts, called a lot, from a producer. A sample of parts will be taken and the number of defective items counted. If the number of defective items is low, the entire lot will be accepted. If the number of defective items is high, the entire lot will be rejected. Correct decisions correspond to accepting a good-quality lot and rejecting a poor-quality lot. Because sampling is being used, the probabilities of inaccurate decisions need to be considered. The error of rejecting a good-quality lot creates a problem for the producer; the probability of this error is called the producer's risk. On the other hand, the error of accepting a poor-quality lot creates a problem for the purchaser or consumer; the probability of this error is called the consumer's risk.

The design of an acceptance sampling plan consists of determining a sample size n and an acceptance number c , where c is the maximum number of defective items that can be found in the sample and the lot still be accepted. The key to understanding both the

producer's risk and the consumer's risk is to assume that a lot has some known percentage of defective items and compute the probability of accepting the lot for a given sampling plan. By varying the assumed percentage of defective items in a lot, several different sampling plans can be evaluated and a sampling plan selected such that both the producer's and consumer's risks are reasonably low.

This paper is organized as follows: section 1 is the conceptual framework of acceptance sampling. Section 2 gives varied references to the acceptance sampling plan and Theory of Runs sampling plan. Based on the Theory of Runs sampling application and procedures are introduced in section 3. The notion and table of the theory of runs sampling plan through MAPD values are in section 4. Section 5 concludes the paper.

2. Existing System of Theory of Runs (TR)

Run statistics and patterns in a sequence of Bernoulli trials have been successfully used in various areas, such as hypothesis testing in Wald and Wolfowitz [48]. The solicitations of success run of theory to quality control application date back to Wolfowitz [49]. The theory of exact and joint distribution for the numbers of runs and patterns in the sequence of multi-state trials by James c Fu [24]. Balakrishnan et.al.[3] in comprehensive the various functions under the Markov model and also consider the cases of corrective actions taken in start-up demonstration tests. Yi-Ling Lin and Ananda Jayawardhana [50] the accessible a sequence of two kinds of symbols and joint distribution to the theory of runs. Mohamad Adam Bujangand FatinEllisya Sapri [35] in this article proposed that the run test has to be performed during a pilot study to ensure whether there is a random order in the age of those patients coming to the clinic. It is not recommended to perform the runs test in full-scale research because the P-value will then become too small if the sample size is large, which will make it unnecessary to conduct the test (since the small P-value is resulting from a large sample size, and is not due to the presence of a real statistical significance).A formal acceptance sampling plan based on the theory of success runs without fixing the number of trials was deliberate by Vance and Donald [46]. Christophe ley and day Paindaveine [8] imitative for the concept of runs to the multivariate runs statistic of the form. This paper considers the multiple-run sampling plan introduced by Govindaraju and Lai [17]. Antzoulakos et.al. [2] in the empirical power of the new randomness test was compared to the empirical power of the randomness test based on $M_{n,k}$. The evaluation of the operational characteristics curves of the test was achieved with the aid of Monte Carlo techniques.

Brad Johnson and James Fu [4] in this document the asymptotic relative error of the normal, Poisson, compound Poisson, and finite Markov chain imbedding and large deviation approximations; and provide some numerical studies to compare these approximations with the exact probabilities for moderately sized n. Both theoretical and numerical results show that, in the relative sense, the finite Markov chain imbedding approximation performs the best in the left tail and the large deviation approximation performs best in the right tail. Christensen Gillingham et.al. [7] in this article proposed in forecasts of long-run economic growth are critical inputs to policy decisions being made today on the economy and the environment. Despite its importance, there is sparse literature on long-run forecasts of economic growth and the uncertainty in such forecasts. David Boyd [9] presented that if one has played a game such as flipping a coin a large number of times, one tends to remember long runs of wins or losses rather than the individual distribution of wins and losses. It may even appear that the game is not fair if a long run of bad luck occurs. For this reason, we were

led to investigate the distribution of the maximum run of losses in a sequence of n Bernoulli trials, given a probability p of a win and $q=1-p$ of a loss at each trial. The distribution function $F(n, k)$ is the probability that there is no losing run of length greater than, k in a sequence of n trials.

