f_γ-PSg-closed sets in Fine Topological Spaces

P.L. Powar[1], J.K. Maitra[1] and Ramratan Kushwaha[1]

[1] Department of Mathematics and Computer Science, R.D. University, Jabalpur, India.
E-mail: kushwaharam786@gmail.com

Abstract: In this paper, we have defined a new class of sets called f_γ-P_S-generalized closed sets using f_γ-P_S-open set and f_γ-P_S-closure of a set in fine topological space. Also, we have defined some new functions, namely f_γ-P_S-g-continuous by using the f_γ-P_S-g-closed set and f_γ-P_S-g-open set in fine topological space with illustrative examples.

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1 Introduction

Kasahara [9] defined the concept of α-closed graphs of an operation on the topology τ defined over X. Later, Ogata [12], renamed the operation α as γ-operation on τ. He defined γ-open sets and introduced the notion of τ_γ which is the class of all γ-open sets in a topological space (X, τ). Further study by Krishnan and Balachandran ([10], [11]) defined two types of sets called γ-preopen and γ-semiopen sets. The notion of α-γ-open set has been defined by Kalaivani and Krishnan[7]. Meanwhile, Basu et al.[5] defined γ-β-open sets by using the γ-operation on τ. Carpintero et al.[6] introduced another notion of γ-open set called γ-b-open sets in topological space (X, τ). Asaad et al.[4] defined the notion of γ-regular-open sets which lies strictly between the classes of γ-open set and γ-clopen set. They introduced a new class of sets called γ-P_S-open sets, and also defined γ-P_S-operations and their properties. Further, introduced a new class of sets called γ-P_S-generalized closed set and investigate some of its properties.

Powar and Rajak [13] have introduced fine-topological space which is a special case of generalized topological space. This new class of fine-open sets contains all α-open sets, β-open sets, semi-open sets, pre-open sets, regular open sets etc. and fine-irresolute mapping includes pre-continuous, semi-continuous, α-continuous, β-continuous, α-irresolute and β-irresolute function.

In this paper, we introduce the concept of f_γ-P_S-generalized closed sets using f_γ-P_S-open set and f_γ-P_S-closure of a set in fine topological space. Also, investigate f_γ-P_S-g-continuous function by utilizing the γ-operation.

2 Preliminaries and Basic Definitions

Throughout this paper, spaces (X, τ) and (Y, σ) always mean topological spaces on which no separation axioms are assumed unless explicitly defined.

Definition 2.1. [12] An operation γ on the topology τ on X is a mapping $\gamma : \tau \rightarrow P(X)$ such that $U \subseteq \gamma(U)$ for each $U \in \tau$, where $P(X)$ is the power set of X and $\gamma(U)$ denotes the value of γ at U.

Definition 2.2. Let (X, τ) be a topological space and γ be an operation on τ. A subset A of X is said to be:

1. γ-open [12] set if for each $x \in A$ there exist an open set U such that $x \in U$ and $\gamma(U) \subseteq A$, τ_γ denotes the set of all γ-open sets in (X, τ). The complement of γ-open set is called a γ-closed set.
2. γ-regular open [4] if $A = \tau_\gamma\text{-}int(\tau_\gamma\text{-}cl(A))$.
3. γ-preopen [10] if $A \subseteq \tau_\gamma\text{-}int(\tau_\gamma\text{-}cl(A))$.
4. γ-semiopen [11] if $A \subseteq \tau_\gamma\text{-}cl(\tau_\gamma\text{-}int(A))$.
5. α-γ-open [8] if $A \subseteq \tau_\gamma\text{-}int(\tau_\gamma\text{-}cl(\tau_\gamma\text{-}int(A)))$.
6. γ-b-open [6] if $A \subseteq \tau_\gamma\text{-}cl(\tau_\gamma\text{-}int(A)) \cup \tau_\gamma\text{-}int(\tau_\gamma\text{-}cl(A))$.
7. γ-β-open [5] if $A \subseteq \tau_\gamma\text{-}cl(\tau_\gamma\text{-}int(\tau_\gamma\text{-}cl(A)))$.
8. γ-clopen [10] if it is both γ-open and γ-closed.
9. γ-dense [12] if $\tau_\gamma\text{-}cl(A) = X$.

Definition 2.3. The complement of γ-regular open, γ-preopen, γ-semiopen, α-γ-open, γ-b-open and γ-β-open set is said to be γ-regular closed [4], γ- preclosed [10], γ-semiclosed [11], α-γ-closed [8], γ-b-closed [6] and γ-β-closed [5], respectively.

Definition 2.4. [1] A γ-preopen subset A of a topological space (X, τ) is called γ-P_S-open set, if for each $x \in A$, there exists γ-semiclosed set F such that $x \in F \subseteq A$. The complement of a γ-P_S-open set is called a γ-P_S-closed.

The class of all γ-P_S-open sets and γ-P_S-closed sets is denoted by τ_γ-$P_SO(X)$ and τ_γ-$P_SC(X)$, respectively.

Definition 2.5. [1] Let A be any subset of a topological space (X, τ) and γ be an operation on τ. Then:

1. The τ_γ-P_S-interior of A is defined as the union of all γ-P_S-open sets of X contained in A and it is denoted by τ_γ-$P_Sint(A)$.

2. The τ_γ-P_S-closure of A is defined as the intersection of all γ-P_S-closed sets of X contained A and it is denoted by τ_γ-$P_Scl(A)$.

3. τ_γ-preclosure and $\tau_{\alpha-\gamma}$-closure of A is defined as the intersection of all γ-preclosed and α-γ-closed sets of X containing A and it is denoted by τ_γ-$pcl(A)$ and $\tau_{\alpha-\gamma}$-$cl(A)$, respectively.

Remark 2.1. [2] Let (X, τ) be a topological space and γ be an operation on τ. For any subset A of a space X. The following statements hold:

1. A is γ-P_S-closed if and only if τ_γ-$P_Scl(A) = A$.

2. A is γ-P_S-open if and only if τ_γ-$P_Sint(A) = A$.

3. τ_γ- $P_Scl(X \backslash A) = X \backslash (\tau_\gamma$-$P_Sint(A))$ and τ_γ- $P_Sint(X \backslash A) = X \backslash (\tau_\gamma$-$P_Scl(A))$

Definition 2.6. [2] Let (X, τ) be a topological space and γ be an operation on τ. A subset A of X is called:

1. γ-pre-generalized closed (γ-pre-g-closed) if τ_γ-$pcl(A) \subseteq G$ whenever $A \subseteq G$ and G is a γ-preopen set in X.

