EFFECT OF BROWNIAN MOTION AND THERMOPHORESIS ON NANOFLUID PAST A STRETCHING SURFACE WITH VARIABLE VISCOSITY AND NEWTONIAN COOLING INSPIRED BY THERMAL RADIATION

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Abstract:
We analyze the combined influence of Brownian motion, thermophoresis, with past a stretching surface with variable viscosity and thermal radiation. The governing equations have been solved by employing fifth-order Runge-Kutta-Fehlberg method along with shooting technique. The effects of various parameters on the velocity, temperature and concentration as well as on the local skin-friction coefficient, local Nusselt number and local Sherwood number are presented graphically and discussed. It is observed that a velocities component increases with Hall parameter\((m)\), Brownian motion parameter \((Nb)\), Radiation parameter \((Rd)\), Viscosity parameter \((\theta r)\), Convective heat transfer constant\((h1)\), reduces with thermophoresis parameter \((Nt)\). Nusselt number increase in \(\theta r\) and reduces Nu and Sh, increase in \(\theta r/h1\) reduces rate of heat transfer and enhances mass transfer.

Keywords: Brownian motion, Thermophoresis, Stretching surface, variable viscosity, thermal radiation, Newtonian cooling.

1. INTRODUCTION
The word “nanofluid” coined by Choi [8] refers to a liquid suspension containing ultra-fine particles (diameter less than 50 nm). The traditional fluids viz., water, mineral oils, ethylene glycol, engine oil with limited heat transfer capabilities are used for heat transfer applications. It has been pointed that contribution of Brownian motion is much lower than other factors such as size effect, clustering of nanoparticles and surface adsorption. The different theories explaining the enhanced heat transfer characteristics of nanofluids have been evaluated by Buongiorno [6]. He developed an analytical model for convective transport in nanofluids which takes into account the Brownian diffusion and thermophoresis.

Anjali Devi and Mekala [4] have analysed discussed the Role of Brownian Motion and Thermophoresis Effects on Hydromagnetic Flow of Nanofluid over a Nonlinearly Stretching Sheet with Slip effects and Solar Radiation. Dulal Pal et al [9] have briefly discussed the thermophoresis and Brownian motion effects on magneto-convective heat
transfer of viscoelastic nanofluid over a stretching sheet with nonlinear thermal radiation. Falana et al. [10] have been described the effect of Brownian Motion and Thermophoresis on a Nonlinearly Stretching Permeable Sheet in a Nanofluid. Kempannagari Anantha Kumar et al. [11] discussed the thermophoresis and brownian motion effects on mhd micropolar nanofluid flow past a stretching surface with non-uniform heat source/sink. Mabood et al. [12] have observed the Framing the features of Brownian motion and thermophoresis on radiative nanofluid flow past a rotating stretching sheet with magnetohydrodynamics. Shobha and Patil Mallikarjun [20] demonstrated the fully developed mixed convection in a vertical channel filled with nanofluids with heat source or sink.

The effect of temperature-dependent viscosity on heat and mass transfer laminar boundary layer flow has been discussed by many authors (Mukhopadhyay and Layek [14], Ali [3], Makinde [13], Prasad et al. [16], Alam et al. [2]) in various situations. They showed that when this effect was included, the flow characteristics might change substantially compared with the constant viscosity assumption. Salem [18] investigated variable viscosity and thermal conductivity effects on MHD flow and heat transfer in viscoelastic fluid over a stretching sheet. Xi-Yan Tian et al. [24] investigated the 2D boundary layer flow and heat transfer in variable viscosity MHD flow over a stretching plate.

Hall currents are important and they have a marked effect on the magnitude and direction of the current density and consequently on the magnetic force term. The problem of MHD free convection flow with Hall currents has many important engineering applications such as in power generators, MHD accelerators, refrigeration coils, transmission lines, electric transformers, heating elements etc., Watanabe and Pop [307], Abo-Eldahab and Salem [261], Rana et al. [292], Shit [301] among others have advanced studies on Hall effect on MHD past stretching sheet. Chamkha et al. [270] analyzed the unsteady MHD free convective heat and mass transfer from a vertical porous plate with Hall current, thermal radiation and chemical reaction effects.

