Mari's algorithm is used to solve a fuzzy transportation problem using Pentagonal Fuzzy numbers

Antonyraj Martin
Assistant Professor, PG & Research Department of Mathematics,
Bishop Heber College, Trichy 620017, Tamil Nadu, INDIA
mathanto90@gmail.com

Abstract

In this article, the researcher studied Mari's Algorithm for solving Transportation problems. This research article discusses a modified solution methodology based on the existing method of solving a Transportation Problem [TP]. The author made an attempt to solve the Fuzzified Transportation Problem [FTP] using Pentagonal Fuzzy numbers by Modified Mari's Algorithm. Algorithm is a very simple and lucid method which passes many times the direct optimal solution. This method ensures that the optimal solution to the given FTP can be obtained with less time.

Keywords: *Transportation Problem, Fuzzy Logic, Optimization*

1. Introduction

The word "transportation" is misleading because it seems to be limited to only transportation systems, but this is not the case. In reality, many resource utilisations issues that arise in manufacturing systems can be treated as transportation issues. Production scheduling, transportation scheduling, and so on are common examples. This problem was first presented by [4] Hitchcock (1941) and [5] Koopmans (1947), and [3] Dantzig (1947) solved it using the revised simplex method.

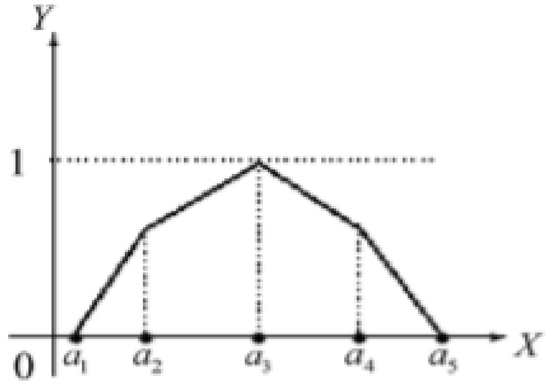
Hitchcock [4] developed the basic transportation problem in 1941, along with a constructive solution process, and Koopmans [5] addressed the problem in depth in 1949. Dantzig [3] reformulated the transportation problem as a linear programming problem and presented a solution method in 1951. Transportation has become a regular application for industrial organisations with multiple production facilities, warehouses, and distribution centres these days. It was necessary to divide the problem into two stages in order to achieve an optimal solution for transportation problems. The initial simple feasible solution (IBFS) was obtained in the first stage by using some of the available methods such as "North West Corner," "Matrix Minima," "Least Cost Method," "Row Minima," "Column Minima," and "Vogel's Approximation Method," among others. The MODI (Modified Distribution) approach was then used in the next and final stage to arrive at an optimal solution. The "Stepping Stone Method" was developed by Charnes and Cooper [2] for finding an optimal solution from IBFS. P. Pandian et al. [8], [9], and Sudhakar et al. [11] suggested two separate approaches for finding an optimal solution directly in 2010 and 2012, respectively. Mari's Algorithm was proposed by Mariappan, P., and Antony Raj M in [1] to solve the Transportation Problem.

Here's a much simpler process. For the Fuzzy Transportation Problem using Trapezoidal Fuzzy Numbers, Mari's Algorithm is used to find an optimal solution directly with fewer steps and using very simple computations. Using a flowchart method, the proposed method's step-by-step process is very clearly illustrated [Refer Flow chart – 3.1]. Example problems are used to demonstrate the proposed method's working methodology. This approach often yields a direct optimal solution. The significance is that this technique can easily skip the time-consuming MODI iterations.

2. Definition: Pentagonal Fuzzy Number

A fuzzy number A on R is said to be a pentagonal fuzzy number (PFN) or linear fuzzy number which is named as $(a_1, a_2, a_3, a_4, a_5)$ if its membership function $\mu_A(x)$ has the following characteristic

$$\begin{cases} 0, if \;\; x < a_1 \\ u_1\left(\frac{x-a_1}{a_2-a_1}\right), if \;\; a_1 \le x \le a_2 \\ 1, if \;\; x = a_3 \\ 1-(1-u_2)\left(\frac{a_4-x}{a_4-a_3}\right), if \;\; a_3 \le x \le a_4 \\ u_2\left(\frac{a_5-x}{a_5-a_4}\right), if \;\; a_4 \le x \le a_5 \\ 0, if \;\; x > a_5 \end{cases}$$