2. α-γ-generalized closed (α-γ-g-closed) if $\tau_{\alpha-\gamma}$-$cl(A) \subseteq G$ whenever $A \subseteq G$ and G is a α-γ-open set in X.

Definition 2.7. [2] Let (X, τ) and (Y, σ) be two topological spaces and γ be an operation on τ. A function $f : (X, \tau) \to (Y, \sigma)$ is called γ-P_S-continuous if the pre-image of every closed set in Y is γ-P_S-closed set in X.

Definition 2.8. [2] Let A be any subset of a topological space (X, τ) with an operation γ on τ is called γ-P_S-generalized closed (γ-P_S-g-closed) if τ_γ- $P_S cl(A) \subseteq G$ whenever $A \subseteq G$ and G is a γ-P_S-open set in X.

The class of all γ-P_S-g-closed sets of X is denoted by τ_γ-$P_S GC(X)$ and the class of all γ-P_S-g-open sets of X is denoted by τ_γ-$P_S GO(X)$.

A set A is said to be γ-P_S-generalized open(γ-P_S-g-open) if its complement γ-P_S-g-closed. Or equivalently, a set A is γ-P_S-g-open if $F \subseteq \tau_\gamma$-$P_S int(A)$ whenever $F \subseteq A$ and F is a γ-P_S-closed set in X.

Definition 2.9. [2] Let (X, τ) and (Y, σ) be two topology spaces and γ be an operation on τ. A function $f : (X, \tau) \to (Y, \sigma)$ is called γ-P_S-g-continuous if the pre-image of every closed set in Y is γ-P_S-g-closed set in X.

Definition 2.10. [13] Let (X, τ) be a topological space we define $\tau(A_\alpha) = \tau_\alpha$(say)$=$ $\{G_\alpha(\neq X) : G_\alpha \cap A_\alpha \neq \phi$, for $A_\alpha \in \tau$ and $A_\alpha \neq \phi, X$. for some $\alpha \in J$, where J is the index set$\}$. Now, define $\tau_f = \{\phi, X\} \cup_\alpha \{\tau_\alpha\}$. The above collection τ_f of subsets of X is called the fine collection of subsets of X and (X, τ, τ_f) is said to be the fine topological space X generated by the topology τ on X.

Definition 2.11. [13] A subset U of a fine topological space X is said to be a fine-open (or f-open) set of X, if U belongs to the collection τ_f and the complement of fine-open set U of X is called the fine-closed (or f-closed) set in X and denote the collection of fine-closed sets by F_{f_τ}.

Remark 2.2. Let (X, τ, τ_f) be a fine topological space, the arbitrary union of fine open set in X is fine open in X.

Remark 2.3. The intersection of two fine-open sets need not be a fine-open set.

Definition 2.12. [13] A fine-open set S of a space (X, τ, τ_f) is called:

1. f_α-open if S is α-open subset of a topological space (X, τ).

2. f_s-open set if S is semi open subset of a topological space (X, τ).

3. f_p-open set if S is pre-open subset of a topological space (X, τ).

4. f_β-open set if S is β-open subset of a topological space (X, τ).

5. f_r-open set if S is regular-open subset of a topological space (X, τ).

6. f-clopen set (fine-clopen) if S is both fine-open and fine-closed subset of a topological space (X, τ).

Definition 2.13. [13] Let A be the subset of a fine space X, the fine interior of A is defined as the union of all fine-open sets contained in the set A i.e. the largest fine-open set contained in the set A and is denoted by $f_{int}(A)$.

Definition 2.14. [13] Let A be the subset of a fine space X, the fine closure of A is defined as the intersection of all fine-closed sets containing the set A i.e. the smallest fine-closed set containing the set A and is denoted by $f_{cl}(A)$.

Definition 2.15. A function $f : (X, \tau, \tau_f) \to (Y, \sigma, \tau_\sigma)$ is called fine-irresolute if $f^{-1}(V)$ is fine-open in X for every fine-open set V of Y.

Definition 2.16. A function $f : (X, \tau, \tau_f) \to (Y, \sigma, \tau_\sigma)$ is called fine-irresolute (f-irresolute) homeomorphism if
(1) f is one-one and onto.
(2) Both the function f and inverse function $f^{-1} : (Y, \sigma, \tau_\sigma) \to (X, \tau, \tau_f)$ are f-irresolute.

3 f_γ-P_S-Generalized Closed sets

In this section, we define a new class of sets called f_γ-P_S-generalized closed sets using f_γ-P_S-open set and f_γ-P_S-closure of set. Also study some generalized results.

Definition 3.1. Let (X, τ, τ_f) be a fine topological space, an operation γ on the fine topology τ_f is a mapping from τ_f on to the power set $P(X)$ of X such that $U \subseteq \gamma(U)$ for each $U \in \tau_f$, where $\gamma(U)$ denotes the value of γ at U.

Example 3.1. Let $X = \{a, b, c\}$ with the topology $\tau = \{\phi, \{b\}, X\}$ and $\tau_f = \{\phi, \{b\}, \{b, c\}, \{a, b\}, X\}$. Define an operation $\gamma : \tau_f \to P(X)$ by $\gamma(A) = cl(A)$ for all $A \in \tau_f$. Then $A \subseteq \gamma(A)$ for all $A \in \tau_f$.

Definition 3.2. Let (X, τ, τ_f) be a fine topological space and γ be an operation on τ_f. A subset A of X is said to be f_γ-open set if for each $x \in A$ there exist an fine-open set U such that $x \in U$ and $\gamma(U) \subseteq A$. The set of all f_γ-open sets in (X, τ, τ_f) is denoted by $f\tau_\gamma$. The complement of f_γ-open set is f_γ-closed set and the collection is denoted by $F\tau_\gamma$.

Example 3.2. Let $X = \{a, b, c\}$ with the topology $\tau = \{\phi, \{b\}, X\}$ and $\tau_f = \{\phi, \{b\}, \{b, c\}, \{a, b\}, X\}$. Define an operation $\gamma : \tau_f \to P(X)$ by $\gamma(A) = A$ for all $A \in \tau_f$. Then, $f\tau_\gamma = \{\phi, \{b\}, \{b, c\}, \{a, b\}, X\}$, $F\tau_\gamma = \{\phi, \{a, c\}, \{c\}, \{a\}, X\}$.