Motivated by the above-mentioned researchers, this paper aims at studying the combined influence of Brownian motion, thermophoresis, with past a stretching surface with variable viscosity and thermal radiation. The governing equations have been solved by employing fifth-order Runge-Kutta-Fehlberg method along with shooting technique. The effects of various parameters on the velocity, temperature and concentration as well as on the local skin-friction coefficient, local Nusselt number and local Sherwood number are presented graphically and discussed.

2. FORMULATION OF THE PROBLEM
We consider the steady free-convective flow, heat and mass transfer of an incompressible, viscous and electrically conducting fluid past a stretching sheet and the sheet is stretched with a velocity proportional to the distance from a fixed origin O (Fig. 1).
The fluid viscosity $\mu_f$ is assumed to vary as a reciprocal of a linear function of temperature given by

$$\frac{1}{\mu_f} = \frac{1}{\mu_\infty}[1 + \gamma_0(T - T_\infty)]$$  \hspace{1cm} (1)

$$\frac{1}{\mu_\infty} = a(T - T_\infty)$$  \hspace{1cm} (2)

Where $a = \frac{\gamma_0}{\mu_\infty}$ and $T_r = T_\infty - \frac{1}{\gamma_0}$

In the above equation both $a$ and $T_r$ are constants, and their values depend on the thermal property of the fluid, i.e., $\gamma_0$. In general $a > 0$ represents for liquids, whereas for gases $a < 0$.

The boundary layer free-convection flow with mass transfer and generalized Ohm’s law with Hall current effect are governed by the following system of equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$  \hspace{1cm} (3)

$$\rho_f \left( \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \mu_f \frac{\partial^2 u}{\partial y^2} + g(1-C_\infty)\rho_f \beta g(T - T_\infty) -$$

$$- (\rho_p - \rho_f)kg(C - C_\infty) - \frac{\sigma_{nf}B_0^2}{1+m^2} (u + mw)$$  \hspace{1cm} (4)

$$\rho_f \left( \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} \right) = \mu_f \frac{\partial^2 w}{\partial y^2} + \frac{\sigma_{nf}B_0^2}{1+m^2} (mu - w)$$  \hspace{1cm} (5)

$$\left( \rho C_p \right)_f \left( \frac{u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y}}{C_p} \right) = k_f \frac{\partial^2 T}{\partial y^2} + \tau \left[ D_H \left( \frac{\partial T}{\partial x} + \frac{\partial C}{\partial x} \right) + \frac{\partial T}{\partial y} \right] \left( \frac{\partial C}{\partial y} \right) +$$

$$\frac{D_T}{T_\infty} \left( \left( \frac{\partial T}{\partial y} \right)^2 + \left( \frac{\partial T}{\partial y} \right)^2 \right) - \frac{\partial (q_R)}{\partial y}$$  \hspace{1cm} (6)

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_H \frac{\partial^2 C}{\partial y^2} - k_0(C - C_\infty) + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2}$$  \hspace{1cm} (7)
where \((u,v,w)\) are the velocity components along the \((x,y,z)\) directions respectively. \(\sigma\) is the effective electrical conductivity, \((\beta)\) is the effective thermal volumetric coefficient of expansion, \(k_0\) is the chemical reaction coefficient, \(D_r\) is the solution diffusivity of the medium, \(K_T\) is the thermal diffusion ratio, \(C_s\) is the concentration susceptibility, \(C_p\) is the specific heat at constant pressure, \(T_\infty\) is the mean fluid temperature and \(q_R\) is the radiative heat flux. \(m = \frac{cB}{en}\) is the Hall parameter.

