B_1 SEMI NEAR RINGS

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ABSTRACT

The theory of semi near rings is an innovative branch of abstract algebra. The theory of semi near rings is different from the near rings of algebra. We justify the above with suitable example. A semi near ring is an algebraic structure similar to a near ring but satisfying fewer axioms. Many semi near ring structures and properties have been established using various sub-structures. Generalised conditions of various structures of semi near rings under certain induced properties like Boolean, regularity are delineated. Injective homomorphism is found to preserve the structure of a B_1 semi near ring added with certain amenities.

Keywords: *Semi near ring theory, Boolean semi near ring, Regular semi near ring, Reduced, Injective homomorphism.*

1. Introduction

Semi near rings has been one of the many concepts of universal algebra which has been proven as a very important and interesting area of study. Applications of semi near rings are plentiful in the mathematical world around us. An example is the set of natural numbers under usual addition and multiplication. Semi near rings appeared implicitly in connection with the study of ideals of a semi near rings. In the Bulletin of American Mathematical Society, the idea of associative algebra or, semi near ring as a set of elements forming a semi group under addition, a semi group under multiplication, and in which the right-and-left distributive laws hold. This became the onset of the study of semi near rings.

2. Preliminaries

Delineation 2.1

A node type set N assimilating together with the binary operations '+' added with '.' is said to be a ***semi near ring*** if:

(i) (N, +) is a semi group

(ii) N is a semi group with respect to multiplication satisfying the distributive laws.

Definition 2.2

N_c = {a ∈ N / a0 = a} = {a ∈ N / aa' = a for all a' a ∈ N} is called the constant part of N. N is called a ***constant semi near ring*** if N = N_c.

Delineation 2.3

A semi near ring N acquaints ***zero-symmetricity*** if n0 = 0 for all n ∈ N.

Delineation 2.4

N undergoes ***regularity*** if for all m ∈ N there exists n ∈ N defining m = mnm.

Definition 2.5

N is said to be ***commutative*** in general if mn = nm for all m, n ∈ N and is said to be ***quasi weak commutative*** if mnx = nmx for all m, n, x ∈ N.

Definition 2.6

An element a in the semi near ring N is said to be ***boolean*** if $y^2 = y$.

Definition 2.7

An element a ∈ N is called ***nilpotent***, if there exists a positive integer k > 1 such that $a^k = 0$.

Definition 2.8

A mapping f: N ⟶ N' is said to be a ***homomorphism***, where N and N' are semi near rings, when the following conditions are satisfied:

i. f(m+n) = f(m) + f(n)

ii. f(mn) = f(m)f(n)

where m, n belongs to the semi near ring N.

Definition 2.9

A semi near ring N is ***N-simple***, if either Nx = N or Nx = {0} for all x ∈ N.

3. Main Results

3.1 B_1 Semi Near Rings

Definition 3.1.1

A semi near ring N is said to be a ***B_1 semi near ring*** if for every a ∈ N, there exists x ∈ N* such that axa = a^2x^2.

Theorem 3.1.2

Let N be a semi near ring. Each of the following statements implies that N is a B_1 semi near ring:

(i) N is a zero symmetric nil semi near ring.

(ii) N is weak commutative.

(iii) N has identity '1'.

Proof:

(i) Let a ∈ N.

When a = 0: For all x ∈ N*, Nax = Nxa = N0 = {0}

If a ∈ N*, then $a^k = 0$.

Let x = $a^{k-1} \neq 0$

Nax = Naa^{k-1} = Na^k = $Na^{k-1}a$ = Nxa = N0 = {0}.

Thus, N is a B_1 semi near ring.

(ii) N is weak commutative

Let a ∈ N.

For all x ∈ N*, y ∈ Nax implies y = nax where n ∈ N.

N is weak commutative, then y = nxa ∈ Nxa.

Therefore, Nax ⊂ Nxa

Similarly, Nxa ⊂ Nax

Thus, Nax = Nxa and N is a B_1 semi near ring.