Definition 3.3. Let (X, τ, τ_f) be a fine topological space and γ be an operation on τ_f. Then, $f\tau_\gamma$-interior of A is defined as the union of all f_γ-open sets contained in A and it is denoted $f\tau_\gamma$-$int(A)$. That is $f\tau_\gamma$-$int(A) = \cup\{U : U$ is a f_γ-open set and $U \subseteq A\}$.

Definition 3.4. Let (X, τ, τ_f) be a fine topological space and γ be an operation on τ_f. Then, $f\tau_\gamma$-closure of A is defined as the intersection of all $f\gamma$-closed sets containing A and it is denoted $f\tau_\gamma$-$cl(A)$. That is $f\tau_\gamma$-$cl(A) = \cap\{F : F$ is a $f\gamma$-closed set and $A \subseteq F\}$.

Example 3.3. Let $X = \{a, b, c\}$ with the topology $\tau = \{\phi, \{b\}, X\}$ and $\tau_f = \{\phi, \{b\}, \{b, c\}, \{a, b\}, X\}$. Define an operation $\gamma : \tau_f \to P(X)$ by $\gamma(A) = A$ for all $A \in \tau_f$. $f\tau_\gamma = \{\phi, \{b\}, \{b, c\}, \{a, b\}, X\}$, $F\tau_\gamma = \{\phi, \{a, c\}, \{c\}, \{a\}, X\}$. If $S = \{b, c\} \subseteq X$, then $f\tau_\gamma$-$int(S) = \{b\}$ and $f\tau_\gamma$-$cl(S) = X$.

Definition 3.5. Let (X, τ, τ_f) be a fine topological space and γ be an operation on τ_f. A subset A of X is said to be:

1. f_γ-regular open if $A = f\tau_\gamma$-$int(f\tau_\gamma$-$cl(A))$.
2. f_γ-preopen if $A \subseteq f\tau_\gamma$-$int(f\tau_\gamma$-$cl(A))$.
3. f_γ-semiopen if $A \subseteq f\tau_\gamma$-$cl(f\tau_\gamma$-$int(A))$.

4. $f_{\alpha-\gamma}$-open if $A \subseteq f\tau_\gamma\text{-}int(f\tau_\gamma\text{-}cl(f\tau_\gamma\text{-}int(A)))$.

5. $f_{\gamma-b}$-open if $A \subseteq f\tau_\gamma\text{-}cl(f\tau_\gamma\text{-}int(A)) \cup f\tau_\gamma\text{-}int(f\tau_\gamma\text{-}cl(A))$.

6. $f_{\gamma-\beta}$-open if $A \subseteq f\tau_\gamma\text{-}cl(f\tau_\gamma\text{-}int(f\tau_\gamma\text{-}cl(A)))$.

7. f_γ-clopen if it is both $f\gamma$-open and $f\gamma$-closed.

8. f_γ-dense if $f\tau_\gamma\text{-}cl(A) = X$.

Example 3.4. Let $X = \{a, b, c\}$, with topology $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ and $\tau_f = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, c\}\}$. Now, define an operation γ on τ_f such that

$$\gamma(A) = \begin{cases} A & \text{if } A = \{a\} \\ A \cup \{c\} & \text{if } A \neq \{a\} \end{cases}$$

for every $A \in \tau_f$. The collection of all f_γ-open sets $f\tau_\gamma = \{\phi, X, \{a\}, \{b, c\}, \{a, c\}\}$ and the collection of all f_γ-closed sets $F\tau_\gamma = \{\phi, X, \{b, c\}, \{a\}, \{b\}\}$. If $A = \{a, b\}$ then $f\tau_\gamma\text{-}int(f\tau_\gamma\text{-}cl(A)) = X \Rightarrow A \subseteq f\tau_\gamma\text{-}int(f\tau_\gamma\text{-}cl(A))$. Hence, A is f_γ-preopen and also $f_{\gamma-\beta}$-open and f_γ-dense set. If $A = \{a\}$ or $\{b, c\}$ both are f_γ-clopen because they are both f_γ-open and f_γ-closed.

Remark 3.1. Let (X, τ, τ_f) be a fine topological space. Then following hold:

1. Every γ-open set is f_γ-open set.

2. Every γ-regular open set is f_γ-regular open set.

3. Every γ-preopen set is f_γ-preopen set.

Example 3.5. Let $X = \{a, b, c\}$ and topology $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$, $\tau_f = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, c\}\}$. Define an operation γ on τ_f such that

$$\gamma(A) = \begin{cases} A & \text{if } A = \{a\} \\ A \cup \{c\} & \text{if } A \neq \{a\} \end{cases}$$

for every $A \in \tau_f$. The set of all f_γ-open sets $f\tau_\gamma = \{\phi, X, \{a\}, \{b, c\}, \{a, c\}\}$ and the collection of all f_γ-closed sets $F\tau_\gamma = \{\phi, X, \{b, c\}, \{a\}, \{b\}\}$. The collection of all f_γ-preopen sets $f\tau_\gamma\text{-}PO(X) = \{\phi, X, \{a\}, \{b, c\}, \{a, c\}, \{c\}, \{a, b\}\}$ and the collection of all f_γ-regular open sets $f\tau_\gamma\text{-}RO(X) = \{\phi, X, \{a\}, \{b, c\}\}$.

Remark 3.2. Referring Example 3.2 of this paper, following results hold:

1. The collection of all γ-open sets $\tau_\gamma = \{\phi, X, \{a\}\}$ then, it is clear that, every γ-open sets are f_γ-open sets but, conversely not true.

2. The collection of all γ-preopen sets τ_γ-$PO(X) = \{\phi, X, \{a\}, \{a, c\}, \{a, b\}\}$ then, it is clear that, every γ-preopen sets are f_γ-preopen sets but, conversely not true.

3. The collection of all γ-regular open sets τ_γ-$RO(X) = \{\phi, X\}$ so, it is clear that, every γ-regular open sets are f_γ-regular open, but not conversely.

Definition 3.6. The complement of f_γ-regular open, f_γ-preopen, f_γ-semiopen, $f_{\alpha-\gamma}$-open, $f_{\gamma-b}$-open and $f_{\gamma-\beta}$-open set is said to be f_γ-regular closed, f_γ-preclosed, f_γ-semiclosed, $f_{\alpha-\gamma}$-closed, $f_{\gamma-b}$-closed and $f_{\gamma-\beta}$-closed, respectively.