Frederik Beaujean and Allen Caldwell [14] described for simplicity, the construct of statistic only for success runs, the same steps can be taken to define an analogous statistic for failure runs as well. Dillard [16] defined the two distribution-free procedures that have been analyzed, and each can be applied in signal detection problems with little loss in signal detectability compared with optimum procedures that require knowledge of the probability distribution of the observed data. The procedures are easily implemented and analyzed and should be applicable in many existing and future navy communications systems. George Dillard and John Rickard [15] in this paper described a distribution-free Doppler processor (DFDP) that applies to the detection of signals with unknown phases. It is assumed that the received signals are Doppler shifted such that the signal phase may change from one observation to the next. The results presented here indicate that the DFDP can be applied effectively to a Doppler radar without seriously degrading radar performance.

James Fu and Koutras [23] simply unified approach to the distribution theory of runs based on the finite Markov chain approach to the exact distributions for the number of specified runs and patterns in the sequence of Bernoulli trials. Mood [1] in this paper consequent distribution of runs of given length both from random arrangements of fixed numbers of elements of two or more kinds of elements and moments for runs of two kinds of elements, asymptotic distributions from the binomial and multinomial population. Prairie et.al. [37] in this article attribute sampling plans are based on the theory of runs in which a batch is if a run of success is observed in fixed trials. Jennifer L. Lawless and Richard Fox [25] although young women are less likely than young men ever to have considered running for office, they are just as likely as men to respond positively to encouragement to run. Early parental support for a political career, therefore, is a vital ingredient for closing the gender gap in political ambition. Karl Pearson [27] regarded the distribution of runs as a special case of the multinomial distribution. Karl Marple [26] derived an expression for the mean of the number of iterations of a given length from a binomial population. Wishart and Hirshfeld [22] in the distribution of the total number of runs in samples from a binomial population showed it was asymptotically normal.

Koutras [30] have developed Run and pattern problems that have attracted the attention of probability and statisticians. The classical framework for a fixed length run-related problem. Lipping Wang et.al. [29] the agricultural management in the drought area for the different regions and seasons. The theory of runs was used to separate the drought duration and severity of the 29 stations. Rosalba Garcia-Millan and Gunnar Pruessner [40] in this paper premeditated a model of active motility known as Run and Tumble (RnT) motion. Derived Doi-Peliti field theory and use it to calculate the entropy production. Reinour Heijungs [38] an ultimate consequence is that such pedigree-based probability distributions are incompatible with large-scale Monte Carlo simulations. Richard arratia et.al. [39] in this article determined an example in which compare segments taken from the DNA sequence of the bacteriophage lambda. Srinivasan [45] in the correspondence is to describe distribution-free processors based on the theory of runs. Detector A and detector B are described. The distribution-free processors are based on the theory of runs. Detector A and detector B are described. The distribution-free method suggested in this correspondence,

besides being extremely general in the sense that it requires no assumption about the functional form of the distribution of noise, is also extremely simple. No complex analysis of distribution theory is needed. The simple binomial, Rayleigh, and Gaussian distributions provide the necessary tools for the analysis. Frank Wilcoxon described the comparison of ranks [13]. Vujicayevjecvich [47] proposed that some elements of run theory may be applied to the estimation of drought likelihood, and he was among the first attempts at blending probabilistic aspects of run theory with the prediction of drought likelihood. Then fluctuations of wet and dry years particularly to the division of the United States into six areas. Michael [33] provided a simple example of the theory of runs. Ignacio Rodriguez-Iturbe [21] presented the application of the theory runs to hydrology. LemuelMoye [28] in the presentation of these computations was tested using Texas precipitation records. MingkeCai and Xiaoling Su [34].Ziluq Zhang and Gunnar Pruessner [51] in this work determined the properties of the potential that optimally rectifies the steady-state current of RnT particles.

2.1.Literature Review of Single Sampling Plan (SSP)