The boundary conditions for the present problem can be written as

\[
\begin{align*}
\frac{\partial T}{\partial y} &= h_j (T_w - T) , \quad C = C_w \quad \text{at} \quad y = 0 \quad (8) \\
\frac{u}{y} &= 0, \quad w = 0, \quad T \to T_\infty, \quad C \to C_\infty \quad \text{at} \quad y \to \infty \quad (9)
\end{align*}
\]

where \(b\) \((> 0)\) being stretching rate of the sheet. The boundary conditions on velocity in Equation (8) are the no-slip condition at the surface \(y = 0\), while the boundary conditions on velocity at \(y \to \infty\) follow from the fact that there is no flow far away from the stretching surface.

The radiation heat term (Brewster[5]) by using The Rosseland approximation is given by

\[
q_r = -\frac{4\sigma^* \partial T'^4}{3\beta_R \partial y} \quad (10)
\]

\[
T'^4 \approx 4TT_\infty^3 - 3T_\infty^4 \quad (11)
\]

\[
\frac{\partial q_R}{\partial z} = -\frac{16\sigma^* T^3 \partial^2 T}{3\beta_R \partial y^2} \quad (12)
\]

To examine the flow regime adjacent to the sheet, the following transformations are invoked

\[
u = bx f'(\eta); \quad v = -\sqrt{bv} f(\eta); \quad w = bx g(\eta); \quad \eta = \frac{b}{\sqrt{v}} y; \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}; \phi = \frac{C - C_\infty}{C_w - C_\infty} \quad (13)
\]

where \(f\) is a dimensionless stream function, \(h\) is the similarity space variable, \(\theta\) and \(\phi\) are the dimensionless temperature and concentration respectively. Clearly, the continuity Eq. (3) is satisfied by \(u\) and \(v\) defined in Eq. (9). Substituting Eq. (10) the Eqs. (4)-(7) reduce to

\[
\begin{align*}
\left(\frac{\partial}{\partial \theta_r}\right) (f'' - f' + f'' - \left(\frac{\partial}{\partial \theta_r}\right)) f'' - \\
\left(\frac{\partial}{\partial \theta_r}\right) (f^g - f^g') + g^* - \left(\frac{\partial}{\partial \theta_r}\right) g^* M^2 \left(\frac{\partial}{\partial \theta_r}\right) \left(\frac{m f' + g}{1 + m^2}\right) = 0
\end{align*}
\]

\[
\begin{align*}
\left(\frac{\partial}{\partial \theta_r}\right) (f^g - f^g') + g^* - \left(\frac{\partial}{\partial \theta_r}\right) g^* M^2 \left(\frac{\partial}{\partial \theta_r}\right) \left(\frac{m f' + g}{1 + m^2}\right) = 0
\end{align*}
\]

\[
\begin{align*}
(1 + \frac{4Rd}{3}) \theta'' + Nb \left(\frac{\partial}{\partial \eta} + \frac{\partial}{\partial \eta} \frac{\partial}{\partial \eta} \phi^* \right) + \frac{\partial}{\partial \eta} \frac{\partial}{\partial \eta} \phi^* + Nt \left(\frac{\partial}{\partial \eta} \phi^* \right)^2 = 0
\end{align*}
\]

\[
\frac{1}{Le} \phi'' - (f\phi' - \gamma \phi) + \left(\frac{Nt}{Nb}\right) (f\phi') = 0
\]