2.1 Arithmetic Operations

Let $\bar{A} = (a_1, b_1, c_1, d_1, e_1)$ and $\bar{B} = (a_2, b_2, c_2, d_2, e_2)$ are two fuzzy numbers where $a_1 \le b_1 \le c_1 \le d_1 \le e_1$ and $a_2 \le b_2 \le c_2 \le d_2 \le e_2$. Then the arithmetic operations are defined as

(i) Addition

$\bar{A} + \bar{B} = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2, e_1 + e_2)$

(ii) Subtraction

$\bar{A} - \bar{B} = (a_1 - e_2, b_1 - d_2, c_1 - c_2, d_1 - b_2, e_1 - a_2)$

(iii) Multiplication

$\bar{A} * \bar{B} = \left(\frac{a_1}{5}\mu_\theta, \frac{b_1}{5}\mu_\theta, \frac{c_1}{5}\mu_\theta, \frac{d_1}{5}\mu_\theta, \frac{e_1}{5}\mu_\theta\right)$ Where $\mu_\theta = (a_2 + b_2 + c_2 + d_2 + e_2)$

(iv) Division

$\bar{A} \div \bar{B} = \left(\frac{5a_1}{\mu_\theta}, \frac{5b_1}{\mu_\theta}, \frac{5c_1}{\mu_\theta}, \frac{5d_1}{\mu_\theta}, \frac{5e_1}{\mu_\theta}\right)$ Where $\mu_\theta = (a_2 + b_2 + c_2 + d_2 + e_2)$

(v) Scalar Multiplication

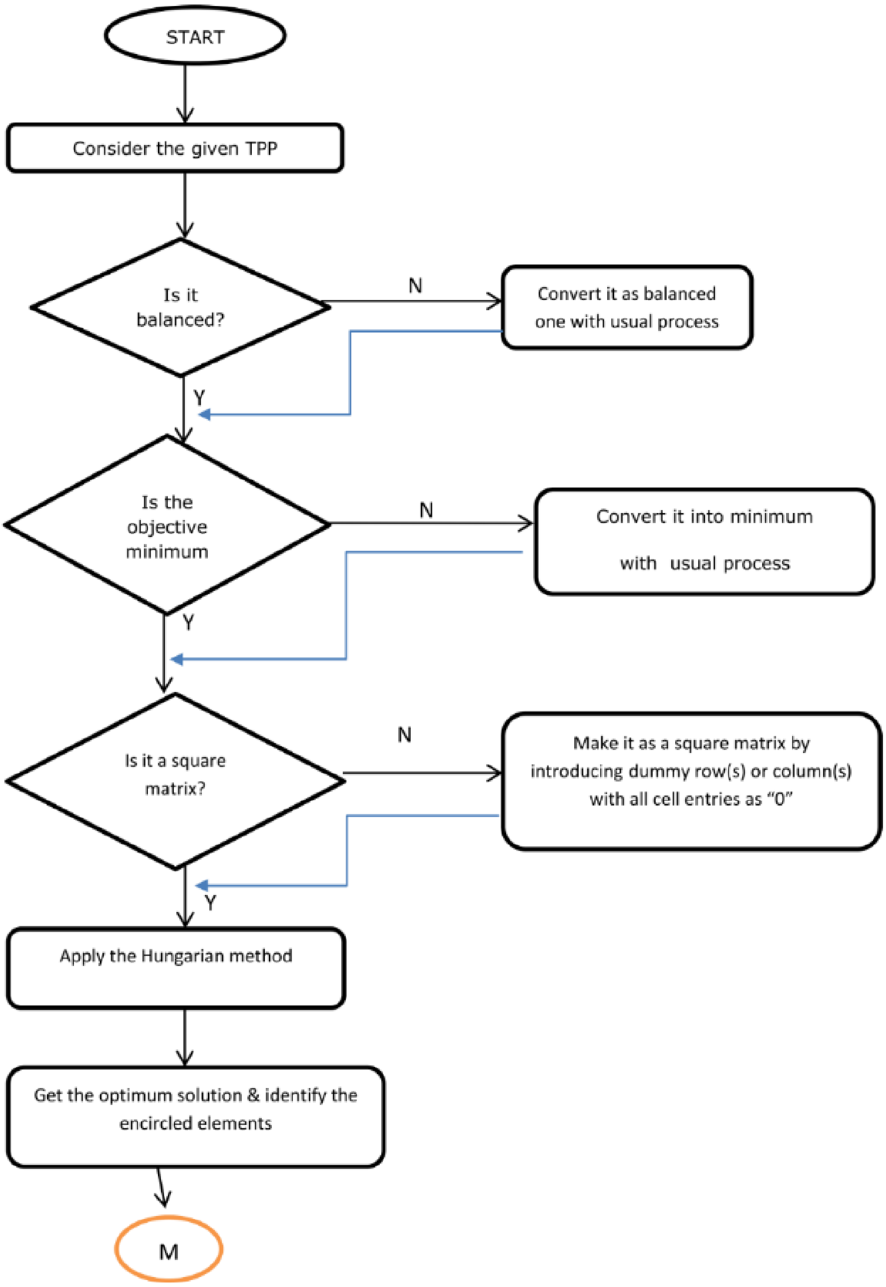
$$K\bar{A} = \begin{cases} (ka, kb, kc, kd, ke) if \;\; k > 0 \\ (ke, kd, kc, kb, ka) if \;\; k < 0 \end{cases}$$