Peach and Littauer[36] have developed an iterative algorithm for finding the parameters of a single sampling plan satisfying the conditions such as $P_a(p_1) \geq 1 - \alpha$ and $P_a(p_2) \leq \beta$ under the condition of Poisson distribution based on the percentiles of the OC function using χ^2 approximation. Such similar algorithms were developed by Gunther [19]-[18]. Burgess [5] presented a graphical method of determining a single sampling plan when it is required that its OC curve should pass through two designated points. Cameron [6] approved the unity value approach under the condition of the Poisson model and the operating ratio for determining the parameters of single sampling plans by attributes and for computing the points for the plot of associated OC curves. Hamaker [20] gave the procedure and tables for finding the sample size and acceptance number of a single sampling plan for given two points on the OC curve $(p_1, 1 - \alpha)$ and (p_2, β) using normal approximation to the binomial distribution. By using this procedure at any point $(p_1, 1 - \alpha)$ and (p_2, β) may be specified and the applicable sample size and acceptance number can be found quite straightforwardly based on the formula for n. Dodge and Romig [11] have proposed a procedure for the selection of a single sampling plan indexed through AOQL by minimizing the Average Total Inspection. This book published by Shilling [41] contains a set of tables for the construction and evaluation of matched sets of single, double, and multiple sampling plans. Soundararajan in [42] has suggested a procedure for the selection of a single sampling plan in terms of AQL and AOQL. Fangyu Liu and Lirong Cui [12] have suggested a new design for an attribute single sampling plan.

2.2. Literature Review of MAPD

The proportion non-conforming corresponding to the inflection point on the OC curve, denoted by p^* and interpreted as the Maximum Allowable Percent Defective (MAPD) by Mayer [32] is also used as the quality standard along with some other conditions for the selection of the sampling plans. Mandelson [31] has explained the desirability of developing a system of sampling plans indexed through MAPD. Soundararajan has constructed tables for the selection of a single sampling plan indexed through MAPD. Suresh

and Ram Kumar [44] have studied the selection of a single sampling plan indexed through Maximum Allowable Average Outgoing Quality (MAAOQ) and MAPD.

3. Theory of Runs (TR) Sampling Plan

A run is defined as a succession of similar events preceded and succeeded by different events; the number of elements in a run will be referred to as its length. The distribution of the theory of runs had a stormy career. The theory seems to have been started toward the end of the nineteenth century rather than in the days of Laplace when there was so much interest in games of chance.

Probability of acceptance of the process

$$P_n(s) = \sum_{r=1}^n x_r = q^s + \sum_{r=s+1}^n pq^s = q^s[(n-s)p + 1] \tag{1}$$

Thus OC curves for the most practical range of TR-1 plans (1) can be easily computed. Examples are given in Figure 1. The generating function D (t) for the probability of a run of length s for the first time at the nth trial.

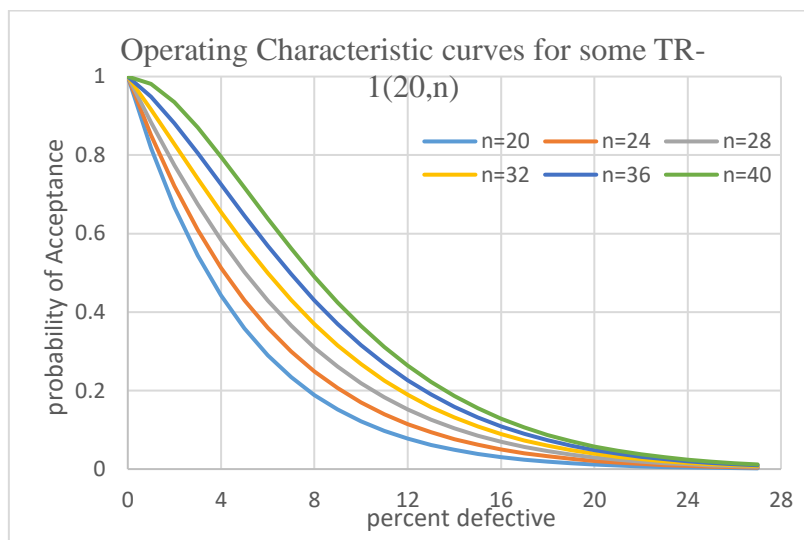


Figure 1 Operating Characteristic curves for some TR-1 (20, n)

$$P_n(s) = \sum_{r=1}^n a_r \quad \text{Where } D(t) = \sum_{r=1}^{\infty} a_r t^r$$

The expected number of trials (ENT) to decide for $n \leq 2s$ is

$$\sum_{r=1}^n r \{ \text{Probability of making a decision at the } r\text{th trial} \}$$

The probability of rejection at the rth trial Y_r is

$$y_r = \begin{cases} 0, & 1 \leq r \leq n - s \\ pq^{r+s-n-1} - \sum_{i=s}^{r-1} pq^{r-i-1}x_i & n - s < r \leq n \end{cases}$$

Inserting the values of x_i

$$y_r = \begin{cases} 0, & 1 \leq r \leq n - s \\ pq^{r+s-n-1} - x_r & n - s + 1 \leq r \leq n, \quad r \neq s \\ 0, & r = s, \quad n = 2s \\ pq^{2s-n-1}, & r = s \quad n < 2s \end{cases}$$

$$ENT = \sum_{r=1}^n r(x_r + y_r) = sq^s + \sum_{r=n-s+1}^n rpq^{r+s-n-1}$$

$$ENT = [q+n + s - 1)p] \left[\frac{1 - q^s}{1 - q} \right] \tag{2}$$

This paper developed the Theory of Runs in (1) and (2) based on the Prairie and Zimmer and Brokehouse (1962).

