Microscopic Body Cosmological Model

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Abstract

In this paper we study Bianchi type-I cosmological model in presence of microscopic body in self-creation theory. We solve field equation by using relation $B = A^m, C = A^n$ *and equation of state of microscopic body. Also we discuss physical and kinematical properties and discussed stability of obtained model.*

Keywords: *Bianchi Model, Microscopic Body, Barber Self Creation Theory.*

1. Introduction

Einstein's general theory of gravitation is a corner stone of modern physics which describes the successful theory of gravitation in terms of geometry. It has also served as a basis for models of the universe. Barber [1] proposed two 'self-creation' theories based on two sets of general relativistic field equations involving matter and a scalar field. In this theory, the gravitational coupling of the Einstein field equations is allowed to be a variable scalar on the space-time manifold. Barber's second theory is a modification of general relativity to include continuous creation and is within observational limits. Thus, it modifies general relativity to become a variable G-theory. In this theory the scalar field does not directly gravitate but simply divides the matter tensor with the scalar acting as a reciprocal gravitational constant.

The Barber field equation in second self-creation theory (Barber, [1]) can be expressed as

$$R_{ij} - \frac{1}{2} R g_{ij} = -8\pi\phi^{-1} T_{ij} \tag{1}$$

and

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$$\Box\phi = \phi^{,k}_{;k} = \frac{8\pi\lambda}{3}T \qquad (2)$$

where ϕ is the Barber's scalar, T_{ij} is the energy momentum tensor,

$\Box$ is the invariant D'Alembertian, T is the trace of energy momentum tensor T_{ij}, λ is a coupling constant to be determined from experiment and $0 \prec |\lambda| \prec 1/10$.

In the limit $\lambda \to 0$, this theory approaches the Einstein's theory in every respect. Due to the nature of the space time Barber's scalar ϕ is a function of 't'.

Reddy [8], Belinchon J.A. et al [5],Bashakara et al[6], Shanti and Rao [7], Mohanty et al [2,10], Adhav et al [4], Ghate [9] etc. are some of the authors who have investigated various aspects of Barber's self-creation theories.

Also, Katore et al [13,17], Parth Shah et al [15], Nasr Ahmed et al [16] have analyze stability of cosmological models.

K.S. Adhav et al [18]have studied Bianchi Type I cosmological model in Lyra manifold. Kandalkar et al[19] have studied homogeneous Bianchi type I cosmological modelfilled with viscous fluid with a varying Λ .S.D.Katore et al [14, 20] have studied Bianchi type I cosmological model.

The purpose of the present work is to obtain Bianchi type-I cosmological model in presence of macroscopic body. Our paper is organized as follows. In section 2, Metric and field Equations**.** Section 3, Solutions of field Equations, Section 4,is mainly concerned with the physical and Kinematical properties of the model and Section 5, Stability Solution. The last section contains some conclusion.

2. Metric and Field Equation

We consider Bianchi type I metric of form

$$ds^2 = dt^2 - A^2\, dx^2 - B^2 dy^2 - C^2\, dz^2 \qquad (3)$$

Where A, B, C are function of t only .

The energy momentum-tensor for a macroscopic body (Landue L. D. and Lifshitz E.M) is given by

$$T^{ik} = (p+\varepsilon)\, u^i u^k - pg^{ik} \qquad (4)$$

Here p is the pressure, ε is the energy density and $u_i$is the four velocity vectors of the distribution respectively. Where u^i will satisfy $u_i u^i = 1$

The energy momentum-tensor of Microscopic Body is given by,

$$T_1^1 = T_2^2 = T_3^3 = -p \quad and \quad T_4^4 = \varepsilon \qquad (5)$$

Using the equations (1), (2) and (4), the field equations of metric (3) are

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC}=-8\pi\phi^{-1}p \tag{6}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC}=-8\pi\phi^{-1}p \tag{7}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}=-8\pi\phi^{-1}p \tag{8}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}=8\pi\phi^{-1}\varepsilon \tag{9}$$

$$\ddot{\phi}+\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)\dot{\phi}=\frac{8\pi\lambda}{3}\left(-3p+\varepsilon\right) \tag{10}$$

Where dot (.) denote differentiate with respect to t .