Similarly, the transformed boundary conditions are given by

\[
f(\eta) = 1, \ f(\eta) = 0, \ g(\eta) = 0, \ \theta(\eta) = 0, \ \phi(\eta) = 1 \quad \text{at} \ \eta = 0 \quad (18)
\]
\[ f(\eta) \to 0, \quad g(\eta) \to 0, \quad \theta(\eta) \to 0, \quad \phi(\eta) \to 0 \quad \text{at} \quad \eta \to \infty \]  \quad (19)

where a prime denotes the differentiation with respect to \( \eta \) only and the dimensionless parameters appearing in the Eqs. (13)-(17) are respectively defined as

\[ \theta_r = \frac{T_r - T_\infty}{T_w - T_\infty} \]

the viscosity parameter,

\[ M = \frac{\sigma B_0^2}{\nu} \]

the magnetic parameter,

\[ P_v = \frac{\rho C_v v}{k_f} \]

the Prandtl number,

\[ G = \frac{g_0 \beta (T_w - T_\infty)}{b^2 x} \]

the local Grashof number,

\[ N = \frac{(\rho_p - \rho_{fr}) (C_w - C_\infty)}{\rho_{fr} (1 - C_\infty)(T_w - T_\infty)} \]

the thermal radiation parameter,

\[ Rd = \frac{4T_\infty^3 \sigma^*}{k_f \beta} \]

the thermal radiation parameter,

\[ Le = \frac{\mu_f}{\rho_x D_m} \]

the Lewis number,

\[ N_b = \frac{2D_B (C_w - C_\infty)}{a} \]

Brownian motion parameter,

\[ N_i = \frac{\pi D_T (T_w - T_\infty)}{a T_w} \]

Thermophoresis parameter.

\[ h_i = \left( \frac{h_i}{k_f} \right) \frac{v}{b} \]

is convective heat transfer constant.

3. METHOD OF SOLUTION

The coupled ordinary differential equations (13)-(17) are of third-order in \( f \), and second-order in \( g \), \( \theta \) and \( \phi \) which have been reduced to a system of nine simultaneous equations of first-order for nine unknowns. In order to solve this system of equations numerically we require nine initial conditions but two initial conditions on \( f \) and one initial condition each on \( g \), \( \theta \) and \( \phi \) are known. However the values of \( f', g, \theta \) and \( \phi \) are known at \( \eta \to \infty \). These four end conditions are utilized to produce four unknown initial conditions at \( \eta = 0 \) by using shooting technique. The most crucial factor of this scheme is to choose the appropriate finite value of \( \eta_c \). In order to estimate the value of \( \eta_c \), we start with some initial guess value and solve the boundary value problem consisting of Eqs. (13)-(17) to obtain \( f'(0), g'(0), \theta'(0) \) and \( \phi'(0) \). The solution process is repeated with another large value of \( \eta_c \) until two successive values of \( f'(0), g'(0), \theta'(0) \) and \( \phi'(0) \) differ only after desired significant digit. The last value of \( \eta_c \) is taken as the final value of \( \eta_c \) for a particular set of physical parameters for determining velocity components \( f(\eta), g(\eta) \), temperature \( \theta(\eta) \) and concentration \( \phi(\eta) \) in the boundary layer. After knowing all the nine initial conditions, we solve this system of simultaneous equations using fifth-order Runge-Kutta-Fehlberg integration scheme with automatic grid generation scheme which ensures convergence at a faster rate. The value of \( \eta_c \) greatly depends also on the set of the physical parameters such as Magnetic parameter, Hall parameter, Prandtl number, thermal radiation parameter, Lewis number, radiation parameter and chemical reaction parameter, convective heat transfer constant so that no numerical oscillations would occur. During the computation, the shooting error was controlled by keeping it to be less than \( 10^{-6} \).
4. SKIN FRICTION, NUSSELT NUMBER AND SHERWOOD NUMBER

The local skin-friction coefficient $C_{fz}$, the local Nusselt number $Nu$ and the local Sherwood number $Sh$ defined by

$$C_{fz} = \frac{\tau_w}{\mu bx \sqrt{\frac{b}{v}}} = f''(0), C_{fz} = g'(0)$$

(20)

where $\tau_w = \mu \left(\frac{\partial u}{\partial y}\right) = \mu bx \sqrt{\frac{b}{v}} f''(0)$. $Nu = \frac{q_w}{k_f \sqrt{\frac{b}{v}} (T_w - T_\infty)}$