3.1. Extension for (1) in the Probability of acceptance under the Theory of Runs sampling (TR) plan is

$$P_n(S) = \begin{cases} 0, & r < s \\ pq^f & r = s \\ q^f(f/s), & s < r < t \end{cases} \tag{3}$$

t= Total number of trials

s=success of lot

f= fraction of defective (0.001 < f < 0.009)

Where the probability of acceptance of reference for a single sampling plan

The reference plane for the SSP plan is

$$p = \sum_{d=0}^c \frac{e^{-np} np^d}{d!}$$

n =sample size and, c=acceptance number

3.2. The procedure of the Theory of Runs Sampling Plan

As the steps of the procedure follow the Theory of Runs Sampling Plan

Step 1: The decision to accept is made as soon as a run of s is attained.

Step 2: The performance of one item is independent of the performance of the preceding items.

Step 3: The quality of the process, defined in terms of the failure rate (p=1-q) remains constant.

Step 4: Start with a normal inspection, using the reference plan.

Step 5: When a pre-specified number, s of consecutive lots is accepted, switch to inspecting only a fraction f of the lots. The selection of the members of that fraction is done at all items.

Step 6: When a lot is rejected solve the product problem then return to normal inspection.

The parameters t, s, and f are essential to calculating the probability of acceptance for a theory of runs sampling plan.

3.3. Application of Theory of Runs Sampling Plan

It is generally agreed that early production of any item should be adequately tested to demonstrate that the design is manufacturable and that the quality requirements of the process can be met. TR-1 plans have effective application in certain aspects of the testing of early production.

Under the condition that the failure rate remains constant, it is possible to find, for each TR-1 plan, in many experimental situations, especially in early production, the assumption that the probability of failure is constant for each successive trial is not very plausible. For example, when a new process is begun, a few minor faults are expected to occur in the first few trials, but as the process continues, the occurrence of such faults should become less frequent. Similarly, a craftsman producing a new type of product would expect to become more expert at the job in later trials and so make fewer and fewer faulty products. It is in these situations that we believe TR-1 plans could find acceptance as a means of evaluating the capability of the process to produce a good quality product. Many of the situations in which the process failure rate changes in early production can be described reasonably well by one of the following two general models:

- 1) The case where the probability of an unsuccessful trial is initially fairly high and reduces monotonically to some final value, and
- 2) The case where the probability of an unsuccessful trial is initially fairly high and remains so until the r^{th} trial, at which point the cause of failure is essentially eliminated and the probability of failure becomes very small.

To give some basis to our belief that TR-1 plans can be used effectively in these situations, a specific model will be chosen for each of the above cases and sampling plans for them investigated.

Illustration 1

To check the trail for 1500 products for the first time makes it difficult hence here for the trail of product cell phones, 300 samples are chosen, if it produces a fraction of defective between $0.001 < f < 0.009$. The trial product will be taken for mass production. If not the error the product will be made for reconstruction and further taken for trials.

Illustration 2

This example gave Wolfowitz 1943 [49], made use of runs of this kind to study what they called "the persistence of weather", i.e., whether dry months tend to follow dry months and wet months to follow wet months. In a long series of weather observations, the months were classified as wet or dry and a four-fold table was constructed of the number of months falling into each of the following categories:

- (a) Wet month following a wet month
- (b) Wet month following a dry month
- (c) Dry month following a wet month
- (d) Dry month following a dry month.

The chi-square test was applied to the four-fold table to test the null hypothesis that the probability of whether a month was wet or dry was independent of what its predecessor had been.

