Bulk Viscous Bianchi Type VI Cosmological Model In Extended Gravity With Exponential Function

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Abstract
We have constructed bulk viscous Bianchi Type VI cosmological model with an exponential scale factor based on $f(R,T)$ gravity. To determine the nature and physical properties of the model, we considered $f(R,T) = R + 2\Lambda_0 + 2\beta T$. We have made use of exponential scale factor to find the physical properties. Evolution of energy conditions is also studied and examined the behavior of that in examined. The kinematical and geometrical parameters are derived and analyzed in detail. The cosmological constant and coupling constant, is likely to experience an expansion at the late time of cosmic acceleration.

Keywords: *Bianchi type-VI, Bulk viscous, Exponential scale factor, Modified gravity, Effective cosmological constant*.

1. Introduction
It is very exclusively well known that modification of Einstein's theory plays an important role in explaining the late time acceleration and negative pressure. It is observed that in recent years, the hot big bang cosmological models have gained a lot of attention because of its simple theoretical structure and its ability to handle the complex observational problems. These types of cosmological models were successful in explaining the observed light element abundances and prediction of relic Cosmic Microwave Background (CMB). The cosmological theories have come up with a number of tested predictions such as relic non baryonic dark matter candidates, extra dimensions, inflation, etc. The recent observational data provided by supernovae cosmology project group and the high z-supernovae group not only bridge the gap between observation and theory of the study but also provide a new avenue to deal with the physics involved in the study of early universe. It is also predicted that an unknown form of energy called DE is responsible for such an expansion. Amidst the debate that, whether dark energy exists or whether there really occurs a substantial cosmic acceleration [1–3], researchers have devoted a lot of time in proposing different dark energy models. These models are also tested against the observational data accumulated over a long period of time. Hence, in order to address the cosmic speed up phenomena, the study on modified theories of gravity becomes important. Among all the constructed models to understand the

cosmic speed up phenomena, geometrically modified gravity theories have attracted substantial research attention.

In an effort to address the cosmic speed up issue, Harko et al. [4] introduced a modified gravity theory known as $f(R,T)$ gravity. Several studies were made in this theory addressing different contexts such as energy conditions (Alvarenga et al. [5]; Kiani & Nozari [6]), wormhole solution (Azizi [7]; Moraes et al. [8]), anisotropy cosmology (Sharif & Zubair [9]; Mishra et al. [10-11]), higher dimensions (Troisi [12]) and non-interacting Chaplygin gas (Shabani [13]; Shabani & Farhoudi [14]). Sharma and Singh [15] have studied the string cosmological model with magnetic field in Bianchi Type II space-time. With a rescaled functional of f (R, T) gravity, extensive investigations were carried out in Bianchi type VIh space-time to understand the dynamical behaviour of the aniotropic universe (Mishra et al. [16-17]). Zubair et al. [18] have investigated the anisotropy source with the dynamical analysis of cylindrically symmetric space-time whereas Mishra and Vadrevu [19] have constructed a cylindrically symmetric model with the exact solution. Aktas and Aygun [20] have shown that magnetized field vanishes in FRW universe for $f(R,T)$ gravity. Many more Bianchi type cosmological models have been developed in recent past (Shamir [21]; Zubair & Hassan [22]; Mishra et al. [23-24]; Chaubey & Shukla [25]). In the context of geometry modification to explain the late time cosmic dynamics and to take into account the cosmic anisotropy, many workers have constructed some Bianchi type cosmological models in $f(R,T)$ gravity [26–29]. However, a lot remain to be explored in this modified gravity theory in the context of different unanswered issues concerning the late time cosmic acceleration and cosmic anisotropy.

The Einstein Hilbert action is modified considering some extra geometrical objects. These models thereby provide a ghost free and stable alternative to GR. Many researchers have used different forms of these functionals to address the issue of mysterious dark energy and the late time cosmic phenomena [30–41]. Alvarenga et al. studied the scalar perturbations [42], Shabani and Ziaie studied the stability of the model [26, 27], Sharif and Zubair investigated the energy conditions and stability of power law solutions [43] in this modified gravity theory. Sharif and Zubair [44] and Jamil et al. [45] have studied thermodynamic aspects of $f(R,T)$ theory. There are some good works available in literature in the context of astrophysical applications of this theory [46–48]. With the advent of recent observations regarding the cosmic anisotropy [49–55], there has been an increase in the interest to investigate on the breakdown of the standard cosmology based on cosmic isotropy [56–61]. In view of this, anisotropic cosmological models that bear a similarity to Bianchi morphology have gained much importance [62–65].