Definition 3.7. A f_γ-preopen subset A of a fine topological space (X, τ, τ_f) is called f_γ-P_S-open set if for each $x \in A$, there exists f_γ-semiclosed set F such that $x \in F \subseteq A$. The complement of a f_γ-P_S-open set is called a f_γ-P_S-closed.

The class of all f_γ-P_S-open set and f_γ-P_S-closed subsets of a fine topological space (X, τ, τ_f) are denoted by $f\tau_\gamma$-$P_SO(X)$ and $f\tau_\gamma$-$P_SC(X)$, respectively.

Definition 3.8. Let A be a subset of a fine space X, we say that a point $x \in X$ is a f_γ-P_S-limit point of A if every f_γ-P_S-open set of X containing x must contains at least one point of A other than x.

Example 3.6. Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$ and fine collection $\tau_f = \{\phi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{c, a\}\}$. Now, define an operation γ on τ_f such that

$$\gamma(A) = \begin{cases} A & \text{if } a \in A \\ cl(A) & \text{if } a \notin A \end{cases}$$

for every $A \in \tau_f$. $f\tau_\gamma = f\tau_\gamma\text{-}PO(X) = f\tau_\gamma\text{-}P_SO(X) = P(X)$

Example 3.7. Let $X = \{a, b, c\}$, with topology $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ and $\tau_f = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, c\}\}$ define an operation γ on τ_f such that

$$\gamma(A) = \begin{cases} A & \text{if } A = \{a\} \\ A \cup \{c\} & \text{if } A \neq \{a\} \end{cases}$$

for every $A \in \tau_f$. The set of all f_γ-open sets $f\tau_\gamma = \{\phi, X, \{a\}, \{b,c\}, \{a,c\}\}$ and set of all f_γ-closed sets $F\tau_\gamma = \{\phi, X, \{b,c\}, \{a\}, \{b\}\}$. The set of all fine γ-preopen sets $f\tau_\gamma$-$PO(X) = \{\phi, X, \{a\}, \{b,c\}, \{a,c\}, \{c\}, \{a,b\}\}$. The set of all fine γ-semiopen sets $f\tau_\gamma$-$SO(X) = \{\phi, X, \{a\}, \{b,c\}, \{a,c\}\}$ and $f\tau_\gamma$-$SC(X) = \{\phi, X, \{b,c\}, \{a\}, \{b\}\}$.

Then the class of all f_γ-P_S-open sets $f\tau_\gamma$-$P_SO(X) = \{\phi, X, \{a\}, \{b,c\}\}$ and the class of all f_γ-P_S-closed sets $f\tau_\gamma$-$P_SC(X) = \{\phi, X, \{a\}, \{b,c\}\}$.

Definition 3.9. Let A be a subset of the fine topological space (X, τ, τ_f) and γ be an operation on τ. Then:

1. The $f\tau_\gamma$-P_S-interior of A is defined as the union of all f_γ-P_S-open sets of X contained in A and it is denoted by $f\tau_\gamma$-$P_Sint(A)$.

2. The $f\tau_\gamma$-P_S-closure of A is defined as the intersection of all f_γ-P_S-closed sets of X contained A and it is denoted by $f\tau_\gamma$-$P_Scl(A)$.

Definition 3.10. Let A be a subset of the fine topological space (X, τ, τ_f) and γ be an operation on τ. Then, $f\tau_\gamma$-preclosure and $f\tau_{\alpha-\gamma}$-closure of A is defined as the intersection of all f_γ-preclosed and $f_{\alpha-\gamma}$-closed sets of X containing A and it is denoted by $f\tau_\gamma$-$pcl(A)$ and $f\tau_{\alpha-\gamma}$-$cl(A)$, respectively.

Theorem 3.1. Let (X, τ, τ_f) be a fine topological space and γ be an operation on τ_f. For any subset A of a space X following statements hold:

1. A is f_γ-P_S-closed if and only if $f\tau_\gamma$-$P_Scl(A) = A$.

2. A is f_γ-P_S-open if and only if $f\tau_\gamma$-$P_Sint(A) = A$.

3. $f\tau_\gamma$-$P_Scl(X \backslash A) = X \backslash (f\tau_\gamma$-$P_Sint(A))$ and $f\tau_\gamma$-$P_Sint(X \backslash A) = X \backslash (f\tau_\gamma$-$P_Scl(A))$.

Proof. **1.** Let A be a f_γ-P_S-closed set and $x \in A \Rightarrow x \in f\tau_\gamma$-$P_Scl(A) \Rightarrow A \subseteq f\tau_\gamma$-$P_Scl(A)$. If $x \in f\tau_\gamma$-$P_Scl(A)$ such that x is f_γ-P_S limit point, since A is f_γ-P_S-closed set then $x \in A \Rightarrow f_\gamma$-$P_Scl(A) \subseteq A$. Hence $A = f_\gamma$-$P_Scl(A)$.

Conversely, if $A = f_\gamma$-$P_Scl(A)$ then, obviously A is f_γ-P_S-closed.

2. Let A be a f_γ-P_S-open and $f\tau_\gamma$-$P_Sint(A)$ is defined as the union of all f_γ-P_S-open subsets of X. Therefore $A \subseteq f\tau_\gamma$-$P_Sint(A)$ and f_γ-$P_Sint(A) \subseteq A$ obvious. Hence, $f\tau_\gamma - P_Sint(A) = A$.

Conversely, if $f\tau_\gamma$-$P_S int(A) = A$ then obviously A is $f\tau_\gamma$-P_S-open set.

3. Since $f\tau_\gamma$-$P_S int(A) \subseteq A$ and let $x \in X\backslash(f\tau_\gamma$-$P_S int(A)) \Rightarrow x \notin (f\tau_\gamma$-$P_S int(A)) \Rightarrow x \in f\tau_\gamma$-$P_S cl(X\backslash A) \Rightarrow X\backslash f\tau_\gamma$-$P_S int(A) \subseteq f\tau_\gamma$-$P_S cl(X\backslash A)$. If $x \in f\tau_\gamma$-$P_S cl(X\backslash A) \Rightarrow x \notin f\tau_\gamma$-$P_S int(A) \Rightarrow x \in X\backslash(f\tau_\gamma$-$P_S int(A)) \Rightarrow f\tau_\gamma$-$P_S cl(X\backslash A) \subseteq X\backslash(f\tau_\gamma$-$P_S int(A))$. Hence $f\tau_\gamma$-$P_S cl(X\backslash A) = X\backslash(f\tau_\gamma$-$P_S int(A))$.