3. Solutions of Field Equations

The equations (6) to (10) is a system of five independent equations with six unknown A, B, C, ϕ, p and ε . Hence to get a determinate solution one has to assume the relation between metric coefficients i.e. $C = A^n$ *and* $B = A^m$ (J.K. Singh and Navin Ku Sharma) and radiation universe.

$\varepsilon = 3p$

we get,

$$\frac{\ddot{A}}{\dot{A}}+k_1\frac{\dot{A}}{A}=0 \tag{11}$$

The above equation admits an exact solution given by

$$A=k_4\left(k_2t+k_3\right)^{\frac{1}{k_1+1}} \tag{12}$$

$$B=k_4^m\left(k_2t+k_3\right)^{\frac{m}{k_1+1}} \tag{13}$$

$$C=k_4^n\left(k_2t+k_3\right)^{\frac{n}{k_1+1}} \tag{14}$$

where $k_4=\left(k_1+1\right)^{\frac{1}{k_1+1}}$

And the scalar field is given by,

$$\phi=\frac{k_7}{\left(k_2t+k_3\right)^{\frac{m+n-k_1}{k_1+1}}} \tag{15}$$

The pressure and energy density is given by,

$$p=\frac{{k_2}^2k_7}{8\pi\,(k_1+1)^2T^{\frac{m+n+k_1+2}{k_1+1}}}\left\{\frac{n\,(n+m+nm)}{(m+n+1)}\right\} \tag{16}$$

$$\varepsilon=\frac{{k_2}^2k_7}{8\pi\,(k_1+1)^2T^{\frac{m+n+k_1+2}{k_1+1}}}\left\{(n+m+nm)\right\} \tag{17}$$

Where $(k_2t+k_3)=T$

Using equations (12), (13) and (14), the cosmological model in equation (3) takes the form,

$$ds^2=dt^2-k_4^2(k_2t+k_3)^{\frac{2}{k_1+1}}dx^2-k_4^{2m}(k_2t+k_3)^{\frac{2m}{k_1+1}}dy^2-k_4^{2n}(k_2t+k_3)^{\frac{2n}{k_1+1}}dz^2 \tag{18}$$

4. Physical and Kinematical Properties

Spatial Volume

$$V=\sqrt{-g}={k_4}^N(k_2t+k_3)^M \tag{19}$$

Scalar Expansion

$$\theta=\frac{1}{3}\frac{Mk_2}{(k_2t+k_3)} \tag{20}$$

Hubble Parameter

$$H=\frac{Mk_2}{(k_2t+k_3)} \tag{21}$$

Shear Scalar

$$\sigma^2=\frac{M^2k_2^2}{54(k_2t+k_3)^2} \tag{22}$$

Average scale factor

$$a(t)=k_4^{\frac{m+n+1}{3}}\,(k_2t+k_3)^{\frac{m+n+1}{3(k_1+1)}} \tag{23}$$

Where $M=\dfrac{m^2+n^2+2mn+2m+2n+1}{m^2+n^2+mn+m+n+1}$, $\qquad N=\dfrac{m+n+1}{k_1+1}$

Also, the expression for the energy density W, energy flow vector S and stress tensor $\sigma_{\alpha\beta}$ are

$$W=\left(\frac{1+\dfrac{\upsilon^2}{3c^2}}{1-\dfrac{\upsilon^2}{c^2}}\right)c\,\frac{{k_2}^2k_7}{8\pi\,(k_1+1)^2(k_2t+k_3)^{\frac{m+n+k_1+2}{k_1+1}}}\left\{(n+m+nm)\right\} \tag{24}$$

$$S = \left(\frac{4\upsilon}{3\left(1 - \frac{\upsilon^2}{c^2}\right)} \right) \frac{{k_2}^2 k_7}{8\pi (k_1 + 1)^2 (k_2 t + k_3)^{\frac{m+n+k_1+2}{k_1+1}}} \{(n + m + nm)\} \quad (25)$$