In connection to the Hilbert Einstein type variational principle, Harko et al. [4] derived the field equations for f (R, T) gravity, whose action can be described as

$$S=\int\left[\frac{1}{16\pi}f(R,T)\sqrt{-g}+L_m\sqrt{-g}\right]d^4x, \tag{1}$$

The field equation in $f(R,T)$ gravity for the choice of the functional $f(R,T) = f(R) + f(T)$ is given by

$$f_R(R)R_{ij}-\frac{1}{2}f(R)g_{ij}-\left(\nabla_i\nabla_j-g_{ij}\Pi\right)f_R(R)=\left(8\pi+f_T(T)\right)T_{ij}+\left(f_T(T)p+\frac{1}{2}f(T)\right)g_{ij}. \tag{2}$$

where ∇_i being the covariant derivative and $f_R=\frac{\partial f(R)}{\partial R}$ and $f_T=\frac{\partial f(T)}{\partial T}$

We wish to consider a functional form of $f(R,T)$ so that the field equations in the modified gravity theory can be reduced to the usual field equations in GR under suitable substitution of model parameters. In this context, we have a popular choice, $f(R,T) = R + 2\beta T$. However, we consider a time independent cosmological constant Λ_0 in the functional so that $f(R,T) = R + 2\Lambda_0 + 2\beta T$. Here β is a coupling constant. For this particular choice of the functional $f(R,T)$, the field equation in the modified theory of gravity becomes

$$R_{ij} - \frac{1}{2}Rg_{ij} = (8\pi + 2\beta)T_{ij} + [(2p + T)\beta + \Lambda_0]g_{ij}. \quad (3)$$

Which can also be written as

$$R_{ij} - \frac{1}{2}Rg_{ij} = (8\pi + 2\beta)T_{ij} + \Lambda(T)g_{ij}. \quad (4)$$

Where $\Lambda(T) = (2p + T)\beta + \Lambda_0$ can be identified as the effective time dependent cosmological constant.

In this work, we have constructed Bianchi type VI cosmological models in $f(R,T)$ gravity. We have adopted a simple approach to the cosmic anisotropy to investigate the effect of anisotropy on cosmic anisotropy. In order to provide some anisotropic directional pressure, we have considered an anisotropic source along x-direction such as the presence of one dimensional cosmic strings. The effect of the coupling constant in the determination of the cosmic evolution has been investigated.

We organise the work as follows: In Section II, some basic equations concerning different properties of the universe are derived for Bianchi VI model in the framework of the modified $f(R,T)$ gravity. The dynamical features of the models are discussed in Section III. We have derived the quark energy density and pressure and their evolutionary behaviour in Section IV. Finally, conclusion is given in Section V.

2. Basic framework with directional Hubble Parameter

Here we consider Bianchi type -VI space time in the following form

$$ds^2 = dt^2 - A^2dx^2 - e^{2x}B^2dy^2 + e^{-2x}C^2dz^2 \quad (5)$$

Where $A = A(t), B = B(t), C = C(t)$ are the metric potentials.

We have considered the energy momentum tensor as

$$T_{ij} = (\rho + p)u_iu_j - pg_{ij} - \xi x_ix_j \quad (6)$$

where $u^iu_i = -x^ix_i = 1$ and $u^ix_i = 0$. In a co moving coordinate system, $u_i$is the four velocity vector and p is the proper isotropic pressure of the fluid. ρ is the energy density and ξ is the string tension density. The strings are considered to be one dimensional and thereby contribute to the anisotropic nature of the cosmic fluid. The direction of the cosmic strings is represented through x_i that are orthogonal to u_i .

Now, the field equations (4) for Bianchi type VI space-time (5) with the energy momentum tensor (6) can be obtained as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} + \frac{1}{A^2} = -\alpha(p - \xi) + \rho\beta + \Lambda_0 \quad (7)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{1}{A^2} = -\alpha p + (\rho + \xi)\beta + \Lambda_0 \quad (8)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = -\alpha p + (\rho + \xi)\beta + \Lambda_0 \quad (9)$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - \frac{1}{A^2} = \alpha\rho - (p - \xi)\beta + \Lambda_0 \quad (10)$$

$$\frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0 \quad (11)$$

A dot over a field variable denotes time differentiation. The directional Hubble rates may be considered as $H_1 = \frac{\dot{A}}{A}, H_2 = \frac{\dot{B}}{B}$ and $H_3 = \frac{\dot{C}}{C}$. It is straightforward to get $H_2 = H_3$. From (11) by suitably absorbing the integration constant. The mean Hubble parameter becomes,

$$H = \frac{\dot{a}}{a} = \frac{1}{3}(H_1 + H_2 + H_3) = \frac{1}{3}(H_1 + 2H_2)$$