To prove, $f\tau_\gamma$-$P_S int(X\backslash A) = X\backslash(f\tau_\gamma$-$P_S cl(A))$, let $B = X\backslash A$. Then, $X\backslash(f\tau_\gamma$-$P_S int(B)) = f\tau_\gamma$-$P_S cl(X\backslash B) \Rightarrow X\backslash(f\tau_\gamma$-$P_S int(X\backslash A)) = f\tau_\gamma$-$P_S cl(X\backslash(X\backslash A)) = f\tau_\gamma$-$P_S cl(A)$. Hence $f\tau_\gamma$-$P_S int(X\backslash A) = X\backslash(f\tau_\gamma$-$P_S cl(A))$ □

Definition 3.11. Let (X, τ, τ_f) be a fine topological space and γ be an operation on τ_f. A subset A of X is called:

1. f_γ-pre-generalized closed (f_γ-pre-g-closed) if $f\tau_\gamma$-$pcl(A) \subseteq G$ whenever $A \subseteq G$ and G is a f_γ-preopen set in X.
2. $f_{\alpha-\gamma}$-generalized closed ($f_{\alpha-\gamma}$-g-closed) if $f\tau_{\alpha-\gamma}$-$cl(A) \subseteq G$ whenever $A \subseteq G$ and G is a $f_{\alpha-\gamma}$-open set in X.

Example 3.8. Let $X = \{a, b, c\}$ with the topology $\tau = \{\phi, \{a\}, X\}$ and $\tau_f = \{\phi, \{a\}, \{a, c\}, \{a, b\}, X\}$. Define an operation $\gamma : \tau_f \to P(X)$ by $\gamma(A) = A$ for all $A \in \tau_f$. $f\tau_\gamma = \{\phi, \{a\}, \{a, c\}, \{a, b\}, X\}$, $F\tau_\gamma = \{\phi, \{a, c\}, \{c\}, \{a\}, X\}$, $f\tau_\gamma$-$PO(X) = \{\phi, \{a\}, \{a, c\}, \{a, b\}, X\}$, $f\tau_\gamma$-$P_S O(X) = \{\phi, X\}$, $f\tau_\gamma$-$P_S C(X) = \{\phi, X\}$ and $f\tau_\gamma$-$P_S GC(X) =$ all subsets of X.

Theorem 3.2. Given a topological space (X, τ), consider a fine space (X, τ, τ_f) generated by τ. Then following hold:

1. Every f_γ-preclosed set is f_γ-pre-g-closed.
2. Every $f_{\alpha-\gamma}$-closed set is $f_{\alpha-\gamma}$-g-closed.

Proof. **1.** Given A be any f_γ-preclosed set in a fine space X and $A \subseteq G$ where G is a f_γ-preopen set in X. We now show that A is a f_γ-pre-g-closed set. By Definition 3.2, set A is said f_γ-pre-generalized closed (f_γ-pre-g-closed) if $f\tau_\gamma$-$pcl(A) \subseteq G$ whenever $A \subseteq G$ and G is a f_γ-preopen set in X. Then $f\tau_\gamma$-$pcl(A) \subseteq G$ since A is f_γ-preclosed set. Therefore, A is f_γ-pre-g-closed.

2. Given A be any $f_{\alpha-\gamma}$-closed set in a fine space X and $A \subseteq G$ where G

is a $f_{\alpha-\gamma}$-open set in X. Now we have to show that A is a $f_{\alpha-\gamma}$-g-closed set. Now applying the Definition 3.2, set A is said $f_{\alpha-\gamma}$-generalized closed ($f_{\alpha-\gamma}$-g-closed) if $f\tau_{\alpha-\gamma}cl(A) \subseteq G$ whenever $A \subseteq G$ and G is a $f_{\alpha-\gamma}$-open set in X. Then $f\tau_{\alpha-\gamma}cl(A) \subseteq G$ and A is $f_{\alpha-\gamma}$-closed set. Therefore, A is $f_{\alpha-\gamma}$-g-closed. □

Theorem 3.3. Given a topological space (X, τ), consider a fine space (X, τ, τ_f) generated by τ. Then following hold:

1. Every γ-open set is fine-open set.
2. Every γ-pre-open set is fine-open set.
3. Every γ-semi-open set is fine-open set.
4. Every α-γ-open set is fine-open set.
5. Every γ-regular-open set is fine-open set.
6. Every f_γ-open set is fine-open set.
7. Every f_γ-regular-open set is fine-open set.
8. Every f_γ-semi-open set is fine-open set.

Proof.

1. Let (X, τ, τ_f) be the fine space with respect to the topological space (X, τ). Consider A be a non-empty γ-open set then, for each $x \in A$ there exists an open set U such that $x \in U$, and $\gamma(U) \subseteq A$. Since each $x \in A$ belong in U_α, so $A \subseteq U_\alpha$ for all $\alpha \in J$. Since U is an open set and $A \cap U \neq \phi$ for $U \in \tau$ and $A \neq \phi \Rightarrow A \in \tau_\alpha = \{A : A \cap U_\alpha \neq \phi \text{ for } U_\alpha \in \tau \text{ and } A \neq \phi\}$ $\Rightarrow \tau_f = \{\phi, X\} \cup \tau_\alpha \Rightarrow A \in \tau_f$.

2. Let A be a non-empty subset of X and $A \notin \tau_f$.
   Claim: A is not γ-pre-open set in X.
   Since, it is given that $A \notin \tau_f \Rightarrow G_\alpha \cap A = \phi \;\; \forall\, \alpha \in J \Rightarrow A \subseteq X - G_\alpha \Rightarrow \tau_\gamma$-$cl(A) \subseteq X - G_\alpha$. Since $G_\alpha \cap X - G_\alpha = \phi$ and τ_γ-$cl(A) \subseteq X - G_\alpha \Rightarrow G_\alpha \cap \tau_\gamma$-$cl(A) = \phi \Rightarrow \tau_\gamma$-$int(\tau_\gamma$-$cl(A)) = \phi \Rightarrow A$ is not subset τ_γ-$int(\tau_\gamma$-$cl(A))$. Hence A is not γ-pre-open set.

3. Let A be a non-empty subset of X and $A \notin \tau_f$.
Claim: A is not γ-semi-open set in X.
Since, it is given that $A \notin \tau_f \Rightarrow G_\alpha \cap A = \phi$, $\forall\ \alpha \in J \Rightarrow \tau_\gamma\text{-}int(A) = \phi \Rightarrow \tau_\gamma\text{-}cl(\tau_\gamma\text{-}int(A)) = \phi \Rightarrow A$ is not subset of $\tau_\gamma\text{-}cl(\tau_\gamma\text{-}int(A))$. Hence A is not γ-semi-open set.