(21)

where $q_w = -k_f \left(\frac{\partial T}{\partial y}\right) = -k_f \sqrt{\frac{b}{v}} (T_w - T_\infty) \theta'(0)$ and

$$Sh = \frac{m_w}{D_m \sqrt{\frac{b}{v}} (C_w - C_\infty)} = -\phi'(0)$$

(22)

where $m_w = -D_m \left(\frac{\partial C}{\partial y}\right) = -D_m \sqrt{\frac{b}{v}} (C_w - C_\infty) \phi'(0)$

5. COMPARISON:

The results of this paper are compared with the results of previous published paper of Shit and Haldar [19] as shown in Table 1 and the outcomes are in good concurrence.

Table 1. Comparison of Nu and Sh at $\eta=0$ with Shit and Haldar [19] with $h_1=0, Nb=Nt=0$

<table>
<thead>
<tr>
<th>M</th>
<th>Rd</th>
<th>$\gamma$</th>
<th>$\theta r$</th>
<th>Shit and Haldar [19] Results</th>
<th>Present Results</th>
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<td></td>
<td></td>
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<td>Sh(0)</td>
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<tr>
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<td>-6</td>
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6. RESULTS AND DISCUSSION

The system of coupled non-linear Eqs. (13)-(17) together with the boundary conditions (15) and (16) have been solved numerically. In our present study the numerical values of the physical parameters have been chosen so that G, M, m, Rd, $\gamma$, nb,Nt,h1 and $\theta r$ are varied over a range which is listed in the following figures. (cf. Watanabe and Pop [21], Salem [18], Shit and Haldar [19], Ozotop and Abu-Nada[15]).

Figs. 2a-8a represent the axial velocity $f'$ and cross flow velocity $g(\eta)$ which is induced due to the presence of the Hall effects. All these figures show that for any particular values of the physical parameters $g$ reaches a maximum value at a certain high above the sheet and beyond which $g(\eta)$ decreases gradually in asymptotic nature for different velocities of G, M, m, N, Rd, $\gamma$, Nb, Nt, $\theta r$ and Pr.
Figs.2(a & b) represent \( f'(\eta) \) and \( g(\eta) \) with Grashof Number G. It is found that the axial and cross flow velocity components enhance with increase in thermal buoyancy force while the cross flow velocity(g) enhance in the flow region. An increase in G reduces the temperature and concentration in the boundary layer (Figs.2(c & d)), respectively. The variation of axial velocity \( f'(\eta) \) and \( g(\eta) \) with magnetic parameter \( M \) shows that both velocities reduce with increase in magnetic parameter \( M \). The variation of temperature \( \Theta \) and concentration \( C \) with \( M \) shows that higher the Lorenz force, larger the temperature and concentration in the boundary layer. This is due to the fact that the thickness of the thermal and molecular boundary layers grows with \( M \) (Figs.2(c & d)) respectively.

Figs.3(a & b) shows that the axial and cross flow velocities increase with increase in Hall parameter. This is due to the fact that as \( m \) increases the Lorentz force which opposes the flow and leads to the degeneration of the fluid motion. The anomalous behavior of \( \Theta \) with variation of \( m \) is observed due to the presence of the Hall Current and there by induces a cross flow velocity component \( g(\eta) \). For an increase in the Hall parameter \( m \) we noticed a reduction in cross flow velocity. From Figs.3(c & d), we find that an increase in the Hall parameter \( m \), results in a depreciation in the temperature and concentration. When the molecular buoyancy force dominates over the thermal buoyancy force the velocities enhance while temperature and concentration decay with increasing values of buoyancy ratio\( (N) \)(figs.3a-3d).