Here, a is the scale factor of the universe. The set of field equations (7)–(11) can be expressed in terms of the directional Hubble parameter as

$$2\dot{H}_3 + 3H_3^2 + \frac{1}{A^2} = -\alpha(p - \xi) + \rho\beta + \Lambda_0 \tag{12}$$

$$\dot{H}_1 + \dot{H}_3 + H_1^2 + H_3^2 + H_1H_3 - \frac{1}{A^2} = -\alpha p + (\rho + \xi)\beta + \Lambda_0 \tag{13}$$

$$H_3^2 + 2H_1H_3 - \frac{1}{A^2} = \alpha\rho - (p - \xi)\beta + \Lambda_0 \tag{14}$$

3. Derivation and analysis of parameters

As the field equations are highly non-linear differential equations, we need some other condition to complete the field equations. We consider the shear scalar (σ) is proportional to the expansion scalar (θ)

So, an algebraic manipulation of the field equations (12)–(14) yields

$$p = \Delta_1(k,\alpha,\beta)\left[\Delta_3(k,\alpha,\beta)\delta_1^2 + m^{-2n}(\alpha - \beta)e^{-\frac{6n\delta_1 t}{n+2}}\right] + \Delta_5(\alpha,\beta)\xi + \Delta_6(\alpha,\beta)\Lambda_0 \tag{15}$$

$$\rho = \Delta_1(k,\alpha,\beta)\left[\Delta_4(k,\alpha,\beta)\delta_1^2 - m^{-2n}(\alpha - \beta)e^{-\frac{6n\delta_1 t}{n+2}}\right] - \Delta_5(\alpha,\beta)\xi - \Delta_6(\alpha,\beta)\Lambda_0 \tag{16}$$

$$\xi = \Delta_1(k,\alpha,\beta)\left[-\Delta_2(k,\alpha,\beta)\delta_1^2 + 2m^{-2n}(\alpha + \beta)e^{-\frac{6n\delta_1 t}{n+2}}\right] \tag{17}$$

Consequently, the equation of state parameter ω and the effective cosmological constant Λ can
be expressed as

$$\omega = \frac{\Delta_1(k,\alpha,\beta)\left[\Delta_3(k,\alpha,\beta)\delta_1^2 + m^{-2n}(\alpha-\beta)e^{-\frac{6n\delta_1 t}{n+2}}\right] + \Delta_5(\alpha,\beta)\xi + \Delta_6(\alpha,\beta)\Lambda_0}{\Delta_1(k,\alpha,\beta)\left[\Delta_4(k,\alpha,\beta)\delta_1^2 - m^{-2n}(\alpha-\beta)e^{-\frac{6n\delta_1 t}{n+2}}\right] - \Delta_5(\alpha,\beta)\xi - \Delta_6(\alpha,\beta)\Lambda_0} \tag{18}$$

$$\Lambda = \Delta_1(k,\alpha,\beta)\beta\left[\Delta_4(k,\alpha,\beta)\delta_1^2 - \Delta_3(k,\alpha,\beta)\delta_1^2 - 2m^{-2n}(\alpha - \beta)e^{-\frac{6n\delta_1 t}{n+2}}\right] - 2\Delta_5(\alpha,\beta)\xi\beta - 2\Delta_6(\alpha,\beta)\Lambda_0\beta + \Lambda_0 \tag{19}$$

Where

$$\Delta_1(k,\alpha,\beta) = \frac{1}{(k+2)^2(\alpha^2 - \beta^2)}$$

$$\Delta_2(k,\alpha,\beta) = \frac{9(\alpha + \beta)(k - 1)}{(k + 2)}$$

$$\Delta_3(k,\alpha,\beta) = \frac{9}{(k+2)^2}[(2k+1)\beta - (k^2 + k + 1)\alpha]$$

$$\Delta_4(k,\alpha,\beta) = \frac{9}{(k+2)^2}[(2k+1)\alpha - (k^2 + k + 1)\beta]$$

$$\Delta_5(\alpha,\beta) = \frac{\beta}{(\alpha+\beta)}$$
$$\Delta_6(\alpha,\beta) = \frac{1}{(\alpha+\beta)}$$

We can now discuss the dynamical behavior of the model from the above equations. In other words, we can say that the formalism described above can help us to study the expansion rate of the universe for an assumed dynamic of the universe. In the above equations, all the physical quantities are expressed with respect to cosmic time. The energy density (ρ) stays in the positive range and decreases to small values at the late times and the isotropic pressure remains negative for all these values throughout the evolution, as expected, which in turn indicates the accelerated expansion of the universe. For all the above-considered values of parameters, ω stays negative. Mostly it remains in the phantom region. The concrete dynamical nature of the model can be observed through the evolution of the Equation of State parameter ω. However, it evolves from a phantom phase to a quintessence phase after crossing the phantom divide. It is clear that ω increases from some higher negative value at early time and overlaps with the ΛCDM model at late times. The time-varying cosmological constant with the considered values of the parameters remains in the negative domain.