4. Let A be a non-empty subset of X and $A \notin \tau_f$.
Claim: A is not α-γ-open set in X.
Since, it is given that $A \notin \tau_f \Rightarrow G_\alpha \cap A = \phi$, $\forall\ \alpha \in J \Rightarrow \tau_\gamma\text{-}int(A) = \phi \Rightarrow \tau_\gamma\text{-}cl(\tau_\gamma\text{-}int(A)) = \phi \Rightarrow \tau_\gamma\text{-}int(\tau_\gamma\text{-}cl(\tau_\gamma\text{-}int(A))) = \phi \Rightarrow A$ is not subset of $\tau_\gamma\text{-}int(\tau_\gamma\text{-}cl(\tau_\gamma\text{-}int(A)))$. Hence A is not α-γ-open set.

5. Let A be a non-empty subset of X and $A \notin \tau_f$.
Claim: A is not γ-regular-open set in X.
Since, it is given that $A \notin \tau_f \Rightarrow G_\alpha \cap A = \phi$, $\forall\ \alpha \in J \Rightarrow A \subseteq X - G_\alpha \Rightarrow \tau_\gamma\text{-}cl(A) \subseteq X - G_\alpha$. Since $G_\alpha \cap X - G_\alpha = \phi$ and $\tau_\gamma\text{-}cl(A) \subseteq X - G_\alpha \Rightarrow G_\alpha \cap \tau_\gamma\text{-}cl(A) = \phi \Rightarrow \tau_\gamma\text{-}int(\tau_\gamma\text{-}cl(A)) = \phi \Rightarrow A \neq \tau_\gamma\text{-}int(\tau_\gamma\text{-}cl(A))$. Hence A is not γ-regular-open set.

6. Let (X, τ, τ_f) be the fine space with respect to the topological space (X, τ). Consider A be a non-empty f_γ-open set then, for each $x \in A$ there exists an f-open set U such that $x \in U$, and $\gamma(U) \subseteq A$. Since each $x \in A$ belong in U_α, so $A \subseteq U_\alpha$ for all $\alpha \in J$. Since U is an f-open set and $A \cap U \neq \phi$ for $U \in \tau_f$ and $A \neq \phi \Rightarrow A \in \tau_\alpha = \{A : A \cap U_\alpha \neq \phi \text{ for } U_\alpha \in \tau_f \text{ and } A \neq \phi\}$ $\Rightarrow \tau_f = \{\phi, X\} \cup \tau_\alpha \Rightarrow A \in \tau_f$.

7. Let A be a non-empty subset of X and $A \notin \tau_f$.
Claim: A is not f_γ-regular-open set in X.
Since, it is given that $A \notin \tau_f \Rightarrow G_\alpha \cap A = \phi$, $\forall\ \alpha \in J \Rightarrow A \subseteq X - G_\alpha \Rightarrow f\tau_\gamma\text{-}cl(A) \subseteq X - G_\alpha$. Since $G_\alpha \cap X - G_\alpha = \phi$ and $f\tau_\gamma\text{-}cl(A) \subseteq X - G_\alpha \Rightarrow G_\alpha \cap f\tau_\gamma\text{-}cl(A) = \phi \Rightarrow f\tau_\gamma\text{-}int(f\tau_\gamma\text{-}cl(A)) = \phi \Rightarrow A \neq f\tau_\gamma\text{-}int(f\tau_\gamma\text{-}cl(A))$. Hence A is not f_γ-regular-open set.

8. Let A be a non-empty subset of X and $A \notin \tau_f$.
Claim: A is not f_γ-semi-open set in X.

Since, it is given that $A \notin \tau_f \Rightarrow G_\alpha \cap A = \phi,\ \forall\ \alpha \in J \Rightarrow f\tau_\gamma\text{-}int(A) = \phi \Rightarrow f\tau_\gamma\text{-}cl(f\tau_\gamma\text{-}int(A)) = \phi \Rightarrow A$ is not subset of $f\tau_\gamma\text{-}cl(f\tau_\gamma\text{-}int(A))$. Hence A is not f_γ-semi-open set.

□

Definition 3.12. Let (X, τ, τ_f) and (Y, σ, σ_f) be two fine topological spaces and γ be an operation on τ_f. A function $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ is called f_γ-P_S-continuous if the pre image of every f-closed set in Y is f_γ-P_S-closed set in X.

Example 3.9. In Example 3.7, we have class of all f_γ-P_S-open sets $f\tau_\gamma\text{-}P_SO(X) = \{\phi, X, \{a\}, \{b, c\}\}$ and the class of all f_γ-P_S-closed sets $f\tau_\gamma\text{-}P_SC(X) = \{\phi, X, \{a\}, \{b, c\}\}$. Let $Y = \{b, c\}$ and $\sigma = \{\phi, Y, \{c\}\}$, $\sigma_f = \{\phi, Y, \{c\}\}$ and $F\sigma_f = \{\phi, Y, \{b\}\}$, define a mapping $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ by $f(a) = b, f(b) = c, f(c) = c$. It is clear that pre image of each f-closed set in Y is f_γ-P_S-closed set in X.

Definition 3.13. Let A be any subset of a fine topological space (X, τ, τ_f) with an operation γ on τ_f is called f_γ-P_S-generalized closed (f_γ-P_S-g-closed) if $f\tau_\gamma\text{-}P_Scl(A) \subseteq G$ whenever $A \subseteq G$ and G is a f_γ-P_S-open set in X.

The class of all f_γ-P_S-g-closed sets of X is denoted by $f\tau_\gamma\text{-}P_SGC(X)$ and the class of all f_γ-P_S-g-open sets of X is denoted by $f\tau_\gamma\text{-}P_SGO(X)$.

A set A is said to be f_γ-P_S-generalized open (f_γ-P_S-g-open) if its complement f_γ-P_S-g-closed. Or equivalently, a set A is f_γ-P_S-g-open if $F \subseteq f\tau_\gamma\text{-}P_Sint(A)$ whenever $F \subseteq G$ and F is a f_γ-P_S-closed set in X.