$$\sigma_{\alpha\beta} = \left(\frac{4\upsilon_\alpha \upsilon_\beta}{c^2 \left(1 - \frac{\upsilon^2}{c^2}\right)} + \delta_{\alpha\beta} \right) \frac{{k_2}^2 k_7}{8\pi (k_1 + 1)^2 (k_2 t + k_3)^{\frac{m+n+k_1+2}{k_1+1}}} \left\{ \frac{n(n + m + nm)}{m + n + 1} \right\} \quad (26)$$

If the velocity of the microscopic motion is small compared with the velocity of the light ,then we have approximately $S = (p + \varepsilon)\nu$.

Since $\frac{S}{c^2}$ is the momentum density and $\frac{(p+\varepsilon)}{c^2}$ plays the role of the massdensity of the body.

From the expression (4), we get $T_i^i = \varepsilon - 3p$ (27)

But, $T_i^i = \sum_a m_a c^2 \sqrt{1 - \frac{\nu_a^2}{c^2}}\, \delta(r - r_0)$ (28)

Compare the relation (26) with the general formula (27) which we saw was valid for an arbitrary system .Since we are at present considering a microscopic body ,the expression (27) must be averaged over all the value of r in unit volume.

We obtained the result

$$\varepsilon - 3p = \sum_a m_a c^2 \sqrt{1 - \frac{\nu_a^2}{c^2}}$$

Here the summation extends over all particles in unit volume. The right side of this equation tends to zero in the ultra-relativistic limit, So in this limit the equation of state of matter is $p = \frac{\varepsilon}{3}$.

The decomposition of time-like tidal tensor is

$$u_{a;b} = \frac{-k_2}{(k_1 + 1)(k_2 t + k_3)} \left[{k_4}^2 (k_2 t + k_3)^{\frac{2}{k_1+1}} + {k_4}^{2m} m (k_2 t + k_3)^{\frac{2m}{k_1+1}} + n{k_4}^{2n} (k_2 t + k_3)^{\frac{2n}{k_1+1}} \right] \quad (29)$$

And,

Vorticity $\omega_{11} = \omega_{22} = \omega_{33} = \omega_{44} = 0$ (30)

Vorticity of model along *x, y, z* and *t*- axes is zero. So, the obtained model is non rotational. Whereas vorticity is nonzero, model is rotating.

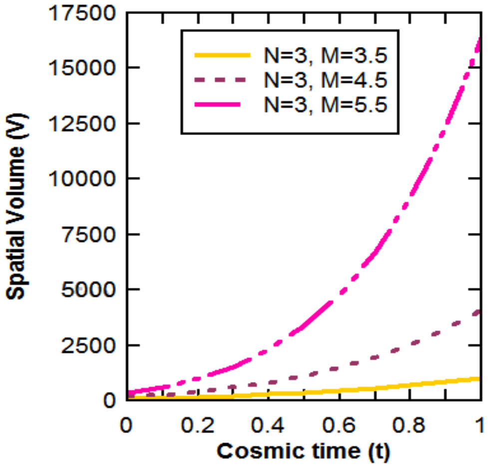

Fig. 1 $k_4 = k_2 = k_3 = 2$

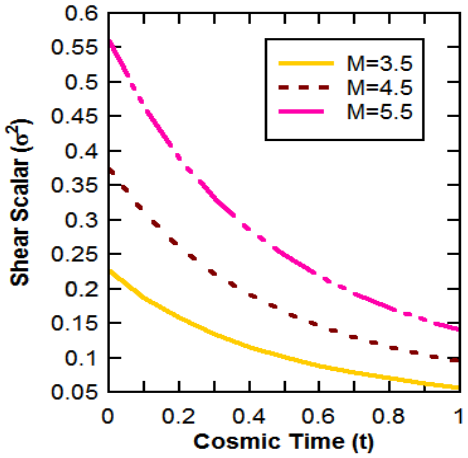

Fig. 2 $k_2 = k_3 = 2$

The behavior of Spatial Volume and Shear Scalar are represented in Fig.(1) and Fig.(2), As cosmic time increase spatial volume increase. But Shear scalar decrease with increase in cosmic time and finally vanish at $t \to \infty$.