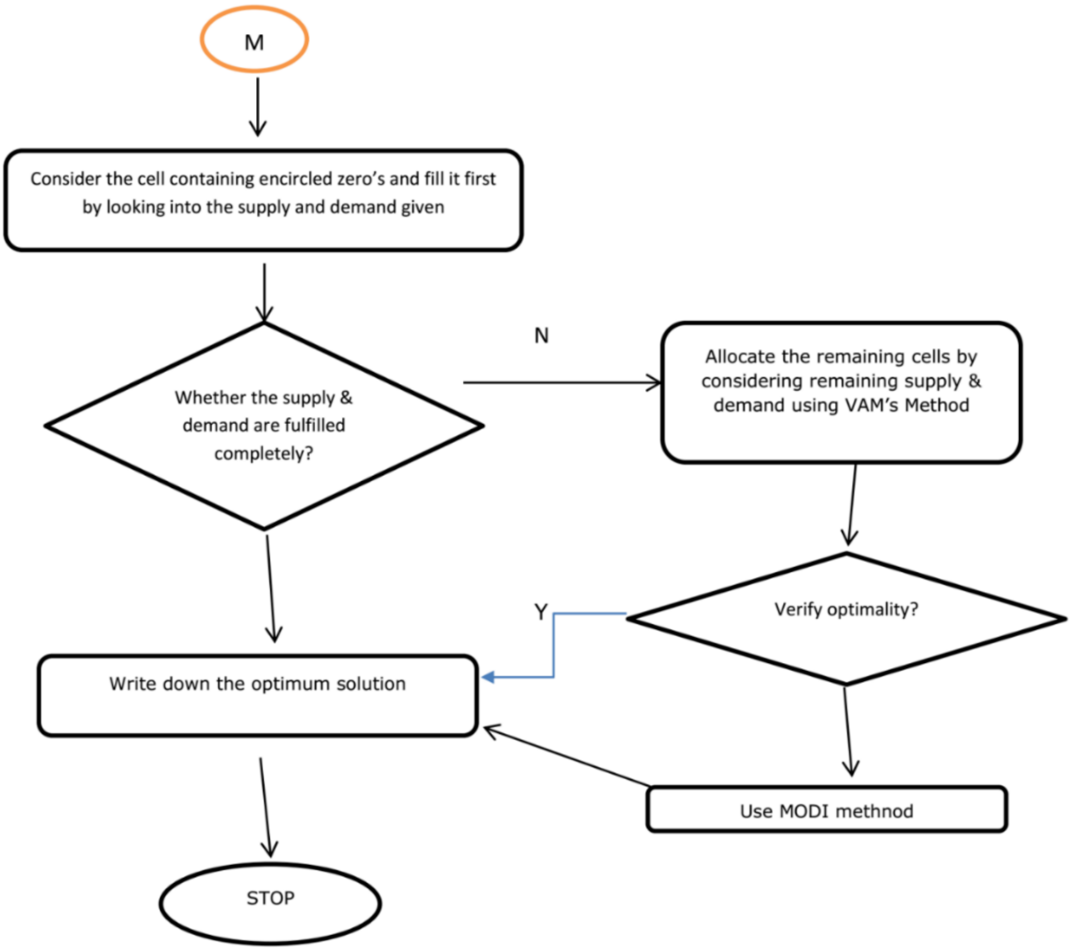
2.2 Mathematical Formulation of Fuzzy Transportation Problem