(iii) N has identity 1.

N is a B_1 semi near ring, for all a ∈ N, there exists x ∈ N* such that Nax = Nxa.

When n = 1, N is B_1.

This completes the proof.

Corollary 3.1.3

Converse of theorem 3.1.2, (ii), Every B_1 semi near ring is weak commutative.

Proof:

Obvious.

Assertion 3.1.4

In a B_1 semi near ring, for all a ∈ N, there exists x ∈ N* such that the following are true:

(i) there exists n ∈ N such that axa = nax.

(ii) Nax ⊂ Na∩Nx

(iii) If N is Boolean, then Naxa = Nxa

Proof:

Let a ∈ N.

Since N is a B_1 semi near ring, then for every a ∈ N, there exists x ∈ N* such that, Nax = Nxa.

(i) axa ∈ Nax

Then, axa = nxa for all n ∈ N.

Which implies, axa = nax and (i) is proved.

(ii) Nax = Nxa ⊂ Na

Obviously, Nax ⊂ Nx

Therefore, Nax ⊂ Na and Nax ⊂ Nx

Thus, Nax ⊂ Na∩Nx.

Hence, (ii) is proved.

(iii) Let N be the Boolean B_1 semi near ring.

Then, $Nxa = Nxa^2$

= (Nxa)a

= (Nax)a

Thus, Nxa = Naxa.

Theorem 3.1.5

In a Boolean semi near ring, (i) $Naxa = Nxa^n$ (ii) $= Na^nx$, when N is weak commutative.

Proof:

Since N is a B_1 semi near ring, then, for every a ∈ N, there exists x ∈ N* such that, Nax = Nxa.

From Theorem 3.1.4, (iii), Naxa = Nxa.

Consider, Nax = Nxa

= Nxa^2

= (Nxa)a

= (Naxa)a

Now consider, Nax = (Naxa)a

(i) Nax = (Naxa)a

= $Naxa^2$

= $(Nxa)a^2$

= Nxa^3

(ii) N is weak commutative and Nax = (Naxa)a

= N(aax)a

= Na^2xa

= Na^2ax

= Na^3x

Continuing this process, we get, Naxa = Na^nx = Nxa^n

Hence the proof.

Theorem 3.1.6

In a regular quasi weak commutative B_1 semi near ring, Nax = Nx^2a^2.

Proof:

Since N is a B_1 semi near ring, then, for every a ∈ N, there exists x ∈ N* such that, Nax = Nxa

= Nx(axa), since N is regular

= Nxaxa

= Nx(xaa)

= Nx^2a^2

And this completes the proof.

Theorem 3.1.7

Structure of a B_1 semi near ring is preserved under isomorphism.

Verification:

Let N be a B_1 semi near ring and let g: N → N' be a homomorphism.

For all a ∈ N, there exists x ∈ N' such that Nax = Nxa

Let a’, x’ ∈ N’.

The elements a and b from N acknowledges g(a) = a’ and g(x) = x’.

Now, nax = nxa for all n ∈ N.

Thus, n’a’x’ = g(n)g(a)g(x)

= g(nax)

= g(nxa)

= g(n) g(x) g(a)

= n’x’a’

Thus, n’a’x’ = n’x’a’.

Hence, isomorphism preserves the structure of a B_1 semi near ring.

Theorem 3.1.8

In a regular B_1 semi near ring,

(i) Na = Naxa

(ii) Na = a^2

Proof:

Since N is a B_1 semi near ring, then, for every a ∈ N, there exists x ∈ N* such that, Nax = Nxa

(i) Since Nax = Nxa,

Then, Naxa = Nxaa

Na = Nxa^2, since N is regular

na = nxa^2, for all n in N

= (nxa)a

= (nax)a

Thus, Na = Naxa.

(ii) From (i), na = (nax)a

= (axa)a

= axa^2

= (axa)a

= a.a

= a^2

Thus, Na = a^2 for all a in N.