Example 3.10. Let $X = \{a, b, c\}$ with topology $\tau = \{\phi, X, \{b\}\}$ and fine space $\tau_f = \{\phi, X, \{b\}, \{a, b\}, \{b, c\}\}$ define an operation γ on τ_f such that

$$\gamma(A) = \begin{cases} A & \text{if } c \in A \\ cl(A) & \text{if } c \notin A \end{cases}$$

for every $A \in \tau_f$. Set of all fine-γ-open sets $f\tau_\gamma = \{\phi, X, \{b, c\}\} = f\tau_\gamma\text{-}SO(X)$. and $f\tau_\gamma\text{-}P_SO(X) = \{\phi, X\}$, $f\tau_\gamma\text{-}P_SGO(X) = P(X)$.

Lemma 3.1. Every f_γ-P_S-closed set is f_γ-P_S-g-closed.

Proof. Given A be any f_γ-P_S-closed set in a fine space X and $A \subseteq G$ where G is a f_γ-P_S-open set in X. We now show that A is a f_γ-P_S-g-closed set. By the Definition

3.13, set A is said f_γ-P_S-generalized closed (f_γ-P_S-g-closed) if $f\tau_\gamma$-$P_Scl(A) \subseteq G$ whenever $A \subseteq G$ and G is a f_γ-P_S-open set in X. Then $f\tau_\gamma$-$P_Scl(A) \subseteq G$ since, A is f_γ-P_S-closed set. Therefore, A is f_γ-P_S-g-closed. □

Theorem 3.4. Let (X, τ, τ_f) be a fine topological space and γ be an operation on τ_f. If a subset A of X is f_γ-P_S-g-closed and f_γ-P_S-open, then A is f_γ-P_S-closed.

Proof. Given A is f_γ-P_S-g-closed and f_γ-P_S-open in X, then by Definition 3.13, $f\tau_\gamma$-$P_Scl(A) \subseteq A$ and $A \subseteq f\tau_\gamma$-$P_Scl(A)$ is obvious. Hence A is f_γ-P_S-closed set. □

Theorem 3.5. Let (X, τ, τ_f) be a fine topological space and γ be an operation on τ_f. If a subset A of X is f_γ-P_S-g-closed and f_γ-P_S-open and F is f_γ-P_S-closed, then $A \cap F$ is f_γ-P_S-closed.

Proof. Let A be f_γ-P_S-g-closed and f_γ-P_S-open in X. Then by Theorem 3.4, if A is f_γ-P_S-g-closed and f_γ-P_S-open, then A is f_γ-P_S-closed and F is also f_γ-P_S-closed i.e., A and F both are f_γ-P_S-closed, then $A \cap F$ is f_γ-P_S-closed. □

Corollary 3.1. If $A \subseteq X$ is both f_γ-P_S-g-closed and f_γ-P_S-open and F is f_γ-P_S-closed, then $A \cap F$ is f_γ-P_S-g-closed.

Proof. Since every f_γ-P_S-closed set is f_γ-P_S-g-closed. F is f_γ-P_S-g-closed then F is also f_γ-P_S-closed set. That means A and F both are f_γ-P_S-closed set, then $A \cap F$ is f_γ-P_S-g-closed. □

Theorem 3.6. Let A be a subset of fine topological space (X, τ, τ_f) and γ be an operation on τ_f. Then A is f_γ-P_S-g-closed if and only if $f\tau_\gamma$-$P_Scl(A) \backslash A$ does not contain any non-empty f_γ-P_S-closed set.

Proof. Let F be a non-empty f_γ-P_S-closed set in fine space X such that $F \subseteq f\tau_\gamma$-$P_Scl(A) \backslash A$. Then, it is clear that $F \subseteq X \backslash A$ implies $A \subseteq X \backslash F$. Since F is f_γ-P_S-closed set implies $X \backslash F$ is f_γ-P_S-open set and A is f_γ-P_S-g-closed set, then $f\tau_\gamma$-$P_Scl(A) \subseteq X \backslash F$. That is $F \subseteq X \backslash (f\tau_\gamma$-$P_Scl(A))$. Hence $F \subseteq X \backslash (f\tau_\gamma$-$P_Scl(A)) \cap (f\tau_\gamma$-$P_Scl(A)) \backslash A = \phi$. This implies that $F \subseteq \phi$ and $\phi \subseteq F$ obvious, consequently $F = \phi$. This is contradiction. Therefore, F is not subset of $f\tau_\gamma$-$P_Scl(A) \backslash A$.

Conversely, let $A \subseteq G$ and G is f_γ-P_S-open set in X. So $X \backslash G$ is f_γ-P_S-closed set in X. Suppose that $f\tau_\gamma$-$P_Scl(A)$ not subset of G, then $f\tau_\gamma$-$P_Scl(A) \cap X \backslash G$

is a non-empty f_γ-P_S-closed set such that $f\tau_\gamma$-$P_S cl(A) \cap X \backslash G \subseteq f\tau_\gamma$-$P_S cl(A) \backslash A$. This is a contradiction to our hypothesis. Hence τ_γ-$P_S cl(A) \subseteq G$ and so A is f_γ-P_S-g-closed set. □

4 f_γ-P_S-g-Continuous Functions

In this section, we have introduced a new class of function called f_γ-P_S-g-continuous by using f_γ-P_S-g-closed set. Some theorems and properties for this function are also studied.

Definition 4.1. Let (X, τ, τ_f) and (Y, σ, σ_f) be two fine topological spaces and γ be an operation on τ_f. A function $f : (X, \tau, \tau_f) \to (Y, \sigma, \sigma_f)$ is called f_γ-P_S-g-continuous if the pre- image of every f-closed set in Y is f_γ-P_S-g-closed set in X.

Example 4.1. Let $X = \{a, b, c\}$ with topology $\tau = \{\phi, X, \{b\}\}$ and fine space $\tau_f = \{\phi, X, \{b\}, \{a, b\}, \{b, c\}\}$. Now define an operation γ on τ_f such that

$$\gamma(A) = \begin{cases} A & \text{if } c \in A \\ cl(A) & \text{if } c \notin A \end{cases}$$

for every $A \in \tau_f$. The set of all f_γ-open sets $f\tau_\gamma = \{\phi, X, \{b, c\}\} \cong f\tau_\gamma$-$SO(X)$. and $f\tau_\gamma$-$P_S O(X) = \{\phi, X\}$, $f\tau_\gamma$-$P_S GO(X) = P(X)$. Suppose that $Y = \{1, 2, 3\}$ and $\sigma = \{\phi, Y, \{1\}, \{1, 3\}\}$, $\sigma_f = \{\phi, Y, \{1\}, \{1, 2\}, \{1, 3\}, \{3\}, \{2, 3\}\}$. Let $f : (X, \tau, \tau_f) \to (Y, \sigma, \sigma_f)$ be a function defined by $f(a) = 1$, $f(b) = 2$, $f(c) = 3$. Then f is f_γ-P_S-g-continuous, but f is not f_gamma-P_S-continuous since $\{2, 3\}$ is f-closed in (Y, σ, σ_f), but $f^{-1}(\{2, 3\}) = \{b, c\}$ is not f_γ-P_S-closed set in (X, τ, τ_f).