Consider a fuzzy transportation problem with m sources and n destinations with pentagonal fuzzy numbers. Let $a_i, (a_i \geq 0)$ be the fuzzy availability at source i and $b_j, \left(b_j \geq 0\right)$ be the fuzzy requirement at destination j. Let c_{ij} be the fuzzy unit transportation cost from source i to destination j. Let x_{ij} denote the number of fuzzy units to be transported from source i to destination j. Then the problem is to find the feasible way of transporting the available amount at each source to satisfy the demand at each destination so that the total transportation cost is minimized.

Flow Chart-3.1

Mari's Algorithm [1]:





The mathematical formulation of the fuzzy transportation whose parameters are pentagonal fuzzy numbers under the case that the total supply is equivalent to the total demand is given by

$$Min\ z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\, x_{ij}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_{ij}, i = 1,2, \dots .m.$$

$$\sum_{i=1}^{m} x_{ij} = b_{ij}, j = 1,2, \dots .n.$$

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j; i = 1,2, \dots .m ; j = 1,2, \dots .n.$$

$$\text{and } x_{ij} \geq 0.$$

The fuzzy transportation problem is explicitly represented by the fuzzy transportation table:

	1	**…**	**n**	**Supply**
1	c_{11}	…	c_{1n}	a_1
…		…	…	…
m	c_{n1}	…	c_{mn}	a_m
Demand	b_1	…	b_n	

2.3 Range Technique

The range is defined as the difference between the maximum value and minimum value.
Range = Maximum amount – Minimum amount

3.0 Numerical Illustrations

Example 3.1

Consider the [12] Balanced fuzzy transportation problem.
A Product is produced by four factories Factory1, Factory2, Factory3, Factory4 production capacity of the four factories are 30, 27,40, and 50 units, respectively. The product is supplied to four stores Store1, Store2, Store3, Store4, the requirements of Demands, which are 20,40,34 and 53, respectively. Here Unit costs of fuzzy transportation are represented as fuzzy pentagonal numbers are given below. Find the fuzzy transportation plan such that the total production and transportation cost is minimum.

	Store 1	**Store 2**	**Store 3**	**Store 4**	**Capacity**
Factory 1	(2,4,6,8,9)	(3,5,7,8,9)	(2,4,5,6,7)	(3,4,6,7,12)	**30**
Factory 2	(0,2,5,6,8)	(4,5,6,8,11)	(2,3,5,7,11)	(1,5,6,9,11)	**27**
Factory 3	(1,2,3,4,5)	(2,3,4,6,8)	(4,5,6,8,9)	(6,7,8,9,13)	**40**
Factory 4	(3,5,6,7,8)	(1,5,6,7,8)	(2,7,8,9,10)	(3,3,4,5,9)	**50**
Demand	**20**	**40**	**34**	**53**	

Solution

The author must transform fuzzy pentagonal numbers into crisp values by using the Range technique.

	Store 1	**Store 2**	**Store 3**	**Store 4**	**Capacity**
Factory 1	7	6	5	9	**30**
Factory 2	8	7	9	10	**27**
Factory 3	4	6	5	7	**40**
Factory 4	5	7	8	6	**50**
Demand	**20**	**40**	**34**	**53**	**147**

2	1	0	4
1	0	2	3
0	2	1	3
0	2	3	1

2	1	0	3
1	0	2	2
0	2	1	2
0	2	3	0

2	1	**0**	3
1	**0**	2	2
0	2	1	2
0	2	3	**0**

7	6	5^{30}	9	**30**
8	7^{27}	9	10	**27**
4^{20}	6^{13}	5^{4}	7^{3}	**40**
5	7	8	6^{50}	**50**
20	**40**	**34**	**53**	**147**

The Transportation cost $Z = (30 * 5) + (27 * 7) + (20 * 4) + (13 * 6) + (4 * 5) + (3 * 7) + (50 * 6) = 838$

3.1 Comparison with Existing Methods

The comparison of the proposed method with the existing process is tabulated below, in which it is clearly shown that the proposed method provides the optimal results.