4. Construction of tables

According to Mendelsohn the MAPD is defined as $p^*=c/n$ when lot quality assumes the theory of runs for the Poisson model. This implies that $np^*=c$

The AOQ is defined as $p Pa(p)$ then, we have

$$MAAOQ=AOQ \text{ at } p=p^*$$

Which is $p^* Pa(p^*)$ and

$$MAAOQ=AOQ =p^*Pa(p^*) \text{ at } p=p^*$$

Which is $p^*Pa(p^*)$, and

One of the desirable properties of an OC curve is the decrease of $Pa(p)$. Should be lower for smaller values of p and steeper for higher values of p , which provides better overall discrimination. Since p corresponds to the inflection point of an OC curve, it implies that,

$$\frac{d^2Pa(p)}{dp^2} < 0 \quad \text{for } p < p^*$$

$$\frac{d^2Pa(p)}{dp^2} = 0 \quad \text{for } p = p^*$$

$$\frac{d^2Pa(p)}{dp^2} > 0 \quad \text{for } p > p^*$$

$$MAAOQ=p^*Pa(p^*),$$

Which is a function of c . then, for any specified values of c , the unique values of R_1, R_2 and R_3 are listed in Table 1.

4.1. For specified MAAOQ and MAPD

Table 1 is used to construct the plans when the MAPD and MAAOQ are specified. For any given values of the MAPD (p^*) and MAAOQ, one can find the ratio $R_1=MAAOQ/MAPD$ which is a function of c alone and strictly decreasing. In this context, SSPs with $c=0$ are not considered, since $c=0$ plans do not involve an inflection point on the OC curve. Find the value in Table 1 under column R_1 which is equal to or just less than the specified ratio. Then the corresponding value of c is noted. From this, one can determine the parameters n and c for the SSP. Parameters t, s , and f for the theory of runs sampling plan.

Table 1. Values of R_1, R_2 and R_3 for specified values of c

t	s	f	c	R_1	R_2	R_3
1500	300	0.001	1	0.0955	0.2113	2.2123
			2	0.0768	0.2366	3.0815
			3	0.0655	0.2577	3.9350
			4	0.0576	0.2804	4.8666
			5	0.0518	0.2998	5.7933
		0.003	1	0.1033	0.2142	2.0732
			2	0.0847	0.2397	2.8307

			3	0.0734	0.2608	3.5545
			4	0.0655	0.2825	4.3124
			5	0.0596	0.3020	5.0638
		0.005	1	0.0200	0.0517	2.5873
			2	0.0925	0.2427	2.6229
			3	0.0812	0.2638	3.2479
			4	0.0734	0.2846	3.8774
			5	0.0675	0.3042	4.5047
		0.007	1	0.0280	0.0515	1.8376
			2	0.1004	0.2456	2.4476
			3	0.0891	0.2668	2.9953
			4	0.0813	0.2866	3.5272
			5	0.0754	0.3064	4.0633
		0.009	1	0.0360	0.0522	1.4490
			2	0.1082	0.2486	2.2977
			3	0.0969	0.2698	2.7840
			4	0.0891	0.2886	3.2389
			5	0.0833	0.3085	3.7052
1500	500	0.001	1	0.0935	0.2104	2.2506
			2	0.0748	0.2358	3.1520
			3	0.0635	0.2568	4.0447
			4	0.0556	0.2797	5.0286
			5	0.0497	0.2990	6.0105
		0.003	1	0.0973	0.2117	2.1751
			2	0.0787	0.2370	3.0133
			3	0.0674	0.2581	3.8307
			4	0.0595	0.2803	4.7094
			5	0.0537	0.2997	5.5854
		0.005	1	0.1012	0.2130	2.1053
			2	0.0825	0.2383	2.8876
			3	0.0712	0.2593	3.6399
			4	0.0634	0.2809	4.4303
			5	0.0575	0.3003	5.2184

Illustration 3

Given MAAOQ = 0.006 and MAPD (p^*)= 0.07, compute the ratio $R_1 = \text{MAAOQ} / \text{MAPD} = 0.0857$ and select the value of R_1 equal to or just less than 0.0857 using Table 1. The corresponding value of R_1 is 0.0847, which is associated with $c=2$. From this, one can find the sample size $n = c/p^* = 2/0.07 = 0.29$. Thus, $n = 29$ and $c = 2$ are the parameters selected for the TR sampling plan, for which the OC curve has an MAPD of 0.0688 and a MAAOQ of 0.0058 defective.