Theorem 4.1. Every f_γ-P_S-continuous function is f_γ-P_S-g-continuous.

Proof. Let $f : (X, \tau, \tau_f) \to (Y, \sigma, \sigma_f)$ be any f_γ-P_S-continuous function. Then, pre-image of each f-closed set in Y is f_γ-P_S-closed set in X. By Lemma 3.1, every f_γ-P_S-closed set is f_γ-P_S-g-closed. Then, pre-image of each f-closed set in Y is f_γ-P_S-g-closed set in X. Then, the function f is f_γ-P_S-g-continuous.

Converge part of this theorem is not true, see Example 4.1, it is clear that function f is f_γ-P_S-g-continuous but not f_γ-P_S-continuous. □

Theorem 4.2. Let γ be an operation on the fine topological space (X, τ, τ_f). If the functions $f : (X, \tau, \tau_f) \to (Y, \sigma, \sigma_f)$ is f_γ-P_S-g-continuous and $g : (Y, \sigma, \sigma_f) \to (Z, \rho, \rho_f)$ is continuous. Then, the composition function $g \circ f : (X, \tau, \tau_f) \to (Z, \rho, \rho_f)$ is f_γ-P_S-g-continuous.

Proof. Obviously. □

Definition 4.2. A function $f : (X, \tau, \tau_f) \to (Y, \tau', \tau'_f)$ is called f_γ-P_S-irresolute map if $f^{-1}(V)$ is f_γ-P_S-open in X for every f_γ-P_S-open set V of Y.

Example 4.2. Let $X = \{a, b, c\}$, with topology $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ and $\tau_f = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, c\}\}$ define an operation γ on τ_f such that

$$\gamma(A) = \begin{cases} A & \text{if } A = \{a\} \\ A \cup \{c\} & \text{if } A \neq \{a\} \end{cases}$$

for every $A \in \tau_f$. Set of all f_γ-open sets $f\tau_\gamma = \{\phi, X, \{a\}, \{b, c\}, \{a, c\}\}$ and the class of all f_γ-P_S-open sets $f\tau_\gamma$-$P_SO(X) = \{\phi, X, \{a\}, \{b, c\}\}$. Let $Y = \{1, 2, 3\}$, with topology $\tau' = \{\phi, Y, \{1\}, \{2\}, \{1, 2\}\}$ and $\tau'_f = \{\phi, Y, \{1\}, \{2\}, \{1, 2\}, \{2, 3\}, \{1, 3\}\}$ define an operation γ on τ'_f such that

$$\gamma(S) = \begin{cases} S & \text{if } S = \{1\} \\ S \cup \{3\} & \text{if } S \neq \{1\} \end{cases}$$

for every $S \in \tau'_f$. Set of all f_γ-open sets $f\tau'_\gamma = \{\phi, Y, \{1\}, \{2, 3\}, \{1, 3\}\}$ and the class of all f_γ-P_S-open sets $f\tau'_\gamma$-$P_SO(Y) = \{\phi, Y, \{1\}, \{2, 3\}\}$. We define a mapping $f : X \to Y$ such that $f(a) = 1, f(b) = 2, f(c) = 3$. It may be checked that the pre-image of f_γ-P_S-open sets of Y viz. $\{1\}, \{2, 3\}$ are $\{a\}, \{b, c\}$ respectively, which are f_γ-P_S-open in X. Therefore f is f_γ-P_S-irresolute, but it is not continuous.

Definition 4.3. A function $f : (X, \tau, \tau_f) \to (Y, \tau', \tau'_f)$ is called f_γ-P_S-irresolute homeomorphism if
(1) f is one-one and onto.
(2) Both f and function $f^{-1} : (Y, \tau', \tau'_f) \to (X, \tau, \tau_f)$ are f_γ-P_S-irresolute.

Example 4.3. Let $X = \{a, b, c\}$, with topology $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ and $\tau_f = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, c\}\}$ define an operation γ on τ_f such that

$$\gamma(A) = \begin{cases} A & \text{if } A = \{a\} \\ A \cup \{c\} & \text{if } A \neq \{a\} \end{cases}$$

for every $A \in \tau_f$. Set of all f_γ-open sets $f\tau_\gamma = \{\phi, X, \{a\}, \{b,c\}, \{a,c\}\}$ and the class of all f_γ-P_S-open sets $f\tau_\gamma$-$P_SO(X) = \{\phi, X, \{a\}, \{b,c\}\}$. Let $Y = \{1,2,3\}$, with topology $\tau' = \{\phi, Y, \{1\}, \{2\}, \{1,2\}\}$ and $\tau'_f = \{\phi, Y, \{1\}, \{2\}, \{1,2\}, \{2,3\}, \{1,3\}\}$ define an operation γ on τ'_f such that

$$\gamma(S) = \begin{cases} S & \text{if } S = \{1\} \\ S \cup \{3\} & \text{if } S \neq \{1\} \end{cases}$$

for every $S \in \tau'_f$. Set of all f_γ-open sets $f\tau'_\gamma = \{\phi, Y, \{1\}, \{2,3\}, \{1,3\}\}$ and the class of all f_γ-P_S-open sets $f\tau'_\gamma$-$P_SO(Y) = \{\phi, Y, \{1\}, \{2,3\}\}$. We define a mapping $f : X \to Y$ such that $f(a) = 1, f(b) = 2, f(c) = 3$. By construction f is one-one and onto. It may be seen that the pre-image of f_γ-P_S-open sets of Y viz. $\{1\}, \{2,3\}$ are $\{a\}, \{b,c\}$ respectively, which are f_γ-P_S-open in X. Therefore f is f_γ-P_S-irresolute. Similarly, it may be checked that the inverse function $f^{-1} : Y \to X$ is also f_γ-P_S-irresolute. Thus, f is f_γ-P_S-irresolute homeomorphism.

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