Figs.4a-d show the variation of \( f',g,\Theta \) and \( C \) with Brownian motion parameter\( (Nb) \) and thermophoresis parameter\( (Nt) \).The velocity components, temperature and concentration experience an enhancement with rising values of \( Nb \) while an increase in thermophoresis parameter\( (Nt) \)decays the velocities, concentration and enhances the temperature in the flow region(figs.4a-d).

Figs.5a-5d represent the effect of chemical reaction\( (\gamma) \)on the flow variables. The velocity components, temperature and concentration decay with degenerating chemical reaction case while in the generating case,all the flow variables accelerate in the flow region(figs.5a-d).

Figs.6(a -d) represent the variation of velocity components, temperature and concentration with radiation parameter \( (Rd) \) and viscosity parameter\( (\theta r) \).It can be seel from the profiles higher the radiative heat flux larger the axial and cross flow velocities. An increase in viscosity parameter\( (\theta r) \) increases the axial velocity and reduces the cross-flow velocities. The temperature and concentration decelerate with higher values of \( Rd \) and \( \theta r \) in the entire flow region(figs.6a-6d). This may be attributed to the fact an increase in \( Rd /\theta r \), leads to a decay in the thickness of the thermal and solutal boundary layers.

The influence of newtonian cooling \( (h1) \) on flow variables can be seen from figs.7a-7d. From the\ profiles we notice an acceleration in the velocities, temperature and reduction in the concentration in the flow region. This may be attributed to the fact that the thickness of
the momentum, thermal boundary layers grows while the solutal layer decays with rising values of convective heat transfer constant($h_1$)(figs.7a-d).

Figs.8(a -d) depict the variation of flow variables $f'$, $g$, and C with Prandtl number(Pr) and Lewis number(Le). It is found that the axial and cross flow velocities, temperature enhance and concentration decelerates with increase in Pr, which is due to the fact that smaller values of Pr are equivalent to larger values of thermal conductivities and therefore heat is able to diffuse towards the stretching sheet. Thus lesser the thermal conductivity larger the axial and cross flow velocities, temperature and smaller the concentration with increase in the Prandtl number Pr. An increase in Lewis number(Le) reduces the velocities, temperature and concentration in the entire flow region(figs.8a-8d).

Fig.2.Variation of [a]axial velocity($f'$), [b]Secondary velocity($g$), [c]Temperature($\theta$), [d] Nano-Concentration($\phi$) with G and M

$m = 0.5, N=0.5, Rd=0.5, \theta_r = -2, \gamma=0.5, Nt=0.1, h_1=0.1, Pr=0.71, Le=2$
Fig. 3. Variation of [a] axial velocity ($f'$), [b] secondary velocity ($g$), [c] temperature ($\theta$), [d] nano-concentration ($\phi$) with $m$ and $N$.

$G=2$, $M=0.5$, $Rd=0.5$, $\theta_r = -2$, $\gamma=0.5$, $Nb=0.1$, $Nt=0.1$, $h_1=0.1$, $Pr=0.71$, $Le=2$

Fig. 4. Variation of [a] axial velocity ($f'$), [b] secondary velocity ($g$), [c] temperature ($\theta$), [d] nano-concentration ($\phi$) with $Nb$ & $Nt$

$G=2$, $M=0.5$, $m = 0.5$, $N=0.5$, $Rd=0.5$, $\theta_r = -2$, $\gamma=0.5$, $h_1=0.1$, $Pr=0.71$, $Le=2$
Fig. 5. Variation of [a] axial velocity \( f' \), [b] secondary velocity \( g \), [c] temperature \( \theta \), [d] nano-concentration \( \psi \) with \( \gamma \)

\( G=2, \, M=0.5, \, m=0.5, \, N=0.5, \, Rd=0.5, \, \theta_r=-2, \, Nb=0.1, \, Nt=0.1, \, h_1=0.1, \, Pr=0.71, \, Le=2 \)

Fig. 6. Variation of [a] axial velocity \( f' \), [b] secondary velocity \( g \), [c] temperature \( \theta \), [d] nano-concentration \( \psi \) with \( \theta_r \) and \( Rd \)