3.2 Unit B_1 Semi Near Rings

Definition 3.2.1

A semi near ring N is said to be a ***Unit B_1 semi near ring*** if for every a ∈ N, there exists u ∈ N such that Nau = Nua.

Assertion 3.2.2

Every unit B_1 semi near ring is a B_1 semi near ring.

Proof:

Obvious.

Axiom 3.2.3

Converse of Assertion 3.2.2 is not valid.

Remark 3.2.4

N is a unit B_1 semi near ring if and only if for all a in N there exists u in N such that $Nuau^{-1} = Na$.

Theorem 3.2.5

Let N be a unit B_1 semi near ring. Then for all a in N, there exists a unit u in N and an element n in N such that:

(i) aua = nua

(ii) N is weak commutative

(iii) Nau = Naua when N is Boolean

Proof:

(i) Let a ∈ N.

Since N is a unit B_1 semi near ring, there exists u ∈ N such that Nau = Nua.

Since aua ∈ Nua, aua = nua for all n ∈ N.

(ii) Let a ∈ N.

y ∈ Nau implies y = nau ∈ Nau

N is a unit B_1 semi near ring, implies, nau = nua for all n ∈ N.

Thus, y = nua ∈ Nua

Thus, nua = nau

Which implies N is weak commutative.

(iii) N is Boolean, then,

$Nua = Nua^2$

= (Nua)a

= (Nau)a

Thus, Nua = Naua.

This completes the proof.

Theorem 3.2.6

In a regular unit B_1 semi near ring,

(i) a = nua

(ii) au = ua^2u when N is Quasi Weak Commutative

Proof:

Since N is a unit B_1 semi near ring, there exists u ∈ N such that Nau = Nua.

(i) By theorem 3.2.5, (i), aua = nua

N is also regular, then, a = aua

Thus, a = nua.

(ii) au = nuau

= nauu

= nau^2

= (nau)u

= (aua)u

= auau

= uaau

= ua^2u

Thus, au = ua^2u and this completes the proof.

3.3 Strong B_1 Semi Near Rings

Definition 3.3.1

A semi near ring N is defined to be a ***strong B_1 semi near ring*** if Nab = Nba for all a, b ∈ N.

Assertion 3.3.2

Every strong B_1 semi near ring is a B_1 semi near ring.

Proof:

Obvious.

Theorem 3.3.3

Every N-simple strong B_1 semi near ring is a strong B_1 semi near ring.

Proof:

Since N is N-simple, either Nx = N (or) Nx = {0} for all x ∈ N.

Let a, b ∈ N.

Case (i): Let Na = N

If Nb = {0}, then Nab = Nba = {0}.

If Nb = N, then Nab = Nba = N.

Case (ii): Let Na = {0}

If Nb = {0}, then Nab = Nba = {0}.

If Nb = N, then Nab = Nba = {0}.

In both cases, N is a strong B_1 near ring.

Theorem 3.3.4

Injective homomorphism preserves the structure of a strong B_1 semi near ring.

Verification:

Let N be a strong B_1 semi near ring and let f : N → N' be a homomorphism.

For all a, b ∈ N, Nab = Nba.

Let a', b' ∈ N'.

The elements a and b from N acknowledges f(a) = a' and f(b) = b'.

Obviously, N'x'y' ⊂ N'y'x'

Similarly, N'y'x' ⊂ N'x'y'

Thus, N'x'y' = N'y'x'

Thus, N is a strong B_1 semi near ring.

Assertion 3.3.5

Every strong B_1 semi near ring is isomorphic to a sub direct product of sub-directly irreducible strong B_1 semi near ring.

Proof:

"Every semi near ring is isomorphic to a sub direct product of sub-directly irreducible semi near ring."

Let N be the strong B_1 semi near ring.

N is isomorphic to a sub direct of sub directly irreducible semi near rings, N_i's, say, Each N_i is a homomorphic image of N under usual projection map $Л_i$.

Desired result follows from Theorem 3.3.4.

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