Spatial volume and Shear scalar are finite at initial state i.e. $t = 0$.

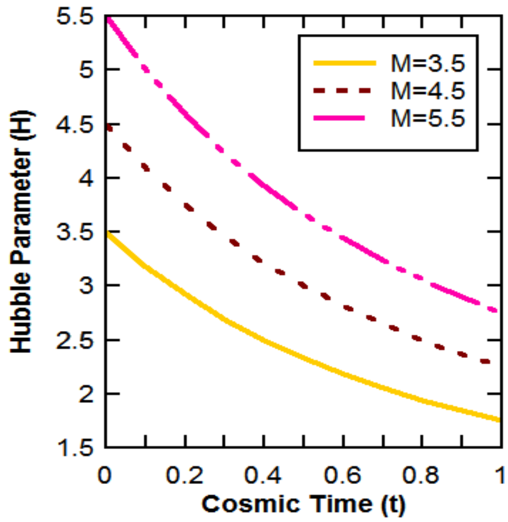

Fig.3 $k_2 = k_3 = 2$

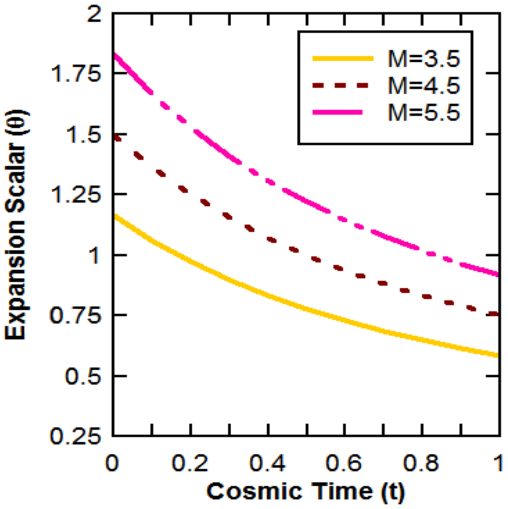

Fig. 4 $k_2 = k_3 = 2$

The behavior of Hubble Parameter and Expansion Scalar are represented in Fig.(3) and Fig.(4), as cosmic time increase Hubble Parameter and Expansion scalar decreases with increase in cosmic time and finally vanishes at $t \to \infty$.

5. Stability Solutions:

We discuss the stability of the model by observing the ratio of sound speed given by $\frac{dp}{d\varepsilon} = {c_s}^2$, when the ratio $\frac{dp}{d\varepsilon}$ is positive i.e. ${c_s}^2 > 0$, we have a stable model. Whereas when the ratio $\frac{dp}{d\varepsilon}$ is negative i.e. ${c_s}^2 < 0$, we have an unstable model.

In this model,

$$\frac{dp}{d\varepsilon} = \frac{n}{m+n+1} \tag{31}$$

From equation (31), it is observed that the ratio of sound speed $\frac{dp}{d\varepsilon}$ is positive for $m, n > 0$, the model is a stable