Methods	**Optimal Solution**
North West Corner Method	898
VAM	886
LCM	844
Zero Suffix Method	1093
Proposed Method	**838**

Example 3.2

Consider the unbalanced fuzzy transportation problem.

There are five factories F1, F2, F3, F4, F5, which supply goods to the four dealers D1, D2, D3, D4. The production capacities of these factories are 30,27,40,50 and 24 units per month, respectively. The requirements from the dealers are 20,40,34 and 53 units per month, respectively. The following table gives the fuzzy unit transportation costs from the factories to the dealers. Find the fuzzy transportation plan such that the total production and transportation cost is minimum.

	D1	**D2**	**D3**	**D4**	**Capacity**
Factory 1	(2,4,6,8,9)	(3,5,7,8,9)	(2,4,5,6,7)	(3,4,6,7,12)	**30**
Factory 2	(0,2,5,6,8)	(4,5,6,8,11)	(2,3,5,7,11)	(1,5,6,9,11)	**27**
Factory 3	(1,2,3,4,5)	(2,3,4,6,8)	(4,5,6,8,9)	(6,7,8,9,13)	**40**
Factory 4	(3,5,6,7,8)	(1,5,6,7,8)	(2,7,8,9,10)	(3,3,4,5,9)	**50**
Factory 5	(1,3,4,6,7)	(0,2,4,3,5)	(1,3,4,6,9)	(2,4,7,8,12)	**24**
Demand	**20**	**40**	**34**	**53**	

Solution

The author must transform fuzzy pentagonal numbers into crisp values by using the Range technique.

	D1	**D2**	**D3**	**D4**	**Capacity**
Factory 1	7	6	5	9	**30**
Factory 2	8	7	9	10	**27**
Factory 3	4	6	5	7	**40**
Factory 4	5	7	8	6	**50**
Factory 5	6	5	8	10	**24**
Demand	**20**	**40**	**34**	**53**	

The given matrix is unbalanced, and We are adding 0 columns to balance the given matrix.

	D1	**D2**	**D3**	**D4**	**D5**	**Capacity**
Factory 1	7	6	5	9	0	**30**
Factory 2	8	7	9	10	0	**27**
Factory 3	4	6	5	7	0	**40**
Factory 4	5	7	8	6	0	**50**
Factory 5	6	5	8	10	0	**24**
Demand	**20**	**40**	**34**	**53**	**24**	**171**

7	6	5	9	0
8	7	9	10	0
4	6	5	7	0
5	7	8	6	0
6	5	8	10	0

3	1	0	3	0
4	2	4	4	0
0	1	0	1	0
1	2	3	0	0
2	0	3	4	0

3	1	**0**	3	0
4	2	4	4	**0**
0	1	0	1	0
1	2	3	**0**	0
2	**0**	3	4	0

7	6	5^{30}	9	0	**30**
8	7^{3}	9	10	0^{24}	**27**
4^{20}	6^{13}	5^{4}	7^{3}	0	**40**
5	7	8	6^{50}	0	**50**
6	5^{24}	8	10	0	**24**
20	**40**	**34**	**53**	**0**	**171**

The Transportation cost $Z = (30 * 5) + (7 * 3) + (24 * 0) + (20 * 4) + (13 * 6) + (4 * 5) + (3 * 7) + (50 * 6) + (24 * 5) = 790$

3.3 Comparison with Existing Methods

The proposed method is compared to the current procedure in the table below, and it is clear that the proposed method produces the best results.

Methods	**Optimal Solution**	
	Example 1	**Example 2**
North West Corner Method	898	898
VAM	886	838
LCM	844	844
Zero Suffix Method	1093	1022
Proposed Method	**838**	**790**

4.0 Conclusion

As a conclusion, Mari's Algorithm is self-evidently the best solution for the Fuzzy Transportation Problem using Pentagonal Fuzzy Numbers. Only the Hungarian algorithm and, on rare occasions, Vogel's Algorithm with MODI algorithms are used in this method. Since the approach is easy to understand and takes less time, it is a good choice for solving Fuzzy Transportation Problem.

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