4. Dynamical Behaviour

The Dynamical Behaviour of the model are obtained as follows:

Spatial Volume $V = a^3 = \left(\delta e^{\delta_1 t}\right)^3$ (20)

Hubble's parameter $H = \delta_1$ (21)

The Expansion Scalar: $\theta = 3H = 3\delta_1$ (22)

The Shear Scalar: $\sigma^2 = \frac{3(n-1)^2}{(n+2)^2}\delta_1^2$ (23)

Anisotropy Parameter: $\mathcal{A} = \frac{2(n-1)^2}{(n+2)^2}$ (24)

Deceleration Parameter $q = -1$ (25)

$\frac{\sigma}{H} = \sqrt{3}\frac{(n-1)}{(n+2)}$ (26)

$\mathcal{A}$ is a measure of deviation from isotropic expansion. The anisotropy of the expansion results in the isotropic expansion of the universe for $\mathcal{A} = 0$. This can be achieved when $n = 1$, which yields $H_1 = H_2 = H_3$. Since the representative value of δ is considered to be positive and the scale factor is exponential, it can be observed that the volume scale factor becomes positive throughout the evolution and increases gradually with increase in time. The scalar expansion constant with increase in time. The shear scalar vanishes for $n = 1$ of the anisotropy parameter $n = 1$. However, it increases everywhere except for $n = 1$

The energy conditions are the co-ordinate invariants that incorporate the common characteristics shared by almost every matter field. As standard matter is assumed to satisfy the necessary energy conditions, for a magnetic fluid distribution, the general inequalities of energy conditions are the following. i) Null Energy Condition (NEC): $\rho + p \geq 0$. ii) Weak Energy Condition (WEC): $\rho + p \geq$

$0, \rho \geq 0$. iii) Strong Energy Condition (SEC): $\rho + 3p \geq 0, \rho + p \geq 0$. iv) Dominant Energy Condition (DEC): $\rho \pm p \geq 0, \rho \geq 0$. The energy conditions for the cosmological model discussed here takes the following form
Without violating the physical parameters behaviour, fixing the relation of the parameters, we have calculated the energy conditions within our formalism of exponential scale factor. Hence, the Null Energy Condition (NEC), Weak Energy Condition (WEC), Strong Energy Condition (SEC) and Dominant Energy Condition (DEC) can be expressed respectively as

$$\rho + p = \Delta_1(k,\alpha,\beta)\delta_1^2[\Delta_3(k,\alpha,\beta) + \Delta_4(k,\alpha,\beta)] \quad (27)$$

$$\rho - p = \Delta_1(k,\alpha,\beta)\left[\Delta_4(k,\alpha,\beta)\delta_1^2 - \Delta_3(k,\alpha,\beta)\delta_1^2 - 2m^{-2n}(\alpha-\beta)e^{-\frac{6n\delta_1 t}{n+2}}\right] - 2\Delta_5(\alpha,\beta)\xi - 2\Delta_6(\alpha,\beta)\Lambda_0 \quad (28)$$

$$\rho + 3p = \Delta_1(k,\alpha,\beta)\left[\Delta_4(k,\alpha,\beta)\delta_1^2 + 3\Delta_3(k,\alpha,\beta)\delta_1^2 + 2m^{-2n}(\alpha - \beta)e^{-\frac{6n\delta_1 t}{n+2}}\right] + 2\Delta_5(\alpha,\beta)\xi + 2\Delta_6(\alpha,\beta)\Lambda_0 \quad (29)$$

5. Conclusion

The cosmological model of the universe is constructed in $f\,(R,T)$ gravity with the use of an exponential scale factor. We assume the functional in the form $R + 2\Lambda_0 + 2\beta T$. We have developed a formalism to find the physical parameters from the field equations and, with the power law assumption of the scale factor, the model is constructed. A more systematic mathematical formulation has been developed to express the physical, kinematical parameters as well the metric potentials involved in the study. The interesting feature of this work is the development of the mathematical expression, where any kind of function can be used for the Hubble parameter and with the physical parameters an accelerating cosmological model can be obtained. It is also worthy to note that in order to maintain a higher rate of anisotropic expansion, additional anisotropic fluid is very much required. All the physical parameters are behaving according to the observational outcomes. An accelerating model is presented here, where the physical parameters are constrained with the representative value. The matter content of the gravitational theory remains the same; however the behavior may change with a very high value.

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