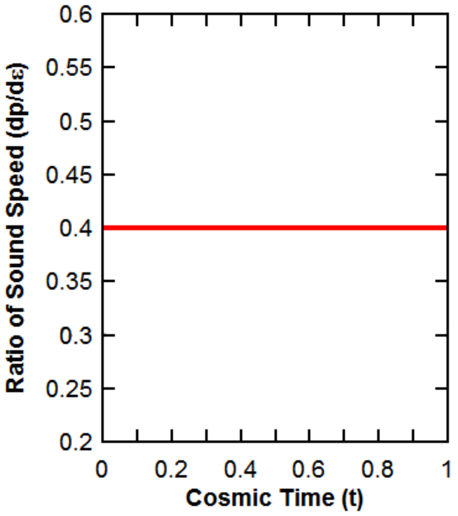

Fig. 5 $m = n = 2$

For any value of m, n model is stable

Universes with Rotation and Shear

In the volume devoted to Einstein's seventieth birthday, Kurt godel [21] proposed the idea of a spinning universe, within the framework of GR, largely to demonstrate the "anti-Machian" result, that in such a universe the distant parts(made of stars, galaxies etc.) rotate with respect to the local inertial frame.In the mid -1950s, Heckmann and Schucking EL[22].looked for spinning models in general with hope that some might turn out to be nonsingular , i.e.without the big bang type singularity. In this they were guided by equation obtained by Raychaudhuri [23]

$$\dot{\theta} + \frac{1}{3}\theta^2 - u^k_{;k} + 2(\sigma^2 - \omega^2) + \frac{1}{2}(\varepsilon + 3p) - \lambda = 0$$

Here σ and ω are respectively ,the shear and spin of the universe θ the rate of volume ε its energy density and p the pressure of cosmic fluid .The velocity vector of the fluid is u^k .It is clear that the spin term goes against the collapse and the singularity while the shear term tends to help them .However ,this equation turns out to not be enough to determine whether the singularity is avoidable: and it was finally established by the singularity theorems in the 1960s that the space time singularity is an inevitable feature of relativistic cosmology, unless one relaxes the so called energy condition ,Hawking and Ellis[24].

The interest in anisotropic models however continued for awhile in the expectation that some large scale observation of the universe would turn up evidence for spin or shear. Observations of distant sources such as galaxies and radio sources show that non rotating with respect to the local inertial frame to within 2.5×10^{-4} arcsecper year, Barbour and Pfister[25]likewise, mappings of the microwave background have put stringent bounds on such anisotropies. Work on Bianchi models has also shown how initial anisotropies are quickly dissipated in an expanding universe. Hence anisotropic models are not currently popular.

Deceleration parameter q: It indicates rate of expansion of our model, it is given as,

$$q=-1+\frac{d}{dt}\left(\frac{1}{H}\right)=-1-\frac{(m+n+1)\,k_2^2}{(k_1+1)(k_2t+k_3)^2} \tag{32}$$

The positive value of q indicates standard decelerating model and negative value of q indicates accelerating expansion of model.

State finder parameter: The state finder parameter is established from space time metric directly which examined the geometrical behavior of the universe. In order to differentiate these model ,a diagnostic proposal that makes use of parameter pair [r,s] called state finder and defined as follows:

$$r\equiv\frac{\dddot{a}}{aH^3}\quad,\qquad s\equiv\frac{r-1}{3(q-\frac{1}{2})} \tag{33}$$

Using equation (21) and (23) in(33) we get,

$$r=\frac{(m+n+1)^2-9k_1(m+n+2+2k_1)+2(m+n)+3}{27(m+n+1)^2} \tag{34}$$

Using equation (32) and(34) in (33)we get,

$$s=\frac{\{(m+n+1)^2-9k_1(m+n+2+2k_1)+2(m+n)+3-27(m+n+1)^2\}\{2(k_1+1)(k_2t+k_3)^2\}}{-81(m+n+1)^2\{(m+n+1)k_2^2+3(k_1+1)(k_2t+k_3)^2\}} \tag{35}$$

Conclusion

At present work, we have considered Bianchi type-I cosmological model in Barber second self-creation theory in existence of macroscopic body. For interpreting the field equations, relation between metric coefficients i.e. and radiation universe are used. The model is expanding, shearing and non-rotating. When $T \to 0$ both density and pressure are infinite. The universe starts expanding with big bang at $T \to 0$.The expansion rate decrease at time proceeds increase when $T \to \infty$ expansion stops where pressure and density become zero.

From equation (30), it is clear that, the ratio of sound speed $\frac{dp}{d\varepsilon}$ is positive for $m \geq 1$ *and* $n > 0$, and the vorticity of model along *x, y, z*

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