Illustration 4

Given $AOQL=1\%$ and $MAPD=4.7\%$ Compute the ratio $R_2 = AOQL/MAPD=0.212$. Select the value of R_2 from Table 1 which is nearest to this ratio. The corresponding values of $s=300, f=0.001$ and $R_2=0.2112, s=500, f=0.001, R_2=0.210$ which are associated with $c=1$ respectively.

4.2. For specified nAOQL or n MAAOQ

Table 2 is used to construct a plan when the sample size n and the AOQL or MAAOQ are specified. Find the value of $nMAAOQ$ or $nAOQL$, which is a monotonic increasing function in c find the value in Table 2 under the $nMAAOQ$ or $nAOQL$ value which is equal to or just less than the calculated value. Then the corresponding value of c is noted. From this, one can determine the parameters $t, s, f,$ and c for the TR.

Table 2 values of n MAAOQ and n AOQL for specified values of s, f, and c

s	f	c	n AOQL	n MAAOQ
300	0.001	1	0.8452	0.3820
		2	1.3665	0.4434
		3	1.9173	0.4872
		4	2.5352	0.5209
		5	3.1755	0.5481
	0.003	1	0.8575	0.4136
		2	1.3845	0.4891
		3	1.9409	0.5460
		4	2.5549	0.5924
		5	3.2001	0.6319
500	0.001	1	0.8419	0.3740
		2	1.3614	0.4319
		3	1.9106	0.4723
		4	2.5285	0.5028
		5	3.1671	0.5269
	0.003	1	0.8474	0.3896
		2	1.3694	0.4544
		3	1.9208	0.5014
		4	2.5348	0.5382
		5	3.1750	0.5684
	0.005	1	0.8529	0.4051
		2	1.3773	0.4769
		3	1.9310	0.5305
		4	2.5411	0.5735
		5	3.1828	0.6099

Table 3 values of MAAOQ, AOQL, and AQL for selected TRs plan in percent defective

n	c=1			c=2			c=3			c=4			c=5		
	AQL	AOQL	MAAOQ	AQL	AOQL	MAAOQ	AQL	AOQL	MAAOQ	AQL	AOQL	MAAOQ	AQL	AOQL	MAAOQ
15	2.510	5.797	2.968	0.108	9.395	3.563	0.421	12.683	4.031	1.031	14.613	4.423	1.892	15.488	4.772
25	1.520	3.404	1.262	0.061	5.547	1.406	0.249	7.965	1.532	0.604	10.404	1.739	1.138	12.783	1.955
50	0.764	1.726	0.533	0.032	2.808	0.626	0.130	3.922	0.707	0.309	5.114	0.778	0.594	6.377	0.856
65	0.584	1.315	0.533	0.023	2.145	0.625	0.096	3.043	0.706	0.235	3.929	0.777	0.437	4.901	0.855
84	0.452	1.035	0.334	0.018	1.669	0.390	0.074	2.338	0.438	0.182	3.064	0.483	0.342	3.838	0.529

Table 3 is used to compare the plans, the MAAOQ is comparatively easy compared with that of the AOQL because the AOQL is a solution of a complicated expression. $MAAOQ < AOQL$ implies that higher consumer protection is guaranteed on a MAAOQ plan. The AOQL is the maximum AOQ over all incoming lots.

Illustration 5

Given $n=65$. $MAAOQ=0.01$, AOQL, compute the values of $nMAAOQ$ and $nAOQL$. Select the respective values from Table 3. The nearest values are $MAAOQ = 0.0053$ and $AOQL=0.0135$, with respective c values 1. Thus, the sampling plan ($n=65$, $c=1$, $f=0.002$, $s=300$, $t=1500$) has $MAAOQ=1\%$.