\( G=2, \, M=0.5, \, m=0.5, \, N=0.5, \, \gamma=0.5, \, Nb=0.1, \, Nt=0.1, \, h_1=0.1, \, Pr=0.71, \, Le=2 \)
Fig. 7. Variation of [a] axial velocity ($f'$), [b] secondary velocity ($g$), [c] temperature ($\theta$), [d] nano-concentration ($\phi$) with $h_1$

$G=2$, $M=0.5$, $m=0.5$, $Rd=0.5$, $\theta_r=-2$, $\gamma=0.5$, $Nb=0.1$, $Nt=0.1$, $Pr=0.71$, $Le=2$

Fig. 8. Variation of [a] axial velocity ($f'$), [b] secondary velocity ($g$), [c] temperature ($\theta$), [d] nano-concentration ($\phi$) with $Pr$ & $Le$

$G=2$, $M=0.5$, $m=0.5$, $Rd=0.5$, $\theta_r=-2$, $\gamma=0.5$, $Nb=0.1$, $Nt=0.1$, $h_1=0.1$
Table - 2
Skin Friction (Cf,ξ), Nusselt number (Nu) and Sherwood number (Sh) at η = 0

<table>
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<tr>
<th>Parameter</th>
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<th>Cξ(1)</th>
<th>Nu(0)</th>
<th>Sh(0)</th>
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The variation of Skin friction coefficients(Cfₓ,ξ,y), Nusselt number(Nu) and Sherwood numbers(Sh) on the wall(η=0) with different parameters is exhibited in table.2. It is found that an increase in Grashof number (G) reduces the Skin friction components(Cfₓ), Cfz and enhances the rate of heat and mass transfer at the wall η=0. Higher the Lorentz force larger Cfₓ,Cfz and smaller the Nusselt and Sherwood number on the wall. An increase the Hall parameter (m) enhances Cfx and Cfz at the wall. The Nusselt number reduces and Sherwood number enhances with rising values of m. When the molecular buoyancy force dominates over the thermal force smaller Cfx,larger Cfz,Nu and Sh on the wall. With reference to the radiation parameter(Rd) we find that an increase in Rd reduces Cfx and enhances Cfz,Nu and Sh at the wall η=0. An increase in variation parameter(θr) grows Cfx and Sh,while decays Cfx,ν on η=0. Increasing Brownian motion parameter(Nb) leads to a reduction in Cfx,ν,Sh and growth in Cfx on η=0.while τₓ,Nu ,Sh grows ,Cfz decays with rising values of thermophoresis parameter(Nt)on the wall. Cfx, Cfz,Nu and Sh experience an enhancement win chemical reaction parameter in the degenerating case on the wall.Higher the values of convective heat transfer parameter(h) smaller Cfx,Cfz,Nu and larger Sh on η=0. Thus the presence of the Newtonian cooling leads to a growth in skin friction coefficients ,Nusselt number and decay in Sherwood number.Lesser the thermal diffusivity(Pr) smaller the skin friction component Cfx,Nu ,Sh and larger Cfx on η=0. An opposite effect is noticed in Cfx, Cfz ,Nu ,Sh with rising values of Lewis number(Le).

7. CONCLUSIONS:

This analysis aims at investigating the effect of variable viscosity on convective heat transfer flow of nanofluid past a stretching surface which is maintained at Newtonian cooling. From the profiles we find that higher the Hall parameter(m), radiation parameter(Rd), variation parameter(θr) larger the velocities, smaller the temperature and concentration. The
velocities, temperature grow, concentration decays with rise in convective heat transfer constant(h1) The velocities rise with Brownian motion parameter (Nb) an depreciate with thermophoresis (Nt). Nusselt number increase in θr and reduces Nu and Sh, Increase in θr/h1 reduce the rate of heat transfer and enhances mass transfer on the wall.

8. REFERENCES