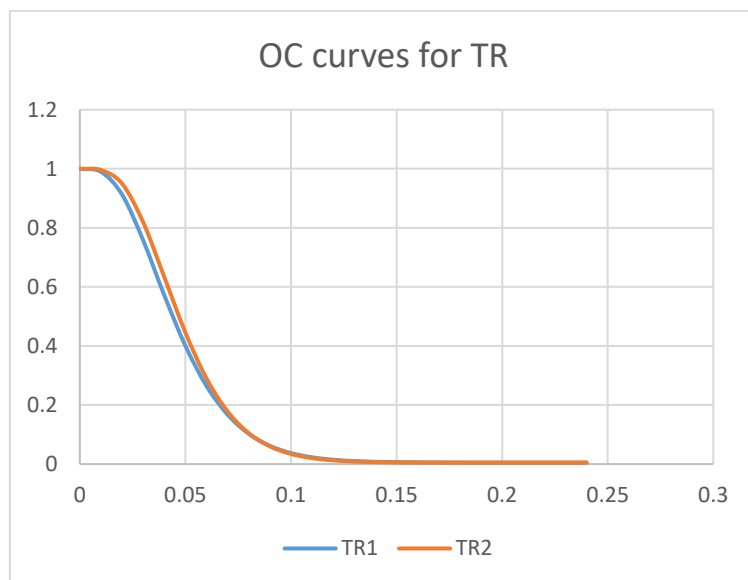


Figure 2: OC curves of the TR with fixed parameters1, TR1. MAPD =0.04, AOQL=0.023. TR2. MAPD=0.04, MAAOQ=0.0048

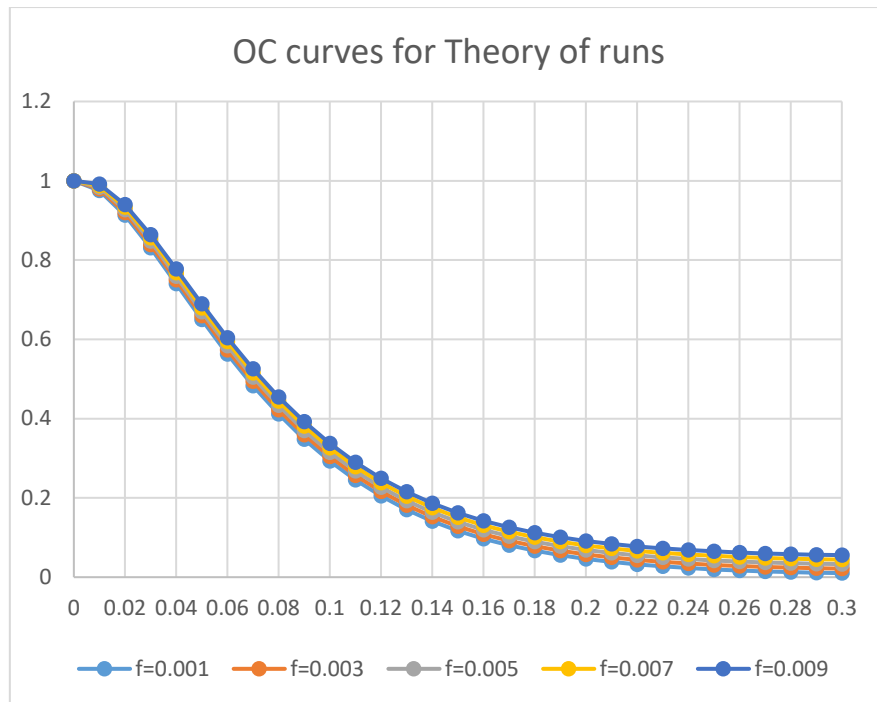


Figure 3: Operating characteristic curves for the theory of runs sampling plan for Comparison of the f values.

Comparison of OC curves:

Figure 2 indicates that the sample size efficiently is increased when the plan is indexed through the MAAOQ rather than the AOQL with the MAPD. It also shows that the probability of acceptance is higher at good quality levels.

Figure 3 shows that for a fixed sample size for (t,s,and c) parameters and different f values for comparison the probability of acceptance is higher at good quality levels.

5. Conclusion

In this work, Theory of Runs sampling plan framework and OC curve equations. For comparison, we have also determined the OC curve for different f values. The OC function allows us to obtain acceptance sampling plans that meet both, the producer's and consumers' risks. The OC curve also helps us estimate the adequate sample sizes n required to obtain with high confidence one or more failures in our experiments. The OC function properties allow us to control the quality of our incoming products and estimate certain parameters of interest such as the required sample sizes, which contribute to the design of better and more efficient experiments. They include the use of OC function tables in the theory of runs sampling plan the calculate table MAAOQ, AOQL, AOQ, and MAPD sample sizes minimized and help avoid the iterative procedure.

The implementation of the OC function has been overviewed for several of the most important discrete distributions. Implementations for other distributions can be done following the same principles described here.

Acknowledgments

Special thanks go to Editor-in-chief and Editorial Board Members. The authors would like to thank the reviewers and the referees for their suggestions, which led to the improvement of this paper.

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